3. (a) Show that radius R of the spherical curvature is given by :

oMeeFS ekeâ ieeueeÙe Je›eâlee e$epÙee R evecve me oe peelee n:

$$R^2 = (P^4\, \underline{r}'''^2 - 1)\, \sigma^2 .$$

(b) Define involute of a curve and find its equation.

Skeâ Je›eâ keâ 'FvJeeuÙeš' keâe heejYeeef<ele keâ peS Deej Gmekeâe meceekeâjCe %eele keâeepeS~

Unit-II / FkeâeF-II 6/11

4. (a) Find the Gaussian Curvature at a point on the surface :

evecve Je›eâ keâ Skeâ eyevo hej ieeemeÙeve Je›eâlee %eele keâeepeS:

$x = a(u+v),\ y = b(u-v),\ z = uv$

(b) Show that family of curves $du^2 - (u^2+c^2)\, dv^2 = 0$ form an orthogonal system on the surface :

oMeeFS ekeâ Je›eâ heejJeej $du^2 - (u^2+c^2)\, dv^2 = 0$ evecve he‰ hej DeeYeueeeyekeâ evekeâeÙe yeveele n :

$x = u \cos v,\ y = u \sin v,\ z = cv.$

A (Printed Pages 7)

Roll No. ____________

S-680

B.A./B.Sc. (Part-III) Examination, 2015

MATHEMATICS-IV-C

Fourth Paper

(*Differential Geometry and Tensor Analysis*)

Time Allowed : Three Hours] ***[Maximum Marks :*** B.A. : 40, B.Sc. : 75

Note : Attempt five questions in all, selecting one question from each Unit. Question No. 1 is compulsory. Symbols have their usual meanings.

ØelÙekeâ FkeâeF me Skeâ ØeMve Ûevele nS, keâue heeÛe ØeMvee keâe nue keâeepeS~ ØeMve me.1 DeeveJeeÙe n~ Øeleekeâe keâ meeceevÙe DeLe n~

1. Attempt all parts : 16/30

meYee KeC[nue keâeepeS :

(a) For a curve, show that :

Skeâ Je›eâ keâ eueS, oMeeFS ekeâ :

$$\underline{r}''.\ \underline{r}''' = KK'$$

(b) If w is the angle between parametric curves on a surface, show that :

Ùeeo Skeâ he‰ hej ØeeÛeue Je›eâe keâ ceOÙe keâe keâeCe w ne lee oMeeFS ekeâ :

$$\tan w = \frac{\sqrt{EG - F^2}}{F}$$

(c) Explain why every helix on a cylinder is geodesic?

Skeâ yeueve hej ØelÙekeâ keâ[euevee DeJeceeleje keÙee nelee n? mecePeeFÙe~

(d) Show that :

oMeeFS ekeâ :

$$\&s^3\ Kg = [\ \hat{N},\ \underline{\&r},\ \underline{\&\&r}\ ]$$

(e) Find the parametric curves on a surface:

evecve he‰ hej ØeeÛeue Je›eâe keâe %eele keâeepeS :

$$x = u \cos v,\ y = u \sin v,\ z = cv.$$

(f) Prove that outer product of two vectors is a second order tensor.

emeæ keâeepeS ekeâ meeoMee keâe JeenÙe ieCeve Skeâ eÉleeÙe keâeeš keâe ØeeoMe nelee n~

(g) Show that $g_{ij}\, dx^i\, dx^j$ is an invariant.

oMeeFS ekeâ $g_{ij}\, dx^i\, dx^j$ Skeâ eveMÛej n~

(h) Show that in a Riemannian space metric tensor is covariantly constant.

oMeeFS ekeâ Skeâ jeceeve mecee° ce oj ekeâ ØeeoMe menÛejeÙe DeÛej nelee n~

(i) Show that :

oMeeFS ekeâ :

$$g_{ij} \begin{Bmatrix} i \\ hk \end{Bmatrix} = [hk, j]$$

(j) If $T^i = g^{ih} T_r$ show that $Ti = g_{ih} T^h$.

Ùeeo $T^i = g^{ih} T_r$ oMeeFS ekeâ $Ti = g_{ih} T^h$.

Unit-I / FkeâeF-I 6/11

2. (a) State and prove necessary and sufficient condition for a curve to be a Helix.

Skeâ Je›eâ keâ keâ[euevee neve keâ eueS DeeJeMÙekeâ SJe heÙeehle Øeeleyebe keâe meeuueKe emeæ keâeepeS~

(b) Prove that :

emeæ keâeepeS :

$$[\ \underline{b}',\ \underline{b}'',\ \underline{b}'''\ ] = \tau^5 \frac{d}{ds}\left(\frac{k}{\tau}\right)$$

5. (a) State and prove necessary and sufficient condition for parametric curves to be lines of Curvature on a surface.

Skeâ he‰ hej ØeeÛeue Je›eâe keâ Je›eâlee jKeeS neve keâ eueS DeeJeMÙekeâ SJe heÙeehle ØeeleyeOee keâe meeuueKe emeæ keâeepeS~

(b) Show that curves $u+v=const.$ are geodesics on a surface with the metric :

oMeeFS ekeâ Je›eâ heejJeej $u+v=const$ Skeâ he‰ epemekeâe ojekeâ evecve n, hej DeJeceelej e n :

$$(1+u^2)\,du^2-2uv\,du\,dv+(1+v^2)\,dv^2.$$

Unit-III / FkeâeF-III 6/11

6. (a) Derive the Weingarten equations in terms of E, F, G, L, M and N.

E, F, G, L, M Deej N keâ heoe ce yeeieešve meceekeâjCee keâe JÙelheVe keâeepeS~

(b) Show that inner product of tensors A^i_j and B^{hk}_p is a tensor of order three.

oMeeFS ekeâ ØeeoMee A^i_j Deej B^{hk}_p keâe Deelej ieCeve Skeâ leleeÙe keâeeš keâe ØeeoMe n~

7. (a) If for all covariant tensor S_{ij}, $T^{ij} S_{ij}$ is an invariant, show that T^{ij} is a second order contravariant tensor.

Ùeeo ØelÙekeâ menÛej ØeeoMe S_{ij} keâ eueS $T^{ij} S_{ij}$ Skeâ eveMÛej n, oMeeFS ekeâ T^{ij} Skeâ eÉleeÙe keâeeš keâe ØeeleÛej ØeeoMe n~

(b) Show that covariant derivative of a tensor of type (1,1) is a tensor of type (1,2).

oMeeFS ekeâ Skeâ (1,1) Øekeâej keâ ØeeoMe keâe menÛejeÙe DeJekeâuepe Skeâ (1,2) Øekeâej keâe ØeeoMe nelee n~

Unit-IV / FkeâeF-IV 6/12

8. (a) Show that the operations of contraction and covariant differentiation on a tensor commute.

oMeeFS ekeâ Skeâ ØeeoMe hej mekeâÛeve SJe menÛejeÙe DeJekeâueve keâe mee›eâÙeeS ›eâceeJeeveecele nelee n~

(b) Show that $R_{jk} = R^h_{jkh}$ is a symmetric covariant tensor.

oMeeFS ekeâ $R_{jk} = R^h_{jkh}$ Skeâ meceeecele menÛej ØeeoMe n~

9. (a) Derive transformation formula for connexion coefficients and show that they are not tensors.

™heeleej Ce me$e keâe JÙelheVe keâjle nS oMeeFS ekeâ keâvekeMeveme ieCeekeâ ØeeoMe vene n~

(b) Show that every 2-dimensional Riemannian space is an Einstein's Space.

oMeeFS ekeâ ØelÙekeâ eÉeJeceeÙe mecee° Skeâ Dee Ševe mecee° nelee n~