Displacement Current :-

The term $\varepsilon_0 \frac{\partial E}{\partial t}$ is called displacement current in vaccum.

This name was first given by Maxwell.

Dimensionally $\varepsilon_0 \frac{\partial E}{\partial t}$ is Current density

The vaccum displacement current density is given by

$$\vec{J}_D = \varepsilon_0 \frac{\partial E}{\partial t}$$

The displacement current through any surface S is given by -

$$I_D = \iint_S \vec{J}_D \cdot d\vec{s}$$

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$$I_D = \iint_S \varepsilon_0 \frac{\partial E}{\partial t} \cdot d\bar{S}$$

$$I_D = \varepsilon_0 \frac{\partial}{\partial t} \iint_S E \cdot dS$$

where $\iint_S \bar{E} \cdot d\bar{S}$ is electric flux ϕ through surface S.

$$\boxed{I_D = \varepsilon_0 \frac{\partial \phi}{\partial t}}$$

This displacement current do not involve the actual transport of current or charge.

Characteristics of displacement Current :-

1. Displacement current is a current that only produce a magnetic field. It is not linked with the motion of charge. It has no other properties of current.

2. The magnitude of displacement current is equal to the rate of change of electric field.

$$\bar{J}_D = \varepsilon_0 \frac{\partial E}{\partial t}$$

3. Displacement current serves the purpose to make the total current continuous across the discontinuity in a conduction current.

4. In a good conductor, displacement current is negligible as compare to conduction current at less than optical frequency 10^{15} Hz.

Maxwell's Equations :-

Integral form of Maxwell's Equation -

1. $\int_S \bar{D} \cdot d\bar{s} = \int_V \rho \, dV$ or $\oint_S \bar{D} \cdot ds = q$

Physical significance - The total flux coming out of a closed surface is equal to the net charge enclosed.

2. $\oint_S \bar{B} \cdot d\bar{s} = 0$

Physical significance - The surface integral of magnetic flux density over a closed surface is zero.

3. $\oint \bar{E} \cdot d\bar{l} = -\frac{\partial}{\partial t} \int_S \bar{B} \cdot d\bar{s}$

Physical significance - The net emf induced in a closed path is equal to the surface integral of negative time rate of change of flux density over the surface bounded by closed path.

4. $\oint \bar{H} \cdot d\bar{l} = \int_S \left(\bar{J} + \frac{\partial \bar{D}}{\partial t} \right) \cdot ds$

Physical significance - The total mmf around any close path must be equal to the surface integral of conduction and displacement densities over the surface bounded by the closed path.

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Differential form of maxwell Equations -

1. $\bar{\nabla}\cdot\bar{D} = \rho$ or $Div\ D = \rho$

2. $\bar{\nabla}\cdot\bar{B} = 0$ or $Div\ B = 0$

3. $\bar{\nabla}\times\bar{E} = -\frac{\partial B}{\partial t}$ or $Curl\ \bar{E} = -\frac{\partial B}{\partial t}$

4. $\bar{\nabla}\times\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$ or $Curl\ \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$

where $\bar{D}$ = Electric displacement Vector $\because D = \varepsilon_0 E$
$\bar{B}$ = Magnetic flux density
$\bar{E}$ = Electric field intensity
J = Current density (Conventional)
ρ = charge density

Derivation of Maxwell's Equation -

1. Maxwell's first Equation -

$$\bar{\nabla}\cdot\bar{D} = \rho \quad or \quad div\ \bar{D} = \rho$$

Since

$$\bar{D} = \varepsilon\bar{E} \text{ for any medium}$$

$$\bar{\nabla}\cdot\bar{E} = \rho/\varepsilon_0$$

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Derivation -

In electrostatic Gauss law is given as -

$$\oint_S \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$$

If ρ is the volume charge density, then

$$q = \int_V \rho\, dV$$

$$\oint_S \vec{E}\cdot\vec{ds} = \frac{1}{\varepsilon_0}\int_V \rho\, dV$$

$$\oint_S \varepsilon_0 \vec{E}\cdot\vec{ds} = \int_V \rho\, dV$$

or

$$\oint_S \bar{D}\cdot\vec{ds} = \int_V \rho\, dV$$

using gauss divergence theorem

$$\oint_S \bar{D}\cdot\vec{ds} = \int_V (\mathrm{div}\, D)\, dV$$

$$\int_V (\mathrm{div}\, \bar{D})\, dV = \int_V \rho\, dV$$

$$\int_V (\mathrm{div}\, D - \rho)\, dV = 0$$

$$\mathrm{div}\, D - \rho = 0$$

$$\mathrm{div}\, \bar{D} = \rho$$

$$\boxed{\bar{\nabla}\cdot\bar{D} = \rho}$$

for free space $\rho = 0$

$$\boxed{\bar{\nabla}\cdot\bar{D} = 0}$$

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2. Maxwell's second Equation :-

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{or} \quad \text{div}\,\vec{B} = 0$$

Derivation -

using gauss law in magnetism

$$\oint \vec{B} \cdot \vec{ds} = 0$$

using gauss divergence thearam

$$\int_V (\text{div}\,B)\, dV = 0$$

$$\text{div}\,B = 0$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

3. Maxwell's third Equation:-

$$\text{Curl}\,\vec{E} = \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Derivation -

The magnetic flux through any closed surface in terms of magnetic flux density $\vec{B}$ is given by

$$\phi_B = \oint_S \vec{B} \cdot \vec{ds} \quad \text{——— (1)}$$

using Faradays law of electromagnetic induction

$$e = -\frac{\partial \phi_B}{\partial t}$$

$$e = -\frac{\partial}{\partial t}\oint_s \vec{B}\cdot\vec{ds}$$

$$e = -\oint_s \frac{\partial \vec{B}}{\partial t}\cdot\vec{ds}$$

$$\oint \vec{E}\cdot dl = e$$

$$\oint \vec{E}\cdot\vec{dl} = e = -\oint_s \frac{\partial B}{\partial t}\cdot\vec{ds}$$

using stoke's theoram

$$\oint_s (\text{Curl } E)\cdot\vec{ds} = -\oint_s \frac{\partial B}{\partial t}\cdot ds$$

$$\text{Curl } E = -\frac{\partial B}{\partial t}$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

4. Maxwell's fourth Equation :-

$$\text{Curl } \bar{H} = \vec{J} + \frac{\partial \bar{D}}{\partial t} \quad \text{or} \quad \text{Curl } \vec{B} = \mu_0\left[\vec{J} + \varepsilon_0\frac{\partial E}{\partial t}\right]$$

Derivation -

Consider a region in which steady flow of current. Consider a closed path in this region bounded by surface s.

The total current I enclosed by this path is given by -

$$I = \oint_s \vec{J}\cdot\vec{ds}$$

According to Ampere's circuital law -

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$$\oint_l \vec{B}\cdot dl = \mu_0 I \quad \text{or} \quad \mu_0 \oint_s \vec{J}\cdot ds$$

using stoke's theoram

$$\oint_s \text{Curl}\,\vec{B}\cdot\vec{ds} = \mu_0 \oint_s J\cdot ds$$

$$\oint_s (\text{Curl}\,B - \mu_0 J)\cdot ds = 0$$

$$\text{Curl}\,\vec{B} = \mu_0 J$$

This equation represent ampere's law is valid only for steady state. But Maxwell's modified ampere's law is valid for varrying electric field which equivalent to a current called displacement current.

This current flows only so long as the electric field is changing so maxwell included a term $\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$ in the expression of curl $\vec{B}$

$$\text{Curl}\,\vec{B} = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\text{Curl}\,\vec{B} = \mu_0 \left[J + \varepsilon_0 \frac{\partial E}{\partial t}\right]$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\left[J + \varepsilon_0 \frac{\partial E}{\partial t}\right]}$$