9. (a) Trace the curve and find the area of the loop :

$$ay^2 = x^2 (a - x)$$

Je›eâ $ay^2 = x^2 (a - x)$ keâe DevejKeCe keâejS leLee Gmekeâ uehe keâe #e$eHeâue evekeâeeueS~

(b) Find the volume of the solid formed by the revolution of the Cardioid $r = a (1 + \cos \theta)$ about initial line.

keâee[DeeF[ $r = a (1 + \cos \theta)$ keâe ceue De#e keâ heejle: heej›eâceCe keâjve me peevele "eme keâe DeeÙeleve %eele keâeepeS~



A (Printed Pages 8)

Roll No. ____________

# S-670

## B.A./B.Sc. (Part-I) Examination, 2015

(Regular)

MATHEMATICS

Second Paper

(Calculus)

***Time Allowed : Three Hours]*** ***[Maximum Marks :*** B.A. : 25, B.Sc. : 50

Note : Attempt five questions in all, choosing one question from each unit. Question No. 1 is compulsory.

ØelÙekeâ FkeâeF me Skeâ ØeMve Ûevele nS, keâue heeÛe ØeMvee keâe nue keâeepeS~ ØeMve meKÙee 1 DeeveJeeÙe n~

1. Attempt all parts : 10/20

meYee Yeeie nue keâeepeS :

(a) Does the limit of f(x) at x = 1 exist?

keÙee f(x) keâe meecee x = 1 hej emLele n?

If Ùeeo

$$f(x) = \begin{cases} x & \text{for } 0 \le x < 1 \\ 3 - x & \text{for } 1 \le x \le 2 \end{cases}$$

(b) Examine the following function for continuity at $x=0$ and $x=1$

evecve Heâueve keâ meelelÙe keâe peeÛe $x=0$ Deej $x=1$ hej keâeepeS :

$$f(x) = \begin{cases} x^2 & \text{for } x \le 0 \\ 1 & \text{for } 0 < x \le 1 \\ \frac{1}{x} & \text{for } x > 1 \end{cases}$$

(c) Find the differential coefficient of the function $f(x)$ at $x = 1$ if $f(x)$ is defined as :

evecve Éeje heejYeeef<ele Heâueve $f(x)$ keâe $x = 1$ hej DeJekeâue ieCeekeâ evekeâeeueS :

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x \le 1 \\ x^3 - x + 1 & \text{if } x > 1 \end{cases}$$

(d) If $y = e^{ax}$, cos bx then prove that

Ùeeo $y = e^{ax}$, cos bx lee emeæ keâeepeS

$$y_2 - 2ay_1 + (a^2+b^2)y = 0$$

(e) Evaluate (ceeve evekeâeeueS) :

$$\lim_{x \to a} \frac{\log(x-a)}{\log(e^x - e^a)}$$





(b) Find the envelope of the family of circles $x^2+y^2-2ax \cos\alpha - 2ay \sin\alpha + c^2=0$ where $\alpha$ is parameter.

Jeòee keâ mecen $x^2+y^2-2ax \cos\alpha - 2ay \sin\alpha + c^2=0$ keâe DevJeeueehe %eele keâeepeS pene $\alpha$ ØeeÛeue n~

7. (a) Find the points of inflexion for the curve

$$y = 3x^4 - 4x^3 + 1$$

Je›eâ $y = 3x^4 - 4x^3 + 1$ keâ veele heejJeleve eyevoDee keâe %eele keâeepeS~

(b) Trace the curve Je›eâ keâe DevejKeCe keâejS :

$$x = (y-1)(y-2)(y-3)$$

Unit - IV 3/7½

FkeâeF-IV

8. (a) Prove that :

emeæ keâeepeS :

$$\int_0^{\pi/2} \cos^m x . \sin nx \, dx = \frac{1}{m+n} + \frac{m}{m+n} \int_0^{\pi/2} \cos^{m-1} x . \sin(n-1)x \, dx$$

(b) Evaluate :

ceeve evekeâeeueS :

$$\lim_{n \to \infty} \left[ \frac{n}{(n+1)\sqrt{2n+1}} + \frac{n}{(n+2)\sqrt{2.(2n+2)}} + \dots + \frac{n}{2n\sqrt{n.3n}} \right]$$

Unit - I 4/7½

FkeâeF - I

2. (a) Examine the continuity and differentiability of following function f(x) at x=2.

eyevo x=2 hej evecve Heâueve f(x) keâ meelelJe Deej DeJekeâuveeÙe neve keâe hej#eCe keâeepeS :

$$f(x) = \begin{cases} -x^2 & \text{if} \quad x \le 0 \\ 5x-4 & \text{if} \quad 0 < x \le 1 \\ 4x^2-3x & \text{if} \quad 1 < x < 2 \\ 3x+4 & \text{if} \quad x \ge 2 \end{cases}$$

(b) State Rolle's theorem and verify it for the function $f(x) = 2x^3+x^2-4x-2$ in interval $[-\sqrt{2}, \sqrt{2}]$.

jeue keâe ØeceÙe keâe keâLeve keâeepeS Deej Dev ejeue $[-\sqrt{2}, \sqrt{2}]$ ce Heâueve $f(x) = 2x^3+x^2-4x-2$ keâ eueS Ghejeòeâ ØeceÙe keâe melÙeehele keâeepeS~

3. (a) Find the $n^{th}$ differential coefficient of $\frac{1}{6x^2-5x+1}$.

$\frac{1}{6x^2-5x+1}$ keâe nJee DeJekeâue ieCeekeâ %eele keâeepeS~





(b) Evaluate (ceeve evekeâeeueS) :

$$\lim_{x \to 1} (1-x^2)^{\frac{1}{\log(1-x)}}$$

Unit - II 4/7½

FkeâeF - II

4. (a) If (Ùeeo) $u = 2(ax+by)^2 - (x^2+y^2)$ and (Deej) $a^2+b^2=1$

then prove that (lee emeæ keâeepeS)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

(b) Show that the equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

transforms to $\frac{d^2y}{dz^2} + y = 0$

on substituting $x = e^z$

oMeeFS ekeâ $x = e^z$ jKeve hej meceekeâjCe

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

meceekeâjCe $\frac{d^2y}{dz^2} + y = 0$ ce ¤heeleeejle ne peelee n~

5. (a) Show that pedal equation of the curve $r = a \sec h\theta$ is of the form $\frac{1}{p^2} = \frac{A}{r^2} + B$.

oMeeFS ekeâ Je›eâ $r = a \sec h\theta$ keâe heeeokeâ meceeekeâjCe

$\frac{1}{p^2} = \frac{A}{r^2} + B$ keâ ¤he keâe n~

(b) Find the asymptotes of the following equation :

evecveeueeKeIe meceekeâjCe keâ DevevIemheMee %eeIe keâeepeS:

$$2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$$

Unit - III 4/7½

FkeâeF - III

6. (a) Prove that the centre of curvature $(\alpha, \beta)$ for the curve $x = 3t;\ y = t^2 - 6$ is given by

$$\alpha = -\frac{4t^3}{3},\ \beta = 3t^2 - \frac{3}{2}.$$

emeæ keâejS ekeâ $\alpha = -\frac{4t^3}{3},\ \beta = 3t^2 - \frac{3}{2}$ Éeje eoÙee ieÙee, Je›eâ $x = 3t;\ y = t^2 - 6$ keâe Je›eâIee keâvõ $(\alpha, \beta)$ n~





(f) Find the value of $\phi$ for the curve

$$r = a\,(1 + \sin\theta)$$

Je›eâ $r = a\,(1 + \sin\theta)$ keâ eueS $\phi$ keâe ceeve evekeâeeueS~

(g) Prove that :

emeæ keâeepeS :

$$r^2 - p^2 = \left(\frac{p}{r}\frac{dr}{d\theta}\right)^2$$

(h) Show that :

emeæ keâeepeS :

$$\int_0^{\pi/2} \phi\,(\sin 2x).\sin x\,dx = \int_0^{\pi/2} \phi\,(\sin 2x).\cos x\,dx$$

(i) If $I_n = \int \operatorname{cosec}^n x\,dx$ then prove that

Ùeeo $I_n = \int \operatorname{cosec}^n x\,dx$ Iee emeæ keâeepeS ekeâ

$$I_n = -\frac{\operatorname{cosec}^{n-2} x.\cot x}{n-1} + \left(\frac{n-2}{n-1}\right) I_{n-2}$$

(j) Find the area bounded by the curve $y = x^3$, the y-axis and the lines $y = 1$ and $y = 8$.

Je›eâ $y = x^3$, y-De#e Deej jKeeDee $y = 1$ Deej $y = 8$ me eIeje #e$eHeâue evekeâeeueS~