(d) Define a convex set and extreme point of a convex set.

Skeâ Gòeue meceÛÛeÙe Deej Gòeue meceÛÛeÙe keâ Ûejce eyevo keâe heejYee<ee oeepeS~

(e) If $AX=b$ has a solution having exactly m non-zero variables and if this solution is unique, then prove that it must be a basic solution. Here A is an $m \times n$ matrx.

Ùeeo $AX=b$ keâe Skeâ nue Smee n epemece ÙeLeeleLÙe m Ûejekeâ MevÙe vene n Deej Ùen nue DeeÉleeÙe n emeæ keâeepeS ekeâ Ùen Skeâ ceeeuekeâ nue n~ Ùene A Skeâ $m \times n$ DeeJÙen n~

(f) Prove that the closed half-space

$S=\{x| \ cx \geq z\}$ is a convex set.

emeæ keâeepeS ekeâ meJele Deæ mecee°

$S=\{x| \ cx \geq z\}$ Skeâ Gòeue meceÛÛeÙe n~

A (Printed Pages 15)

Roll No. ____________

S-682

B.A. / B.Sc. (Part-III) Examination, 2015

MATHEMATICS-IV

Fourth Paper

(Linear Programming)

Time Allowed : Three Hours] ***[Maximum Marks :*** B.A. : 40, B.Sc. : 75

Note : Attempt five questions in all, selecting one question from each unit. Question No. 1 is compulsory. Symbols have their usual meanings.

ØeIÙekeâ FkeâeF me Skeâ ØeMve Ûevele nS, keâue heeÛe ØeMvee keâe nue keâeepeS~ ØeMve me. 1 DeeveJeeÙe n~ Øeleekeâe keâ meeceevÙe DeLe n~

1. Attempt all parts : 16/30

(a) A goldsmith manufactures necklaces and rings. The total number of necklaces and rings that he can make per day is at most 48. It takes one hour to make a necklace and half an hour to make a ring. He can work for is hours a day. The profit on a necklace is Rs.400 and on a ring is Rs. 100. Formulate this as a linear programming problem to maximize the profit.

Skeâ meveej nej Deej Deiee"Ùee yeveelee n~ Skeâ eove ce Jen DeeOekeâlece 48 nej Deej Deiee"Ùee yevee mekeâlee n~ Skeâ nej yeveeve ce Skeâ leŠe Deej Skeâ Deie"e yeveeve ce DeeOee leŠe ueielee n~ Skeâ eove ce Jen 15 leŠ keâece keâj mekeâlee n~ Skeâ nej hej ®. 400 Deej Skeâ Deie"e hej ®. 100 keâe ueeYe nelee n~ Fme Skeâ jeKekeâ Øee«eeeceie mecemÙee keâ ¤he ce eve¤ehele keâj epememe ekeâ ueeYe keâe DeeOekeâeeOekeâ ekeâÙee pee mekeâ~

(b) Show that the feasible solution $x_1=1$, $x_2=0, x_3=1$ to the system of equations $x_1+x_2+x_3=2$, $x_1-x_2+x_3=2$, $x_1, x_2, x_3 \geq 0$ is not basic.

emeæ keâeepeS ekeâ meceekeâjCee keâ evekeâeÙe

$x_1+x_2+x_3=2$, $x_1-x_2+x_3=2$, $x_1, x_2, x_3 \geq 0$

keâe meYeJe nue $x_1=1$, $x_2=0, x_3=1$ ceeeueekeâ vene n~

(c) Convert the following LPP into standard form.

evecveeueeKele LPP keâe ceevekeâ ¤he ce yeoeueS :

Min $z=12x_1+5x_2$ subject to

(Ùeeo) $5x_1+3x_2 \geq 15$

$7x_1-2x_2 \leq 14$

$x_1 \geq 0$, x_2 unrestricted.

x_2 ØeelejeevOele vene n~

(b) Find all the basic solutions for the following system :

evecve eveke‚eÙe ke‚ meYee ceeeueke‚ nue %eele ke‚eeepeS :

$$x_1 + 2x_2 + x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5$$

3. (a) Determine two different basic feasible solutions of the LPP :

evecve LPP ke‚ oe eYeVe cee

ke‚eeepeS :

$$2x_1 + 3x_2 + 4x_3 + x_4 = 6$$

$$x_1 + 2x_2 + 2x_3 \quad -x_5 + x_6 = 3$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

(g) The optimal simplex table for a LPP is given below. Find the range of variation in c_1 which is consistent with the optimal solution.

Ske‚ LPP ke‚e F° lece emecheuekeme meejCee veeÛe oe ieF n~

c_1 ke‚ Øeke‚e jevlej ke‚e hejeme %eele ke‚eepeS pee eke‚ F° lece nue ke‚ memeiele ne~

		C_j	5	3	0	0
C_B	B	XB	a_1	a_2	a_3	a_4
3	$b_1 = a_2$	$\frac{45}{19}$	0	1	5/19	–3/19
5	$b_2 = a_1$	$\frac{20}{19}$	1	0	–2/19	5/19
	$z_j = c_j$	$z = \frac{235}{19}$	0	0	$\frac{5}{19}$	$\frac{16}{19}$

(h) Find the dual of the LPP :

evecve LPP keâe Éle %eele keâeepeS :

Min $z = 2x_1 + 2x_2 + 4x_3$ subject to

(Ùeeo) $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_1, x_3 \geq 0$

(i) State and prove the strong duality theorem.

meyeue Élelee ØeceÙe keâe keâLeve keâeepeS Je Gme emeæ keâeepeS~

(j) Find an initial basic feasible solution to the following transportation problem by North-West Corner rule :

evecve heejJenve mecemÙee keâe Skeâ ØeejecYekeâ ceeeuekeâ meYeJe nue Gòej-heeMÛece Úej eveÙece Éeje %eele keâeepeS :

	D_1	D_2	D_3	D_4	a_i
S_1	13	11	15	20	2
S_2	17	14	12	13	6
S_3	18	18	15	12	7
b_j	3	3	4	5	15

Unit-I 6/11

FkeâeF- I

2. (a) Solve graphically :

DeeueKee eJeeOe me nue keâeepeS :

Max $z = 3x_1 + 4x_2$ subject to

(Ùeeo) $5x_1 + 4x_2 \leq 200$

$3x_1 + 5x_2 \leq 150$

$5x_1 + 4x_2 \geq 100$

$8x_1 + 4x_2 \geq 80$

$x_1, x_2 \geq 0$

(b) Solve the following LPP by revised simplex method :

evecveeueeKele LPP keâe meMeeeOele emecheuekeme eJeeOe Éeje nue keâeepeS :

Max $z = x_1 + 2x_2$ subject to

(Ùeeo) $x_1 + x_2 \le 3$

$x_1 + 2x_2 \le 5$

$3x_1 + x_2 \le 6$

$x_1, x_2 \ge 0$

7. (a) Solve the following LPP by revised simplex method :

evecveeueeKele LPP keâe meMeeeOele emecheuekeme eJeeOe Éeje nue keâeepeS :

Min $z = 5x_2$ subject to

(Ùeeo) $x_1 + x_2 \le 2$

$x_1 + 5x_2 \ge 10$

$x_1, x_2 \ge 0$

(b) Reduce the following LPP to standard form and also give its matrix form :

evecveeueeKele LPP keâe ceevekeâ ¤he ce eve¤ehele keâeepeS Deej Fmekeâe DeeJÙen ¤he Yee oeepeS :

Min $Z = 2x_1 + x_2 + 4x_3$ subject to

(Ùeeo) $-2x_1 + 4x_2 \le 4$

$x_1 + 2x_2 + x_3 \ge 5$

$2x_1 + 3x_2 \le 2$

$x_1, x_2 \ge 0$, x_3 unrestricted

x_3 Øeeleyeevelele vene n~

Unit-II 6/11

FkeâeF-II

4. (a) Prove that an extreme point of $S = \{x \mid Ax = b, x \ge 0\}$ is a basic feasible solution of $Ax = b, x \ge 0$.

emeæ keâeepeS ekeâ $S = \{x \mid Ax = b, x \ge 0\}$ keâe Skeâ Ûejce eyevo $Ax = b, x \ge 0$ keâe Skeâ ceeeueekeâ meYeJe nue nelee n~

(b) Solve the following LPP by Simplex method :

evecveeueeKele LPP keâe emecheuekeme eJeeOe Éeje nue keâeepeS :

Max $z = 2x_1 + x_2$ subject to

(Ùeeo) $x_1 - x_2 \leq 10$

$2x_1 - x_2 \leq 40$

$x_1, x_2 \geq 0$

5. (a) Solve the following LPP by Big-M method :

evecve LPP keâe oele-M eJeeOe Éeje nue keâeepeS :

Max $z = x_1 + 6x_2$ subject to

(Ùeeo) $x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_1, x_2 \geq 0$

(b) Solve the following LPP by Two-Phase Method :

evecve LPP keâe eÉÛejCe eJeeOe Éeje nue keâeepeS-

Max $Z = 3x_1 - x_2$ subject to

(Ùeeo) $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 2$

$x_2 \leq 4$

$x_1, x_2 \geq 0$

Unit-III 6/11

FkeâeF-III

6. (a) Solve the LPP :

evecve LPP keâe nue keâeepeS :

Max $z = 3x_1 + 9x_2$ subject to

(Ùeeo) $x_1 + 4x_2 \leq 8$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

(b) Solve the following LPP and discuss the effect of changing b_1 to 30 and b_2 to 20 :

evecveeueeKele LPP keâe nue keâeepeS Deej b_1 keâe 30 Je b_2 keâe 20 ce yeoueve keâ Demej keâe eJeJeÛevee keâeepeS :

Max $z = 6x_1 + 8x_2$ subject to

(Ùeeo) $5x_1 + 10x_2 \leq 60 = b_1$

$4x_1 + 4x_2 \leq 40 = b_2$

$x_1, x_2 \geq 0$

Unit-IV 6/12

FkeâeF-IV

8\. (a) Solve the following LPP by using the principle of duality :

evecve LPP keâe Élelee keâ emeæevle keâe GheÙeeie keâjle nS nue keâeepeS :

Min $z = 3x_1 + x_2$ subject to

(Ùeeo) $2x_1 + 3x_2 \geq 2$

$x_1 + x_2 \geq 1$

$x_1, x_2 \geq 0$

(b) Solve the following transportation problem :

evecveeueeKeÀe heejJenve mecemÙee keâe nue keâeepeS :

	D_1	D_2	D_3	D_4	a_i
S_1	1	2	1	4	30
S_2	3	3	2	1	50
S_3	4	2	5	9	20
b_j	20	40	30	10	100

9. (a) Solve the following assignment problem :

evecveeueeKeÀe eveÙeÀeve mecemÙee keâe nue keâeepeS :

Job → / ↓Agencies	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

(b) Solve the following integer programming problem by using Gomory cut :

ieeceje keâeš keâe GheÙeeie keâjle nS evecveeueeKeÀe heCeekeâ Øee«eeeceie mecemÙee keâe nue keâeepeS :

Max $z = x_1 + 2x_2$ subject to

(Ùeeo) $x_1 + x_2 \le 7$

$2x_1 \le 11$

$2x_2 \le 7$

$x_1, x_2 \ge 0$ and integer.

$x_1, x_2 \ge 0$ Deej heCeekeâ n~