heejYeee<eIe keâeepeÙe~ Fme mJe®he keâe keâme nue keâjIe n?

Unit-I / FkeâeF-I 4/7½

2. (a) Reduce the matrix A to the normal form & hence find the rank of the matrix where

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$$

evecveeueeKeIe DeeJÙen keâe veeceue ®he ce heejJeeIeIe keâjkeâ Gmekeâe keâeeš %eeIe keâeepeÙe pene

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$$

(b) Check the consistency of the following system of equations & if consistent find the solution :

evecveeueeKeIe meceeekeâjCe evekeâeÙe keâ DeeJejeOee neve keâe peeÛe keâeepeÙe~ Ùeeo Ùe DeeJejeOee n Iee F eâe nue evekeâeeueÙe :

$2x-y+3z-9=0;\ x+y+z-6=0;$

$x-y+z-2=0;\ x+y-z=0$

3. (a) Prove that every square matrix with complex entries can be uniquely written as A+iB, where A & B are Hermitian matrices.

A (Printed Pages 8)

S-671

B.A./B.Sc. (Part-I) Examination, 2015

(Regular)

MATHEMATICS

Third Paper

(Matrices, Vectors & Differential Equations)

Time Allowed : Three Hours] [Maximum Marks : B.A. : 25, B.Sc. : 50

Note : Attempt five questions in all, choosing one question from each unit and Question No. 1 is compulsory.

ØeIÙekeâ FkeâeF me Skeâ ØeMve Ûeveve nS, keâue heeÛe ØeMvee keâe nue keâeepeS IeLee ØeMve me. 1 DeeveJeeÙe n~

1. Attempt all parts : 10/20

meYee Yeeie nue keâeepeÙe :

(a) Prove that $B = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is Unitary.

emeæ keâeepeÙe ekeâ $B = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ SkeâIceekeâ DeeJÙen n~

(b) Show that the rank of a matrix and its transpose matrix are equal.

oMeeFÙe ekeâ ekeâmee DeeJÙen Deej Gmekeâ heejJele DeeJÙen keâe keâeeš meceeve neiee~

(c) Prove that characteric roots of a skew Hermitian matrix are either zero or purely imaginary number.

emeæ keâeepeÙe ekeâ Skeâ Je<e-neceeMeÙeve Dee en keâ DeeYeuee#eeCekeâ Ùee Iee MevÙe neie DeLeJee Meæ DeeOekeâeuhele meKÙee neiee~

(d) A vector $\overline{u}$ is always normal to a given closed surface S. Show that :

$\iiint_{Vs} \text{curl}\ \overline{u}\ dV = \overline{O}$, where V is the region bounded by S.

meeoMe $\overline{u}$ nceMee oe ieF yevo melen hej DeeYeuecye n~

oMeeFÙe : $\iiint_{Vs} \text{curl}\ \overline{u}\ dV = \overline{O}$,

pene V, S Éeje Iej ieÙe #e$e keâe eoKeelee n~

(e) Show that : $u = x^2 - y^2 + 4z$; is a harmonic function.

oMeeFÙe ekeâ : $u = x^2 - y^2 + 4z$; Skeâ neceeevekeâ (njelcekeâ) Heâueve n~

(f) The acceleration of a particle at any time $t \geq 0$ is given by :

$\overline{a} = 12\cos 2t\ \hat{i} - 8\sin 2t\ \hat{j} + 16t\ \hat{k}$. If the velocity $\overline{v}$ is zero at $t = 0$, find $\overline{v}$ at any time.

ekeâmee meceÙe $t \geq 0$ hej Skeâ keâCe keâe IJejCe n :

$\overline{a} = 12\cos 2t\ \hat{i} - 8\sin 2t\ \hat{j} + 16t\ \hat{k}$, Ùeeo $t = 0$ hej ieele $\overline{v}$ MevÙe ne Iee ekeâmee meceÙe hej $\overline{v}$ helee ueieeFÙe~

(g) State two rules of finding the integrating factor of any non exact differential eq^n of first order & first degree.

ekeâmee 'Skeâ keâeeš Skeâ Ieele' DeJekeâueve meceekeâjCe pee Skepekeš ve ne, keâe meceekeâueveeÙe ieCeekeâ Øeehle keâjve keâ oe eveÙece yeleeFÙe~

(h) Solve nue keâeepeÙe :

$$\frac{dx}{dt} = \frac{t\,(2\log t + 1)}{\sin x + x\cos x}$$

(i) Find the complementary function of :

evecve keâe hejkeâ Heâueve %eele keâeepeÙe :

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = x^2$$

(j) Define Clairaut's form of a differential equation of first order. How will you solve this form?

Skeâ keâeeš keâe keuejepe ®heeÙe DeJekeâueve meceekeâjCe

(b) Solve nue keâeepeÙe :

emeæ keâeepeÙe ekeâ ØelÙekeâ meecceße DeJeÙeJe eue Jeie DeeJÙen keâe A+iB keâ ®he ce DeeÉleeÙe Øekeâej me eueKee pee mekeâlee n, pene A leLee B nceeeMeÙeve DeeJÙen n~

(b) Determine the eigen values & the eigen vectors of the smallest eigen value of the following matrix :

evecveeueeKele DeeJÙen keâ DeeYeuee#eeCekeâ ceeve leLee vÙevelece DeeYeuee#eeCekeâ ceeve keâ eueÙe DeeYeuee#eeCekeâ meeoMe %eele keâeepeÙe :

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

Unit-II / FkeâeF-II 4/7½

4. (a) Let $\overline{u}$ & $\overline{v}$ be two Vector point functions differentiable in a certain region, then show that :

Ùeeo $\overline{u}$ & $\overline{v}$ oe DeJekeâueveeÙe meeoMe eyevo Heâue ekeâmee #e$e ce n lee oMeeFÙe :

$$\text{curl}\,(\overline{u}\times\overline{v}) = (\overline{v}\,.\,\nabla)\,\overline{u} - \overline{v}\ \text{div}\ \overline{u} - (\overline{u}\,.\,\nabla)\,\overline{v} + \overline{u}\ \text{div}\ \overline{v}$$

(b) A particle moves along the curve : $x=t^3+1$, $y=t^2$, $z=2t+5$; where t is the time. Find the components of its velocity & acceleration at t = 1 in the direction $i+j+3k$.

Skeâ keâCe Je›eâ $x=t^3+1,\ y=t^2,\ z=2t+5;$ ce YeceCe keâjlee n, pene t meceÙe n~ meceÙe t = 1 hej Gmekeâ Jeie SJe lJejCe keâe leŠkeâ $i+j+3k$ keâe eoMee ce %eele keâeepeÙe~

5. (a) Verify divergence theorem for

$\overline{F} = (x^2 - yz)\ \hat{i} + (y - zx)\ \hat{j} + (z^2 - xy)\ \hat{k}$; taken over the rectangular parallelopiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$

$\overline{F} = (x^2 - yz)\ \hat{i} + (y - zx)\ \hat{j} + (z^2 - xy)\ \hat{k}$; keâ euueS [eFJepeme ØeceeÙe mLeeehele keâeepeÙe pee DeeÙelee keâej hejueueeheeFh[keâ Thej eueÙee ieÙee n~

(b) Show that : $\overline{F} = ze^x\ \hat{i} + 2yz\hat{j} + (e^x + y^2)\ \hat{k}$; is a conservative field and find the function Q s.t. $\overline{F} = \overline{\nabla} Q$

oMeeFÙe ekeâ : $\overline{F} = ze^x\ \hat{i} + 2yz\hat{j} + (e^x + y^2)\ \hat{k}$; Skeâ emLeele Ûeeueekeâ #e$e n~ Heâueve ϕ keâe ceeve %eele keâeepeÙe peyeekeâ $\overline{F} = \overline{\nabla} Q$ n~

Unit-III / FkeâeF-III 4/7½

6. (a) Solve nue keâeepeÙe :

$$\frac{dy}{dx} = \frac{4x + 6y + 5}{3y + 2x + 4}$$

(b) Solve nue keâeepeÙe :

$$(2x + y - 3)\,\frac{dy}{dx} - x - 2y + 3 = 0$$

7. (a) Solve nue keâeepeÙe :

$$\frac{dy}{dx} = e^{x-y}(e^x - e^y)$$

(b) Solve nue keâeepeÙe :

$$y^2 \log y = xpy + p^2$$

Unit-IV / FkeâeF-IV 3/7½

8. (a) Show that the system of confocal conics : $\frac{x^2}{a^2 + k} + \frac{y^2}{b^2 + k} = 1$, where k is a parameter is self orthogonal.

oMeeFÙe ekeâ meveeeYe MekeâJeekeâej evekeâeÙe :

$\frac{x^2}{a^2 + k} + \frac{y^2}{b^2 + k} = 1$, pene k keâeF ØeeÛeue n, mJe uecyekeâeCeeÙe n~

(b) Find the general solution and the singular solution of the differential equation :

$$y = 2xp - y\,p^2$$

DeJekeâue meceekeâjCe $y = 2xp - y\,p^2$ keâe meeceevÙe nue SJe eJeeÛe$e nue %eele keâeepeÙe~

9. (a) Solve nue keâeepeÙe :

$$\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = x^2 \cos x$$