Unit - I 4/7½

FkeâeF - I

2. (a) Show that $\frac{x}{1+x} < \log(1+x) < x$, for x > 0.

emeæ keâeepeS $\frac{x}{1+x} < \log(1+x) < x$, for x > 0.

(b) If f is continuous on [a,b] and $f'(x) \geq 0$ in]a,b[, then show that f is an increasing in [a,b].

Ùeeo f Skeâ [a,b] DevleJeue ce Skeâ melele Heâueve ne Deej $f'(x) \geq 0$;]a,b[ce, lee emeæ keâeepeS f Skeâ DeJejene n [a,b] ce~

3. (a) State and prove Rolle's theorem.

jeume ØeceÙe keâe keâLeve eueKele nS Fme emeæ keâeepeS~

(b) Show that Lagrange mean value theorem does not hold for function $f(x) = |x|$ in interval [–1,1].

emeæ keâeepeS ekeâ Heâueve $f(x) = |x|$, DevleJeue [–1,1] ce ue«eevpe ØeceÙe keâe heeueve vene keâjlee n~

A (Printed Pages 8)

Roll No. ____________

S-673

B.A./B.Sc. (Part-II) Examination, 2015

MATHEMATICS

First Paper

(Advanced Calculus)

Time Allowed : Three Hours] *[Maximum Marks :* B.A. : 25, B.Sc. : 50

Note : Attempt five questions only, choosing one question from each unit. Question No. 1 is compulsory.

ØelÙekeâ FkeâeF me Skeâ ØeMve Ûevele nS, keâJeue heeÛe ØeMvee keâe nue keâeepeS~ ØeMve meKÙee 1 DeeveJeeÙe n~

1. (a) Find the value of $\int_0^{\pi/2} \sin^4 x \cos^2 x \, dx$.

$\int_0^{\pi/2} \sin^4 x \cos^2 x \, dx$ keâe ceeve yeleeFS~ 10/20

(b) Show that $\overline{|(n)} = \int_0^1 \left(\log \frac{1}{y}\right)^{n-1} dy, \quad n > 0$.

eoKeeFS $\Gamma(n) = \int_0^1 \left(\log \frac{1}{y}\right)^{n-1} dy, \quad n > 0$

(c) Show that the function

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}$$

is not continuous at (0,0).

emeæ keâeepeS ekeâ Heâueve

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}$$

(0,0) hej melele vene n~

(d) Examine the convergence of $\int_0^\infty \frac{dx}{\sqrt{x}}$.

$\int_0^\infty \frac{dx}{\sqrt{x}}$ keâe DeeYemeeejlee yeleeFS~

(e) State Darboux theorem.

[jyeekeme keâe ØeceÙe keâe keâLeve eueeKeS~

(f) Evaluate $\int_0^a \int_0^b (x^2+y^2)\, dx\, dy$.

$\int_0^a \int_0^b (x^2+y^2)\, dx\, dy$ keâe ceeve yeleeFS~

(g) Find first three terms of the expansion of the function $e^x \log(1+y)$ in a Taylor Series in n.n.n. of (0,0).

Heâueve $e^x \log(1+y)$ keâe Šuej ßeCee Éeje ØeLece leeve heo, (0,0) keâ meceehe eueeKeS~

(h) If $u = e^x \sin y$, $v = x + \log \sin y$, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

Ùeeo $u = e^x \sin y$, $v = x + \log \sin y$, leye $\frac{\partial(u,v)}{\partial(x,y)}$ keâe ceeve yeleeFS~

(i) Find the envelope of the family of the curve $y = mx + am^3$, where m is a the parameter.

Je›eâ mecen $y = mx + am^3$ keâe DevJeeueehe %eele keâeepeS, pene m Skeâ ØeeÛeue n~

(j) Show that the function $f(x) = x^2$ is uniformly continuous on [–1,1].

emeæ keâeepeS ekeâ Heâueve $f(x) = x^2$ Devlejeue [–1,1] ce Skeâ meceeve melele n~

(b) Find the value of $\iiint \log(x+y+z)\, dxdydz$

where, x>0, y>0, z >0 and x+y+z <1.

$\iiint \log(x+y+z)\, dxdydz$,

pene x>0, y>0, z >0 Deej x+y+z <1.

Unit - II 4/7½

FkeâeF - II

4. (a) Show that function

$$f(x,y) = \begin{cases} \dfrac{x^3y^3}{x^2+y^2} & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}$$

is continuous at (0,0).

emeæ keâeepeS Heâueve $f(x,y) = \begin{cases} \dfrac{x^3y^3}{x^2+y^2} & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}$

(0,0) hej melele n~

(b) Find the envelope of the family of the curve $x^2\sin\alpha + y^2\cos\alpha = a^2$, where α is the parameter.

Je›eâ mecen $x^2\sin\alpha + y^2\cos\alpha = a^2$, pene α Skeâ heeÛeue n keâe DevJeeueehe %eele keâeepeS~

5. (a) Prove that $f(x,y) = x^2 - 2xy + y^2 + x^4 + y^4$ has a minima at the origin.

emeæ keâeepeS ekeâ $f(x,y) = x^2 - 2xy + y^2 + x^4 + y^4$ ceue eyevo hej evecveeve° n~

(b) If $u = \frac{x+y}{1-xy}$, v = $\tan^{-1}x + \tan^{-1}y$, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

Ùeeo $u = \frac{x+y}{1-xy}$, v = $\tan^{-1}x + \tan^{-1}y$, leye $\frac{\partial(u,v)}{\partial(x,y)}$ keâe ceeve evekeâeeueS~

Unit - III 4/7½

FkeâeF - III

6. (a) Prove that $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r^2 \cos\theta \, d\theta \, dr = \frac{5\pi a^3}{8}$.

emeæ keâeepeS ekeâ $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r^2 \cos\theta \, d\theta \, dr = \frac{5\pi a^3}{8}$

(b) Evaluate $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}} dxdy$.

ceeve %eele keâeepeÙe $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}} dxdy$

7. (a) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dzdydx$.

$\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dzdydx$ keâe ceeve evekeâeeueS~

(b) Show that $\beta(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1} dx$.

emeæ keâeepeS ekeâ $\beta(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1} dx$.

Unit - IV 3/7½

FkeâeF - IV

8. (a) Test convergence of the integral.

meceekeâue keâe DeeYemeeejlee keâe heje#ee keâeepeS~

$\int_0^{\infty} \frac{\cos m_x}{x^2 + a^2} dx$

(b) Show that the integral $\int_a^{\infty} \frac{dx}{x\sqrt{(1+x^2)}}$ converges, when a>0.

emeæ keâeepeS ekeâ meceekeâueve $\int_a^{\infty} \frac{dx}{x\sqrt{(1+x^2)}}$ DeeYemeeejkeâ n peye a>0.

9. (a) Test the convergence of the Gamma function $\int_0^{\infty} x^{n-1} e^{-x} \, dx$.

ieecee Heâueve $\int_0^{\infty} x^{n-1} e^{-x} \, dx$ keâ DeeYemeeejlee keâe heje#eCe keâejS~