Unit-IV

FkeâeF-IV

8. Obtain the least square estimate of $\underset{\sim}{\beta}$ in the model $\underset{\sim}{y}_{(n\times1)} = X_{(n\times k)}\,\underset{\sim}{\beta}_{(k\times1)} + \underset{\sim}{u}_{(n\times1)}$ and show that it is best linear unbiased estimator of $\underset{\sim}{\beta}$

$\underset{\sim}{\beta}$ keâe vÙevelece Jeie Deekeâuekeâ cee[ue

$\underset{\sim}{y}_{(n\times1)} = X_{(n\times k)}\,\underset{\sim}{\beta}_{(k\times1)} + \underset{\sim}{u}_{(n\times1)}$ ce Øeehle keâeepeÙe Deej eoKeeFS

ekeâ Ùen $\underset{\sim}{\beta}$ keâe DeÛÚe jeKekeâ DeveeYevele Deekeâuekeâ n~

9. Present a brief account of tests of hypothesis concerning $\underset{\sim}{\beta}$ in the model $\underset{\sim}{y} = x\underset{\sim}{\beta} + \underset{\sim}{u}$ under the normality assumptions.

cee[ue $\underset{\sim}{y} = x\underset{\sim}{\beta} + \underset{\sim}{u}$ me mecyeevOele keâuheveeDee keâ hejer#eCe mee#ehle ce yeleeFS peyeekeâ Øemeeceevûelee ne~

A (Printed Pages 8)

Roll No. ____________

S-697

B.A. (Part-III) Examination, 2015

STATISTICS

First Paper

(Non-parametric Methods and Regression Analysis)

Time Allowed : Three Hours] ***[Maximum Marks : 35***

Note : Attempt five questions in all. Question No. 1 is compulsory. Rest attempt one questions from each unit.

keâue heeÛe ØeMvee keâ Gòej oeepeS~ ØeMve me.1 DeeveJeeÛe n~ Fmekeâ Deeleejòeâ ØelÙekeâ FkeâeF me Skeâ ØeMve keâeepeS~

1. (a) Let $f(x,y) = \begin{cases} k\ ;\ 0 \le x \le y \le 1 \\ 0\ ;\ \text{other wise} \end{cases}$

Find :

(i) k

(ii) f(x)

(iii) f(x/y)

Ùeeo $f(x, y) = \begin{cases} k \; ; 0 \le x \le y \le 1 \\ 0 \quad ; \text{ DevÙeLee} \end{cases}$

lee Øeehle keâeepeÙe :

(i) k

(ii) $f(x)$

(iii) $f(x/y)$

(b) Define and differentiate between univariate and multivariate normal distribution.

SkeâueÛejeÙe SJe yenÛejeÙe ØemeeceevÙe yeŠve keâe heejYee<ee okeâj Devlej yelee FS~

(c) Given that $\underset{\sim}{x}(p \times 1)$ is normally distributed, Write down the distribution of the subvector $\underset{\sim}{x}^{(1)}(q \times 1)$ when the other subvector $\underset{\sim}{x}^{(2)}(p - q) \times 1$ is held fixed.

Ùeeo $\underset{\sim}{x}(p \times 1)$ keâe yenÛej ØemeeceevÙe yeŠve ne, lee GhemeeoMe $\underset{\sim}{x}^{(1)}(q \times 1)$ keâe yeŠve eueeKeÙe peyeekeâ omeje GhemeeoMe $\underset{\sim}{x}^{(2)}(p - q) \times 1$ emLej ceevee ieÙee n~

(d) Obtain the distribution of minimum and maximum of n order statistics for a ran-

Ùeeo Skeâ melele ÙeeÂeÛÚkeâ Ûej x keâe ØeeeÙekeâlee levelJe heâueve $f(x)$ Deej yeŠve heâueve $F(x)$ n, lee emeæ keâeepeÙe ekeâ $Z = F(x)$ keâe yeŠve Skeâ meceeve n~

Unit-III

FkeâeF-III

6. Discuss the Kolmogorov-Smirnov test of goodness of fit and compare it with the χ^2– test of goodness of fit.

Deemepeve mee…Je hej#eCe keâ eueÙe keâeucee ieejeJe-emcejveeJe hej#eCe mecePeeFS leLee Fmekeâe leuevee Deemepeve mee…Je keâ χ^2–hej#eCe me keâeepeÙe~

7. Describe any two of the following tests:

evecveeueeKele ce me ekeâvne oe hej#eCee keâe JÙeeKÙee keâeepeÙe:

(a) Mann-Whitney test

ceeve-e£Švee hej#eCe~

(b) Median test

ceeeOÙekeâe hej#eCe

(c) Sukhatme test

meKeelce hej#eCe

(j) Define general linear regression model along with assumptions usually made.

meeceevÙe jeKekeâ cee[ue mecyeevOeleâ keâuheveeDee meenle heejYeee<ele keâeepeS~

Unit-I

FkeâeF- I

2. Let a p-dimensional vector $\underset{\sim}{x}$ has the probability density function:

$$f(\underset{\sim}{x}) = K \exp\left[-\frac{1}{2}(\underset{\sim}{x}-\underset{\sim}{b})' A (\underset{\sim}{x}-\underset{\sim}{b})\right]$$

then obtain the constant K and interpret the parameters $\underset{\sim}{b}$ $(p\times 1)$ and A $(p\times p)$

ceevee ekeâ p- eJeceeÙe Ûej $\underset{\sim}{x}$ keâe ØeeeÙekeâlee levelJe heâueve :

$$f(\underset{\sim}{x}) = K \exp\left[-\frac{1}{2}(\underset{\sim}{x}-\underset{\sim}{b})' A (\underset{\sim}{x}-\underset{\sim}{b})\right]$$

nlee DeÛej K keâe ceeve evekeâeeueÙe leLee ØeeÛeuee $\underset{\sim}{b}$ $(p\times 1)$ Deej A $(p\times p)$ keâe JÙeeKÙee keâeepeÙe~

3. Obtain the maximum likelihood estimator of parameters $\underset{\sim}{\mu}$ and Σ in $N_p(\underset{\sim}{\mu}, \Sigma)$ on the basis of a random sample of size N.

yenÛejeÙe ØemeeceevÙe yešve $N_p(\underset{\sim}{\mu}, \Sigma)$ me eueÙe ieÙe N Deekeâej keâ ÙeeÂeÛÚkeâ Øeeleoeme keâ DeeOeej hej ØeeÛeuee $\underset{\sim}{\mu}$ leLee Σ keâ DeeOekeâlece mecYeeeJelee Deekeâuekeâ %eele keâeepeÙe~

Unit-II

FkeâeF- II

4. (a) Explain order statistics. Obtain the probability density function of r^{th} order statistic and the joint probability density function of r^{th} and s^{th} order statistics, where $r<s$.

›eâecele Øeeleoepe keâe mecePeeFS~ rJe ›eâecele Øeeleoepe keâe ØeeeÙekeâlee levelJe heâueve leLee rJe SJe sJe ›eâecele Øeeleoepe keâe meÙeòeâ ØeeeÙekeâlee levelJe heâueve ØeehleâkeâeepeÙe, pene $r<s$ n~

(b) If x is a continuous random variable with distribution function F(x) prove that:

$$E[x_{(r)}] = \frac{n!}{(r-1)!(n-r)!}\int_0^1 y^{r-1}(1-y)^{n-r}\, h(y)\, dy,$$

Where $h(y)=F^{-1}(y)$.

Ùeeo x keâe melele ÙeeÂeÛÚkeâ yeŠve heâueve F(x) ne, lee emeæ keâeepeÙe ekeâ :

$$E[x_{(r)}] = \frac{n!}{(r-1)!(n-r)!} \int_0^1 y^{r-1} (1-y)^{n-r} \, h(y) \, dy,$$

pene $h(y) = F^{-1}(y)$

5. (a) Let $y_1 < y_2 < y_3 < y_4$ denote the order statistics of a random sample of size 4 from the population with probability function:

$f(x) = 2x, \; 0 \le x \le 1$

$= 0$, elsewhere obtain the $P\left(\frac{1}{2} < y_3\right)$

ceevee ekeâ $y_1 < y_2 < y_3 < y_4$ Deekeâej 4 keâ ›eâecele ØeeleoMepe n peyeekeâ mecee„ keâe ØeeeÙekeâlee levelJe heâueve :

$f(x) = 2x, \; 0 \le x \le 1$

$= 0$, DevÙeLee n lee $P\left(\frac{1}{2} < y_3\right)$ keâe ceeve %eele keâeepeÙe~

(b) If x is a random variable of continuous type having probability density function F(x), Then prove that Z=F(x) has a uniform distribution.

dom sample of size n from a distribution of continuous type.

melele Øekeâej keâ yeŠve me Øeehle n Deekeâej keâ ÙeeÂeÛÚkeâ ØeeleoMe keâ eueÙe n ›eâecele meeKÙekeâÙee keâ vÙevelece SJe DeeOekeâlece ›eâecele keâe yeŠve Øeehle keâeepeÙe~

(e) Define quantile of order p.

p-›eâce Jeeue eJeYeepeve keâe heejYeee<ele keâeepeÙe~

(f) What are the parametric and non-parametric approaches in statistical inference.

meeKÙekeâe eve<keâ<e ce ØeeÛeue Je DeØeeÛeue Deei ve keâe eJeeOeÙee keÙee n?

(g) What do you understand by a test for goodness of fit? Explain.

Deemepeve ße…lee me Deehe keÙee mecePele n? mecePeeFS~

(h) Describe runs and write down its distribution.

jvee keâe JeCeve keâeepeÙe leLee Fmekeâe yeŠve eueeKeÙe~

(i) Explain the concept of regression.

meceeßeÙeCe me Deehe keÙee mecePele n?