Probability & Probability Distributions in Statistics

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What is Statistics?

"Statistics is a way to get information from data"

Data → **Statistics** → **Information**

Data: Facts, especially numerical facts, collected together for reference or information.

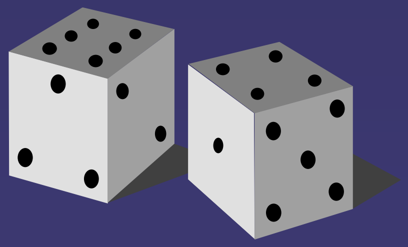
Information: Knowledge communicated concerning some particular fact.

Statistics is a ***tool*** for creating ***new understanding*** from a set of numbers.

Statistics is a science of getting informed decisions.

Topics

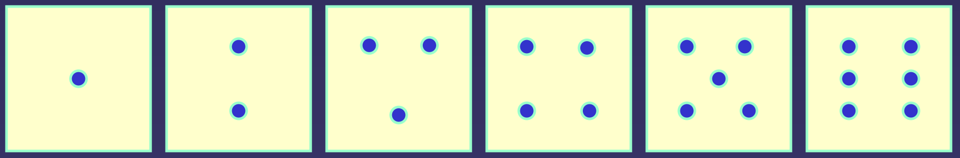
- Basic Probability Concepts: **Certainty/uncertainty**
- Random & Non-Random Experiments:
- Sample Spaces and Events :**Mutually Exclusive, Equally Likely and Exhaustive Events**
- Conditional Probability
- Random Variables
- Probability Distributions
- Binomial, Poisson and Normal



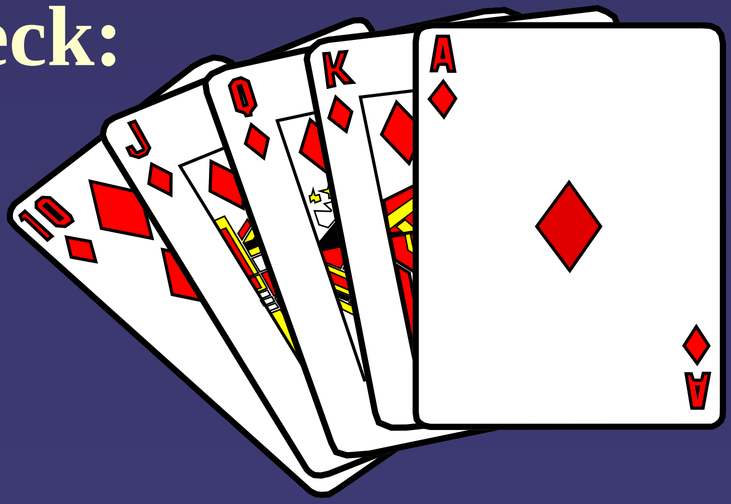
Sample Spaces

Collection of all Possible Outcomes

e.g. All 6 faces of a die:



e.g. All 52 cards of a bridge deck:



Events

- **Simple Event: Outcome from a Sample Space with 1 Characteristic**

  e.g. A *Red Card* from a deck of cards.

- **Joint Event: Involves 2 Outcomes Simultaneously**

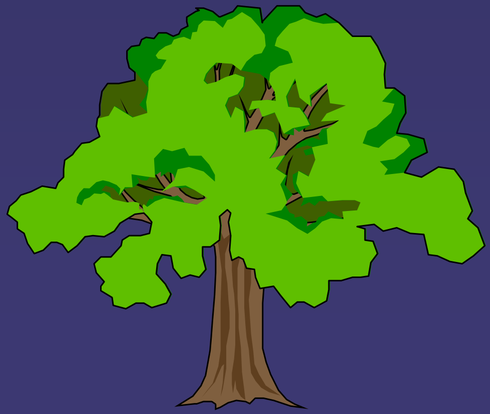
  e.g. An *Ace* which is also a *Red Card* from a deck of cards.

Visualizing Events

- Contingency Tables

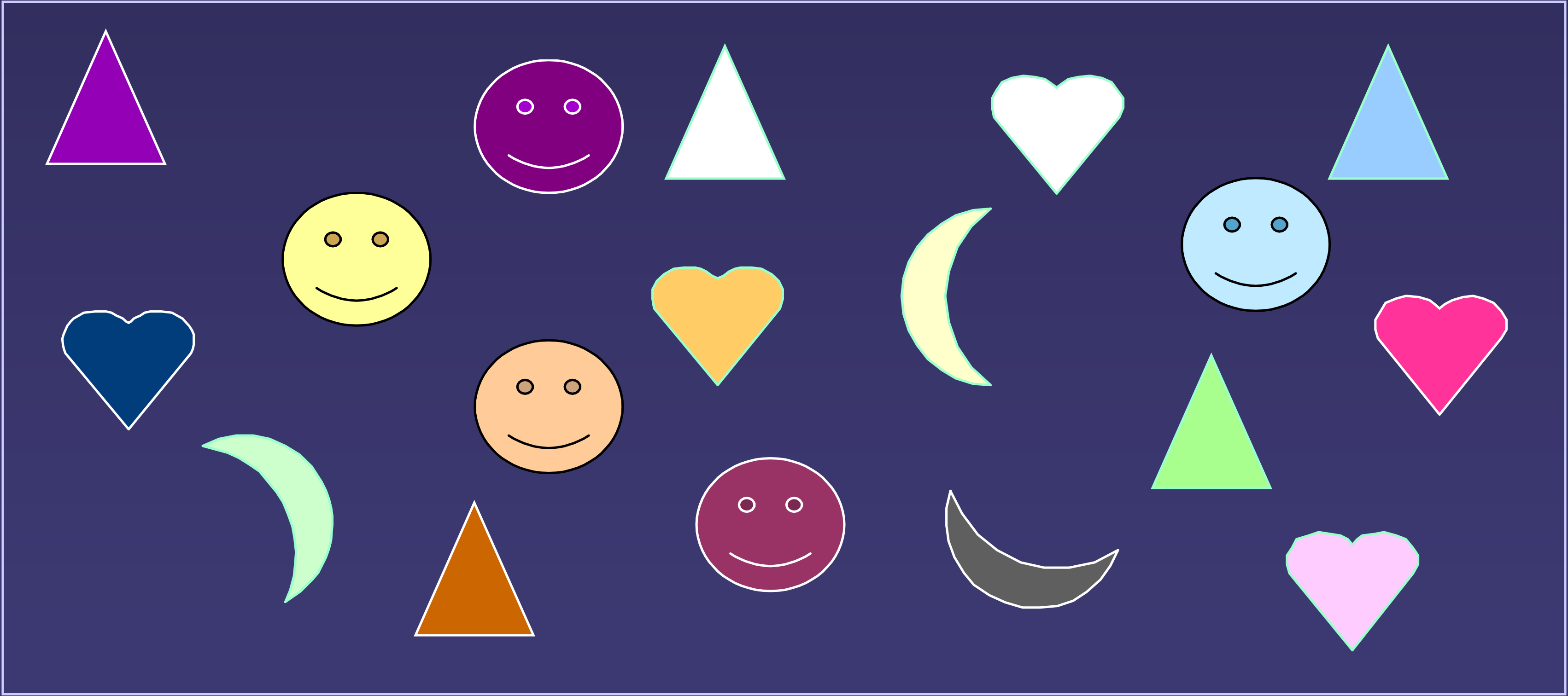
	Ace	Not Ace	Total
Black	2	24	26
Red	2	24	26
Total	4	48	52

- Tree Diagrams



Simple Events

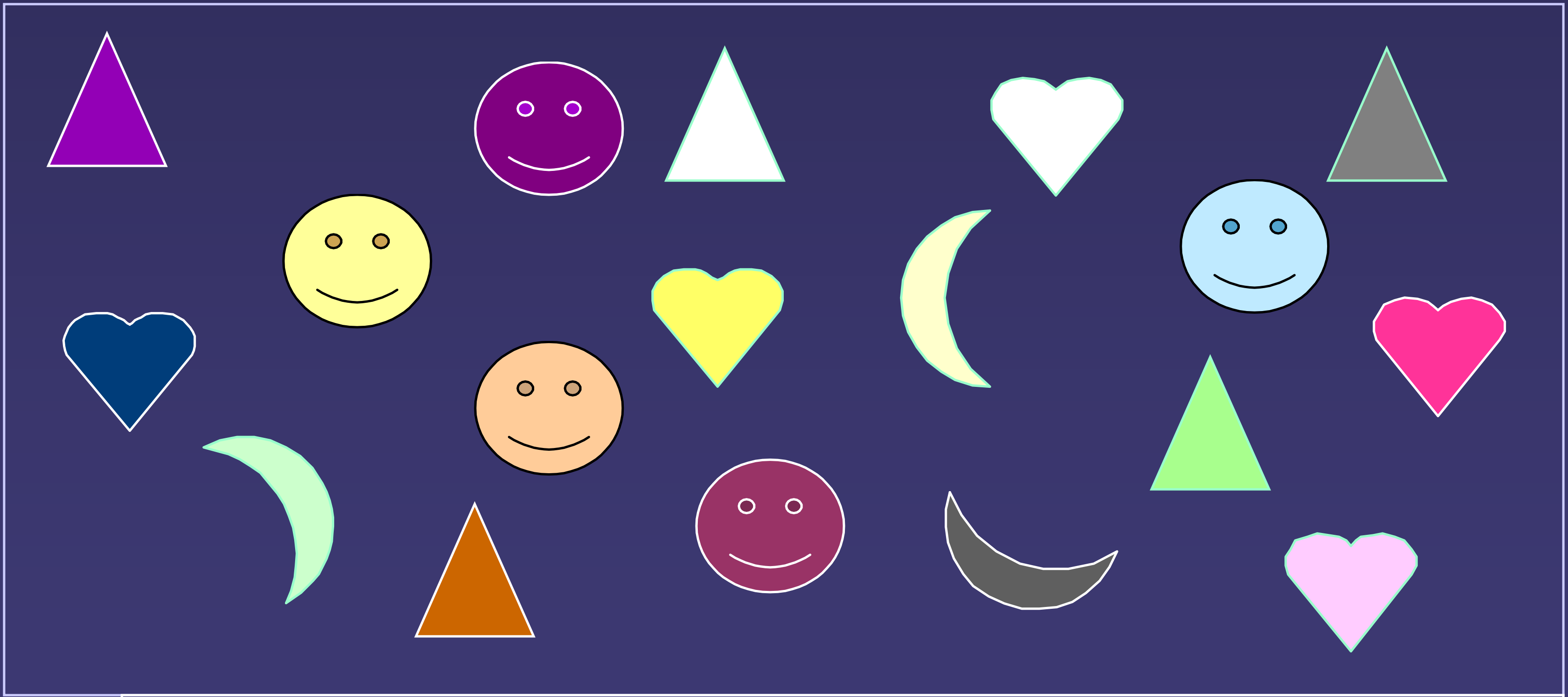
The Event of a Happy Face



There are 5 happy faces in this collection of 18 objects

Joint Events

The Event of a Happy Face *AND* Light Colored



3 Happy Faces which are light in color

Special Events

Null event

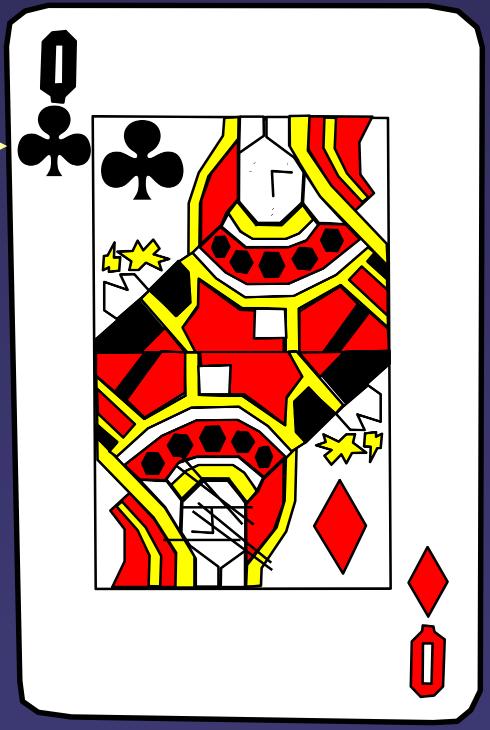
Club & diamond on 1 card draw

Complement of event

For event A,

All events not In A: A'

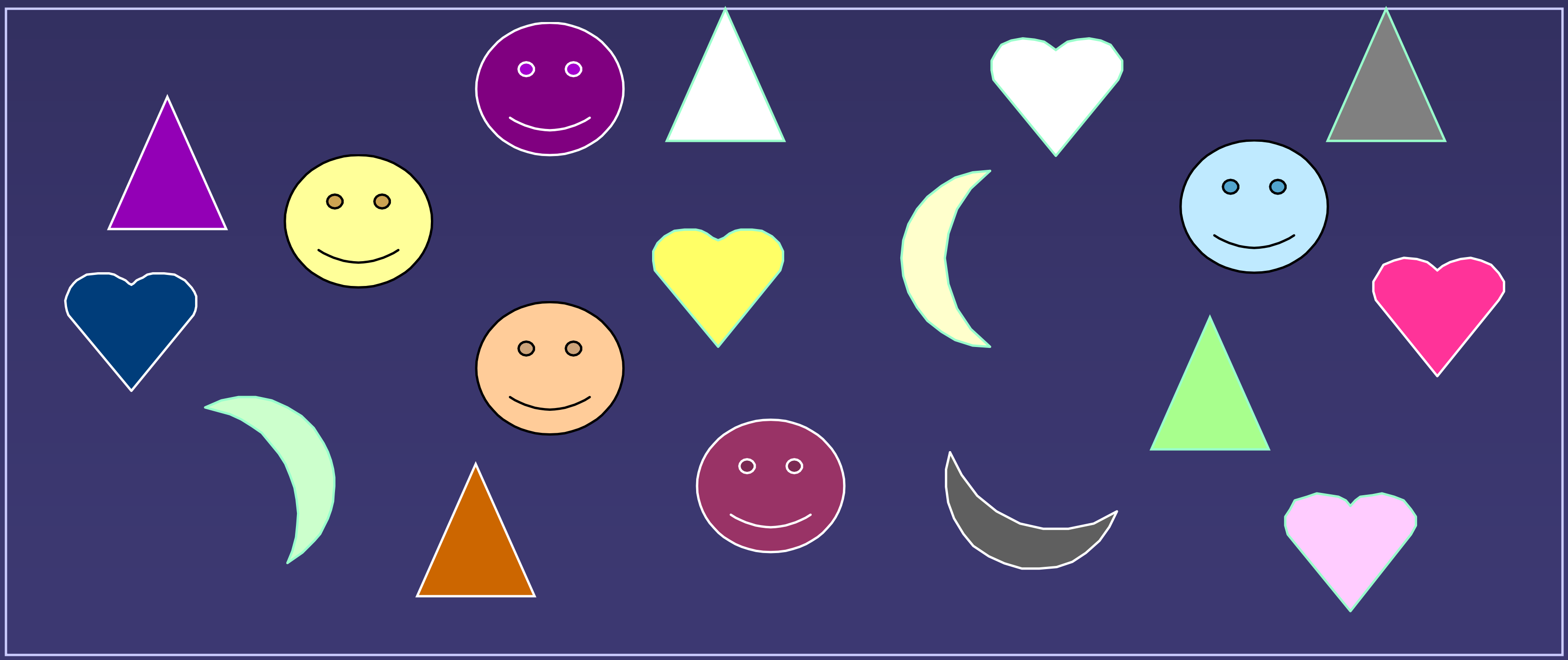
Null Event



Dependent or Independent Events

The Event of a Happy Face *GIVEN* it is Light Colored

E = Happy Face | Light Color



3 Items: 3 Happy Faces Given they are Light Colored

Contingency Table

A Deck of 52 Cards

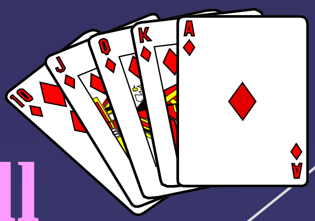
Red Ace

	Ace	Not an Ace	Total
Red	2	24	26
Black	2	24	26
Total	4	48	52

Sample Space

Tree Diagram

Event Possibilities



Full
Deck
of Cards

Red
Cards

Ace

Not an Ace

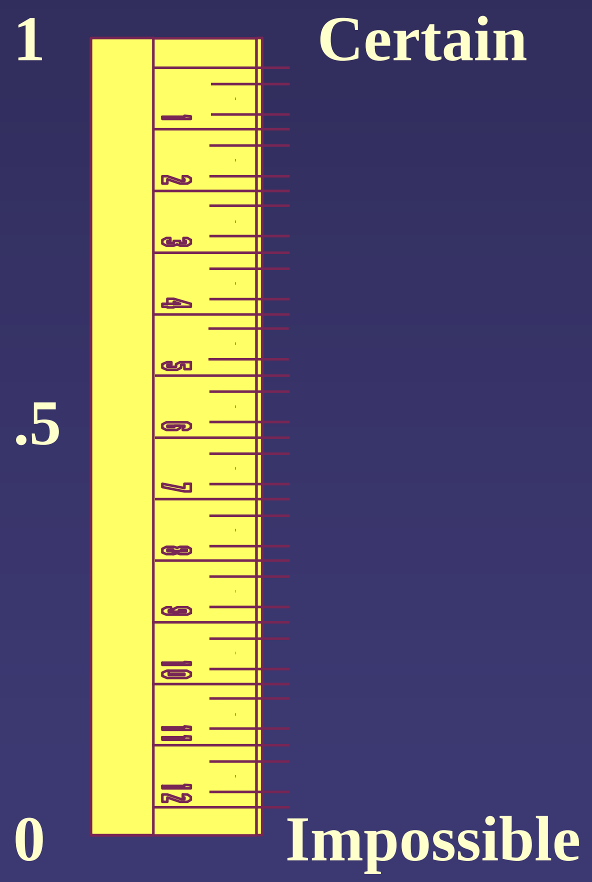
Black
Cards

Ace

Not an Ace

Probability

- Probability is the likelihood that the event will occur.
- Two Conditions:
  - Value is between 0 and 1.
  - Sum of the probabilities of all events must be 1.



Probability: Three Ways

- ***First:*** **Process Generating Events is Known:**
Compute using classical probability definition

Example: rolling dice

- ***Second:*** **Relative Frequency:**
Compute using empirical data

Example: Rain Next day based on history

- ***Third:*** **Subjective Method:**
Compute based on judgment

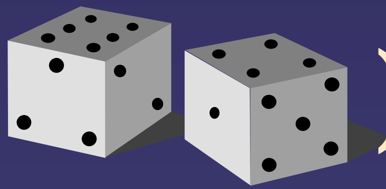
Example: Analyst predicts forest region will increase by 10% over next year

Computing Probability

- The Probability of an Event, E:

$$P(E) = \frac{\text{Number of Event Outcomes}}{\text{Total Number of Possible Outcomes in the Sample Space}}$$

$$= \frac{X}{T}$$

e.g. P() = 2/36

(There are 2 ways to get one 6 and the other 4)

- Each of the Outcome in the Sample Space equally likely to occur.

Random Variable

A numerical description of the outcome of an experiment

Discrete Random Variable:

Obtained by Counting (0, 1, 2, 3, etc.)

(Usually finite by number of different values)

Example: **Toss a coin 5 times & Count the number of tails.**

(0, 1, 2, 3, 4, or 5 times)

Continuous Random Variable:

A numerical description of the outcome of an experiment

Example:

- **The Value of the height, weight etc.**
- **Time to repair a failed machine**

RV Given by Capital Letters X & Y Specific Values Given by lower case

Probability Distribution

Characterization of the possible values that a RV may assume along with the probability of assuming these values.

Discrete Probability Distribution

- List of all possible [x_i, $p(x_i)$] pairs

  X_i = value of random variable

  $P(x_i)$ = probability associated with value

- Mutually exclusive (nothing in common)
- Collectively exhaustive (nothing left out)

$$0 \leq p(x_i) \leq 1$$

$$\Sigma P(x_i) = 1$$

Weekly Demand of a Slow-Moving Product

Probability Mass Function

Demand, x	Probability, p(x)
0	0.1
1	0.2
2	0.4
3	0.3
4 or more	0

Weekly Demand of a Slow-Moving Product

A Cumulative Distribution Function:

Probability that RV assume a value <= a given value, x

Demand, x	Cumulative Probability, P(x)
0	0.1
1	0.3
2	0.7
3	1

Two or More Random Variables

Frequency of applications during a given week

Number of Applications					
Interest Rate	5	6	7	8	Total
7.00%	**3**	**4**	**6**	**2**	**15**
7.50%	**2**	**4**	**3**	**1**	**10**
8%	**3**	**1**	**1**	**0**	**5**
Total	**8**	**9**	**10**	**3**	**30**

Two or More Random Variables

Joint Probability Distribution

Number of Applications					
Interest Rate	5	6	7	8	Total
7.00%	0.100	0.133	0.200	0.067	0.500
7.50%	0.067	0.133	0.100	0.033	0.333
8.00%	0.100	0.033	0.033	0.000	0.167
Total	0.267	0.300	0.333	0.100	1.000

Two or More Random Variables

Joint Probability Distribution

Number of Applications					
Interest Rate	5	6	7	8	Total
7.00%	0.100	0.133	0.200	0.067	0.500
7.50%	0.067	0.133	0.100	0.033	0.333
8.00%	0.100	0.033	0.033	0.000	0.167
Total	0.267	0.300	0.333	0.100	1.000

Marginal Probabilities

Computing Joint Probability

The Probability of a *Joint Event,* A *and* B:

P(A and B)

$$= \frac{\textit{Number of Event Outcomes from both A and B}}{\textit{Total Number of Possible Outcomes in Sample Space}}$$

e.g. P(Red Card and Ace)

$$= \frac{\text{2 Red Aces}}{\text{52 Total Number of Cards}} = \frac{1}{26}$$

Joint Probability Using Contingency Table

Event	Event		
	B_1	B_2	Total
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	$P(A_1)$
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	$P(A_2)$
Total	$P(B_1)$	$P(B_2)$	1

Joint Probability

Marginal (Simple) Probability

Computing Compound Probability

The Probability of a *Compound Event*, A *or* B:

$$P\left(\text{A or B}\right) = \frac{\text{Numer of Event Outcomes from Either A or B}}{\text{Total Outcomes in the Sample Space}}$$

e.g.

P(Red Card *or* Ace)

$$= \frac{\text{4 Aces + 26 Red Cards } - \text{ 2 Red Aces}}{\text{52 Total Number of Cards}} = \frac{28}{52} = \frac{7}{13}$$

Compound Probability Addition Rule

$$P(A_1 \text{ or } B_1) = P(A_1) + P(B_1) - P(A_1 \text{ and } B_1)$$

Event	Event		Total
	B_1	B_2	
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	$P(A_1)$
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	$P(A_2)$
Total	$P(B_1)$	$P(B_2)$	1

For Mutually Exclusive Events: $P(A \text{ or } B) = P(A) + P(B)$

Computing Conditional Probability

The Probability of Event A given that Event B has occurred:

$$P(A \mid B) = \frac{P(A \textit{ and } B)}{P(B)}$$

e.g.

P(Red Card ***given that it is*** **an Ace) =** $\dfrac{2 \text{ Red Aces}}{4 \text{ Aces}} = \dfrac{1}{2}$

Conditional Probability Using Contingency Table

Conditional Event: Draw 1 Card. Note Kind & Color

Type	Color: Red	Color: Black	Total
Ace	2	2	4
Non-Ace	24	24	48
Total	26	26	52

Revised Sample Space

$$P(\text{Ace} \mid \text{Red}) = \frac{P(\text{Ace AND Red})}{P(\text{Red})} = \frac{2/52}{26/52} = \frac{2}{26}$$

Conditional Probability and Statistical Independence

Conditional Probability:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Multiplication Rule:

$$P(A \text{ and } B) = P(A \mid B) \bullet P(B)$$
$$= P(B \mid A) \bullet P(A)$$

Conditional Probability and Statistical Independence *(continued)*

Events are Independent:

$$P(A \mid B) = P(A)$$

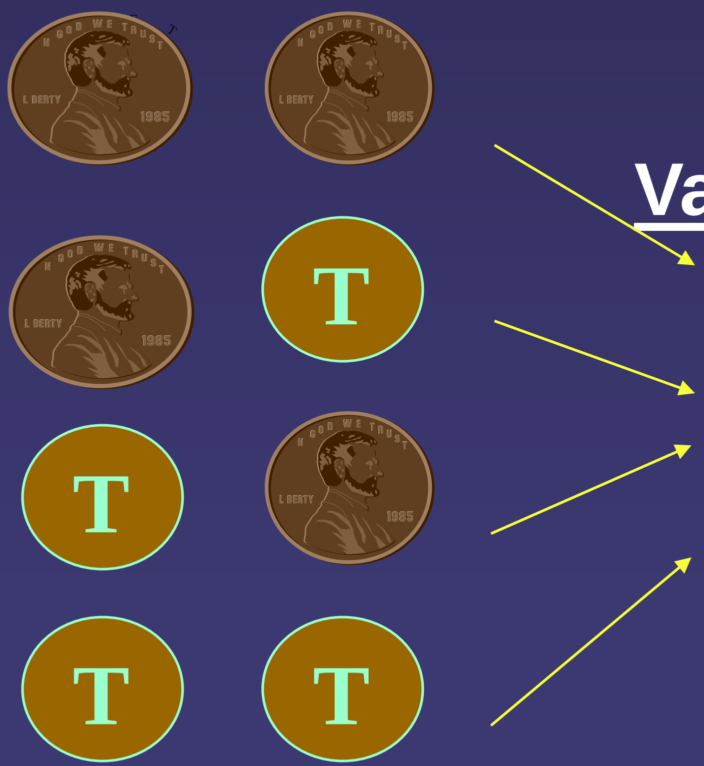
Or, $$P(B \mid A) = P(B)$$

Or, $$P(A \text{ and } B) = P(A) \cdot P(B)$$

Events A and B are *Independent* when the probability of one event, A is *not affected* by another event, B.

Discrete Probability Distribution Example

Event: Toss 2 Coins. Count # Tails.



Probability distribution

Values	probability
0	1/4 = .25
1	2/4 = .50
2	1/4 = .25

Important Discrete Probability Distribution Models

Discrete Probability Distributions

Binomial

Poisson

Bernoulli Distribution

- Two possible mutually exclusive outcomes with constant probabilities of occurrence

“Success” (x=1) or “failure” (x=0)

Example : Response to telemarketing

The probability mass function is

- p(x) = p if x=1
- P(x) = 1- p if x=0

Where p is the probability of success

Binomial Distribution

- ‘N’ identical trials
  - Example: 15 tosses of a coin, 10 light bulbs taken from a warehouse
- 2 mutually exclusive outcomes on each trial
  - Example: Heads or tails in each toss of a coin, defective or not defective light bulbs

Binomial Distributions

- Constant Probability for each Trial
  - Example: Probability of getting a tail is the same each time we toss the coin and each light bulb has the same probability of being defective
- 2 Sampling Methods:
  - Infinite Population Without Replacement
  - Finite Population With Replacement
- Trials are Independent:
  - The Outcome of One Trial Does Not Affect the Outcome of Another

Binomial Probability Distribution Function

$$P(X) = \frac{n!}{X!(n-X)!} p^X (1-p)^{n-X}$$

$P(X)$ = probability that X successes given a knowledge of n and p

X = number of 'successes' in sample, (X = 0, 1, 2, ..., n)

p = probability of each 'success'

n = sample size

Tails in 2 Tosses of Coin

X	$P(X)$
0	1/4 = .25
1	2/4 = .50
2	1/4 = .25

Binomial Distribution Characteristics

Mean

$$\mu = E(X) = np$$

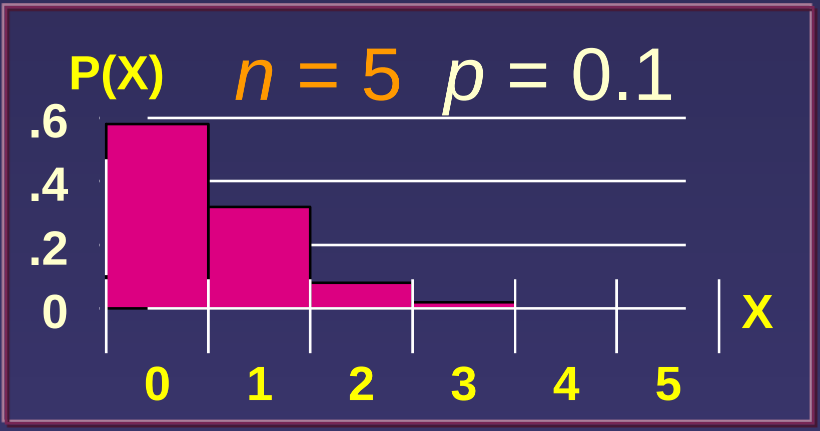
e.g. $\mu = 5\,(.1) = .5$

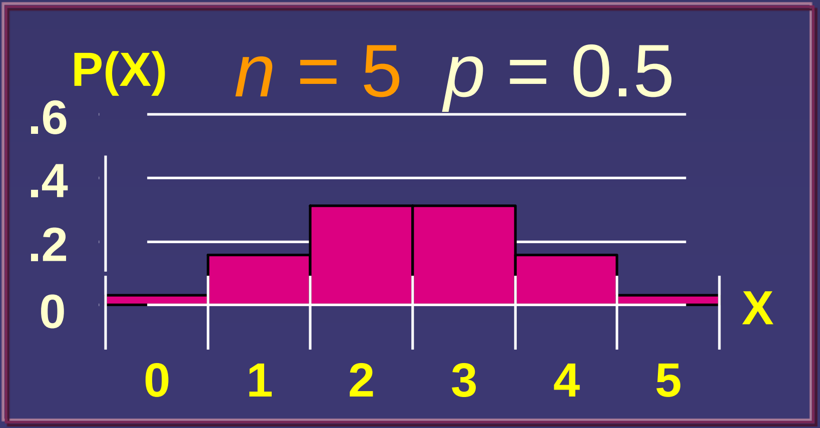
Standard Deviation

$$\sigma = \sqrt{np(1-p)}$$

e.g. $\sigma = \sqrt{5(.5)(1 - .5)}$

$= 1.118$





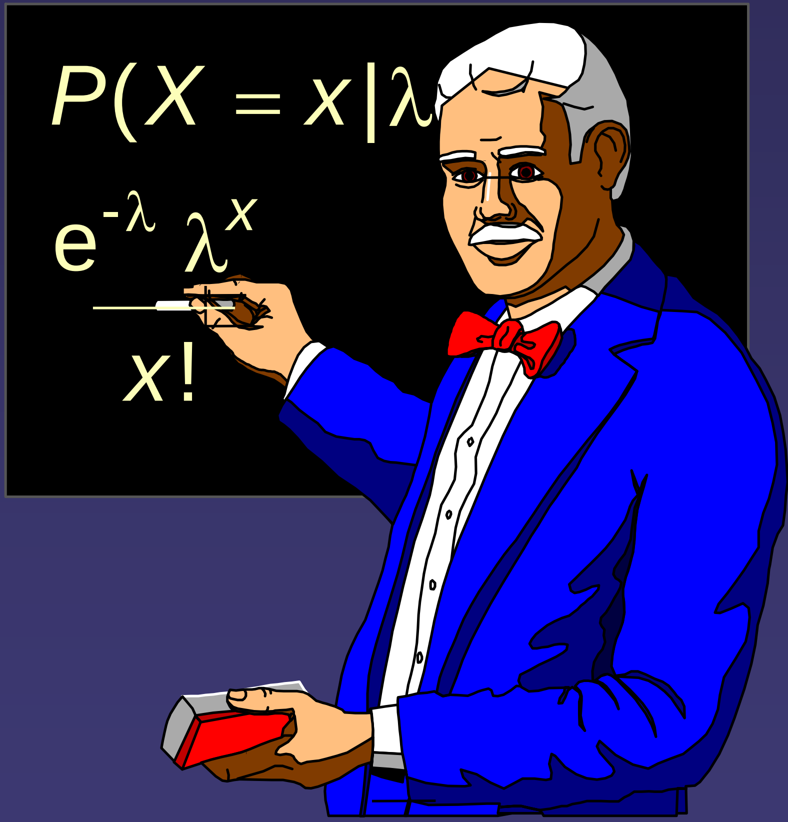
Poisson Distribution

Poisson process:

- **Discrete events in an 'interval'**
  - **The probability of one success in an interval is stable**
  - **The probability of more than one success in this interval is 0**
- **Probability of success is Independent from interval to Interval**

Examples:

- # Customers arriving in 15 min
- # Defects per case of light bulbs



Poisson Distribution Function

$$P(X) = \frac{e^{-\lambda}\lambda^{X}}{X!}$$

P*(*X*) = probability** of ***X successes given λ

λ = expected (**mean**) number of 'successes'

e = 2.71828 (base of natural logs)

X = number of **'successes' per unit**

e.g. Find the probability of 4 customers arriving in 3 minutes when the mean is 3.6

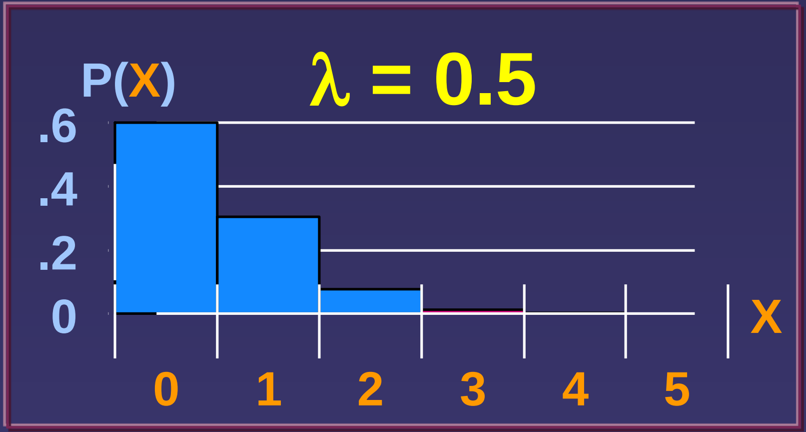
$$P(X) = \frac{e^{-3.6}\,3.6^{4}}{4!} = .1912$$

Poisson Distribution Characteristics

Mean

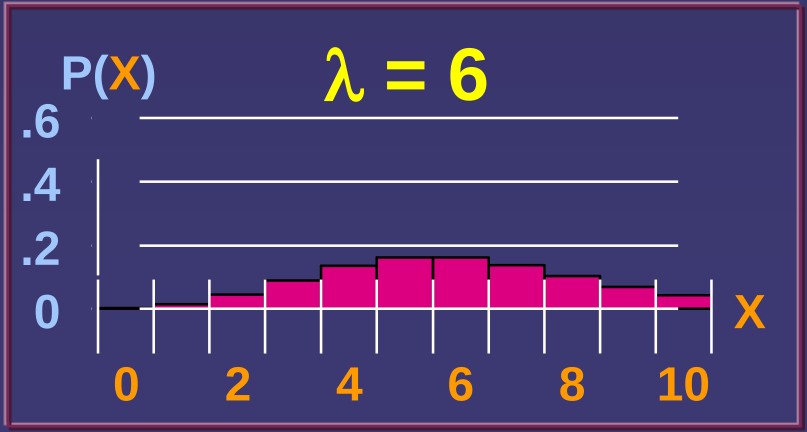
$$\mu = E(X) = \lambda$$

$$= \sum_{i=1}^{N} X_i P(X_i)$$



Standard Deviation

$$\sigma = \sqrt{\lambda}$$

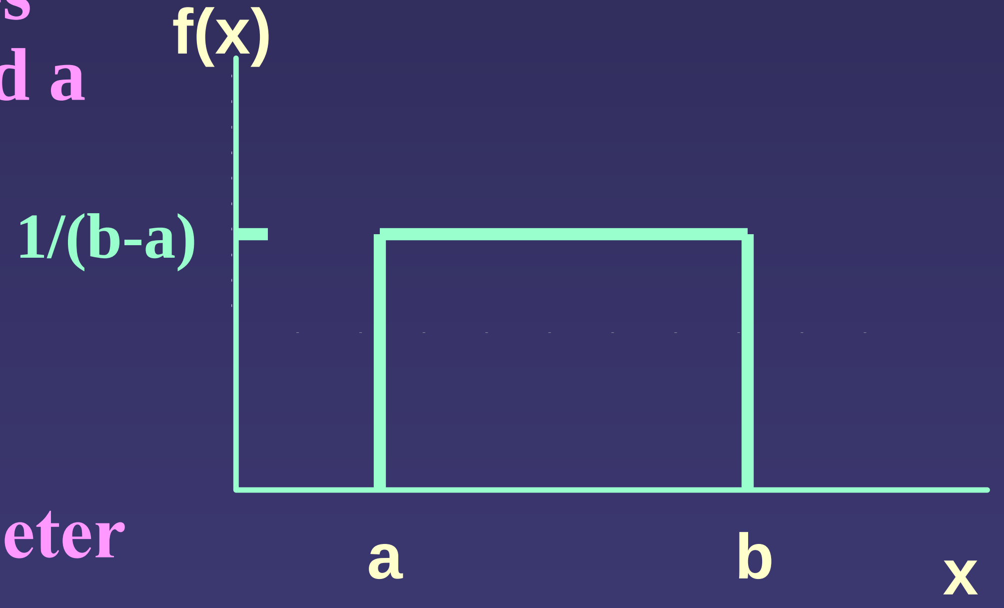


Continuous Probability Distributions

- Uniform
- Triangular
- Normal
- Exponential

The Uniform Distribution

- Equally Likely chances of occurrences of RV values between a maximum and a minimum
- Mean = $(b+a)/2$
- Variance = $(b-a)^2/12$
- ‘a’ is a location parameter
- ‘b-a’ is a scale parameter
- no shape parameter



The Uniform Distribution

Probability Density Function

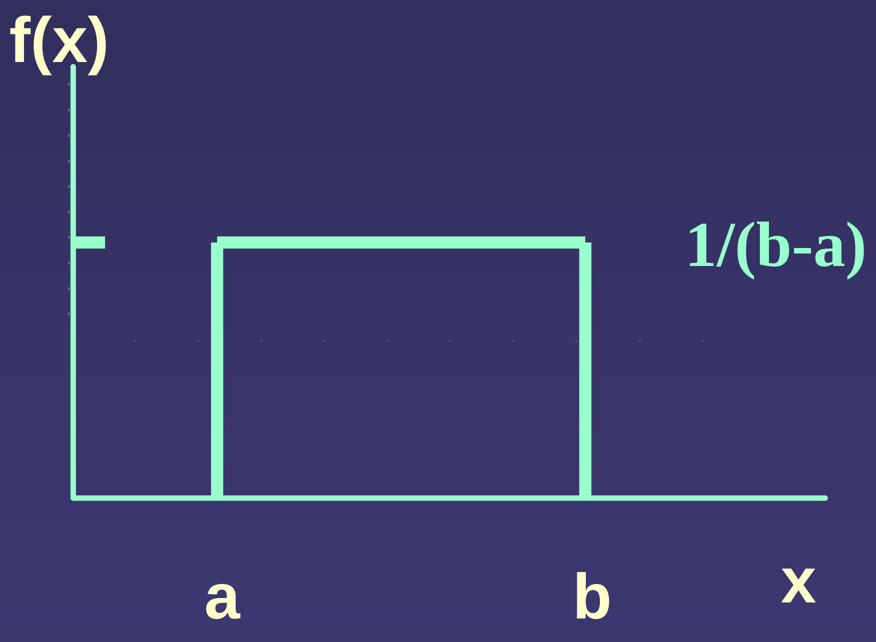
$$f(x) = \frac{1}{b-a} \qquad if \;\; a \le x \;\; \le \mathrm{b}$$

Distribution Function

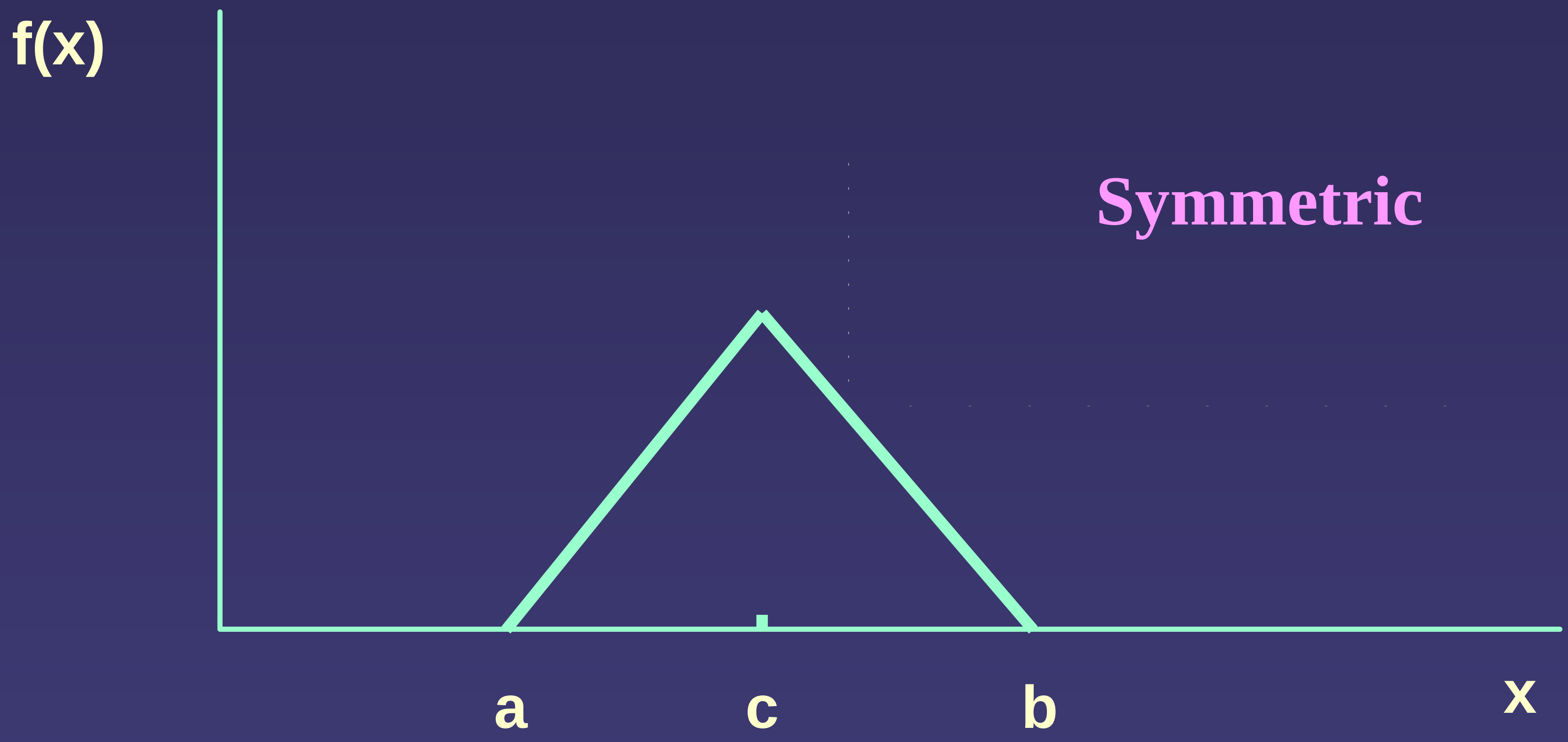
$$F(x) = 0 \qquad if \;\; x < \mathrm{a}$$

$$F(x) = \frac{x-a}{b-a} \qquad if \;\; a \le x \;\; \le \mathrm{b}$$

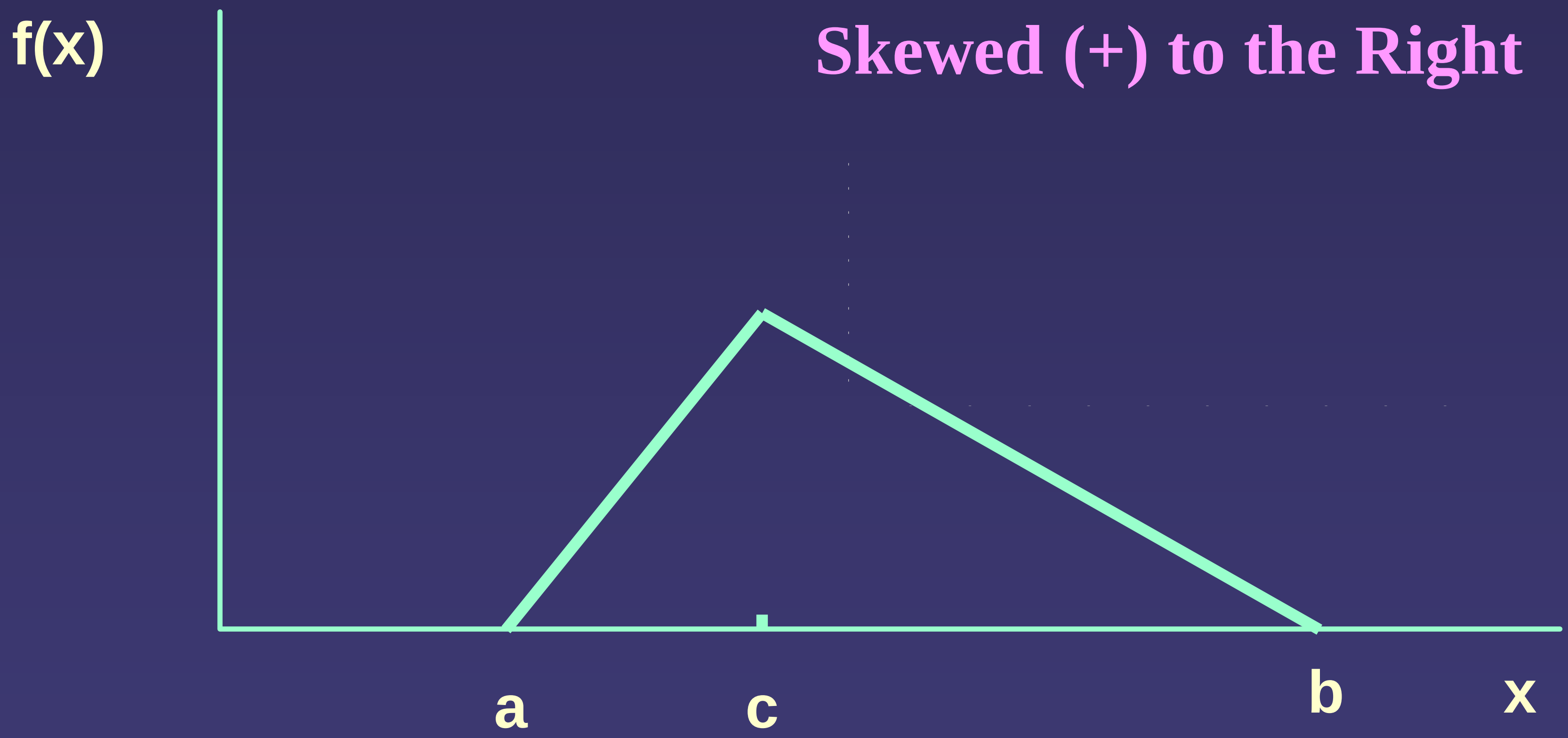
$$F(x) = 1 \qquad if \;\; b < x$$



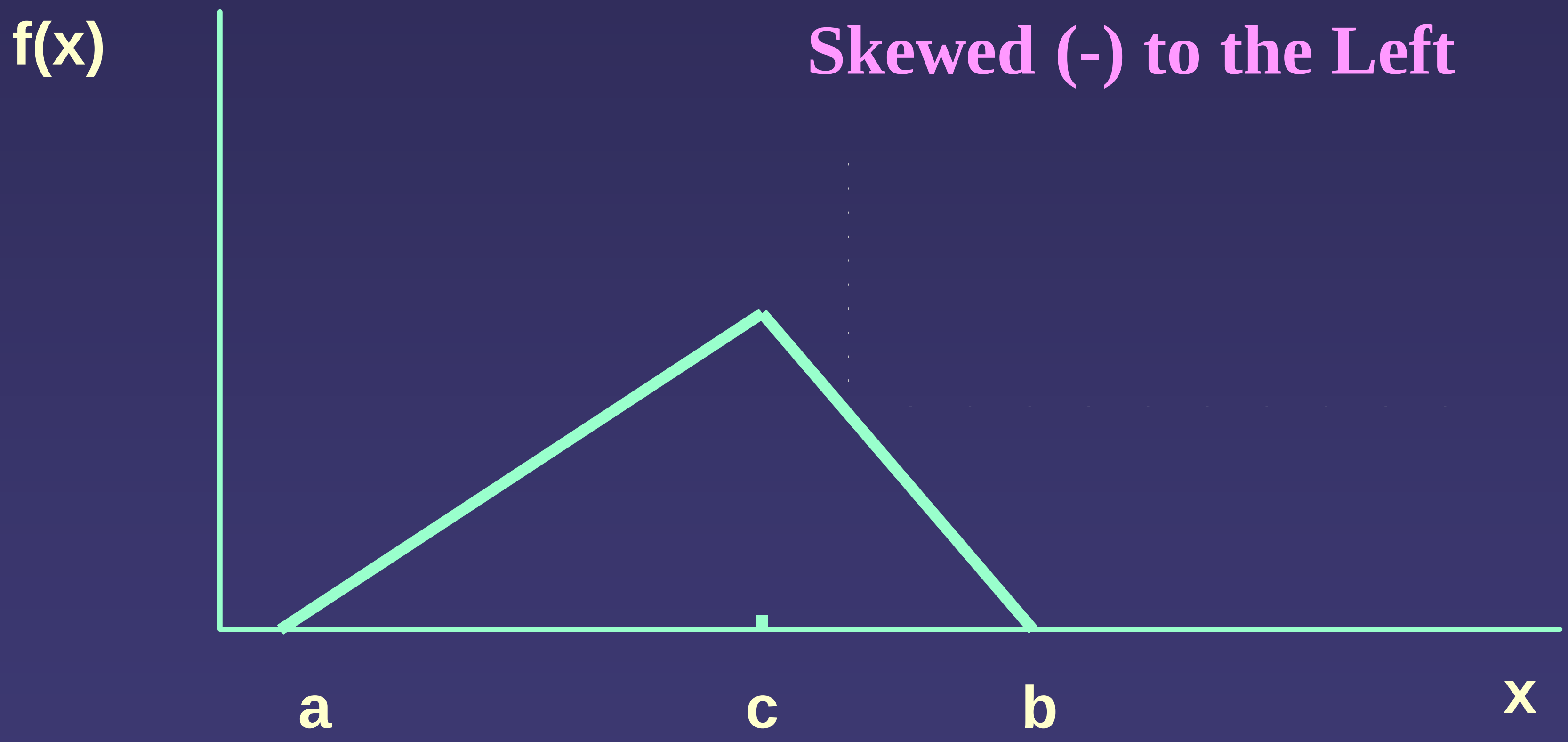
The Triangular Distribution



The Triangular Distribution



The Triangular Distribution



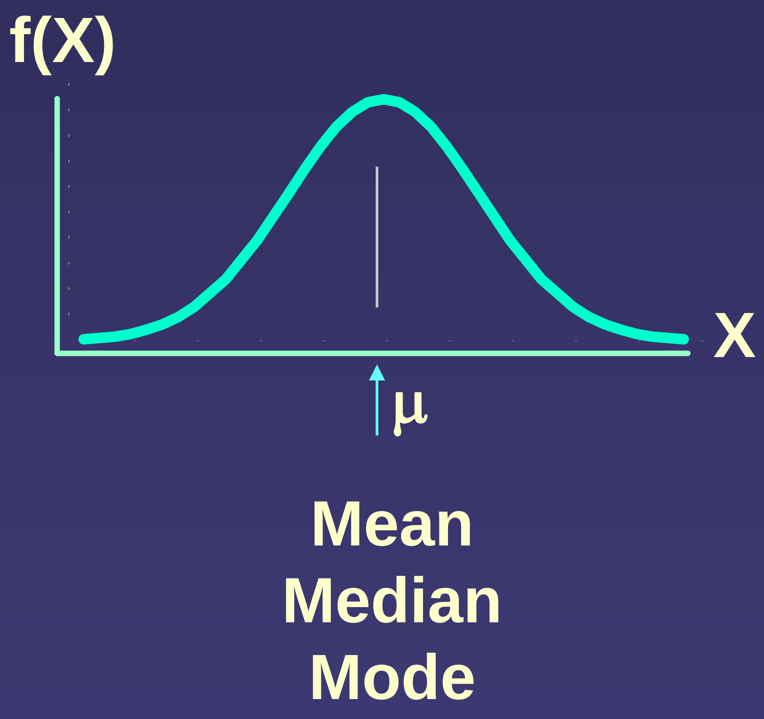
The Normal Distribution

- ‘Bell Shaped’
- Symmetrical
- Mean, Median and

  Mode are Equal
- ‘Middle Spread’

  Equals 1.33 σ
- Random Variable has

  Infinite Range

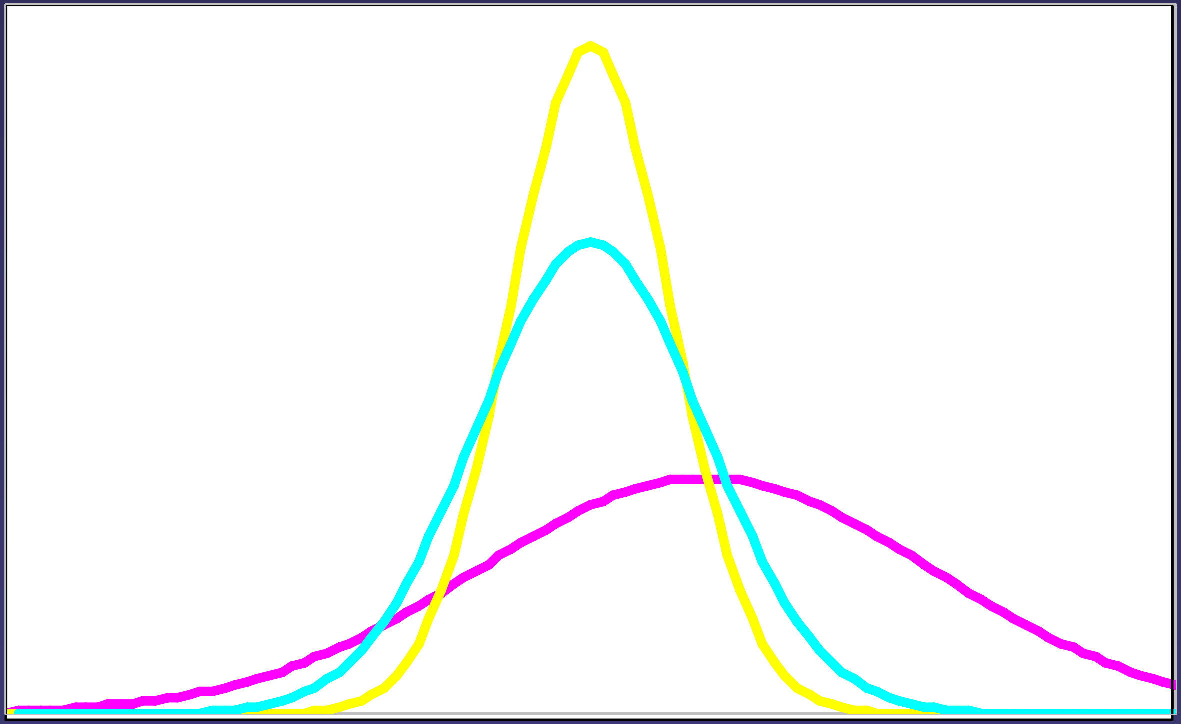


The Mathematical Model

$$f(X) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(X-\mu)^2}$$

$f(X)$	=	frequency of random variable X
π	=	3.14159; e = 2.71828
σ	=	population standard deviation
X	=	value of random variable ($-\infty < X < \infty$)
μ	=	population mean

Many Normal Distributions

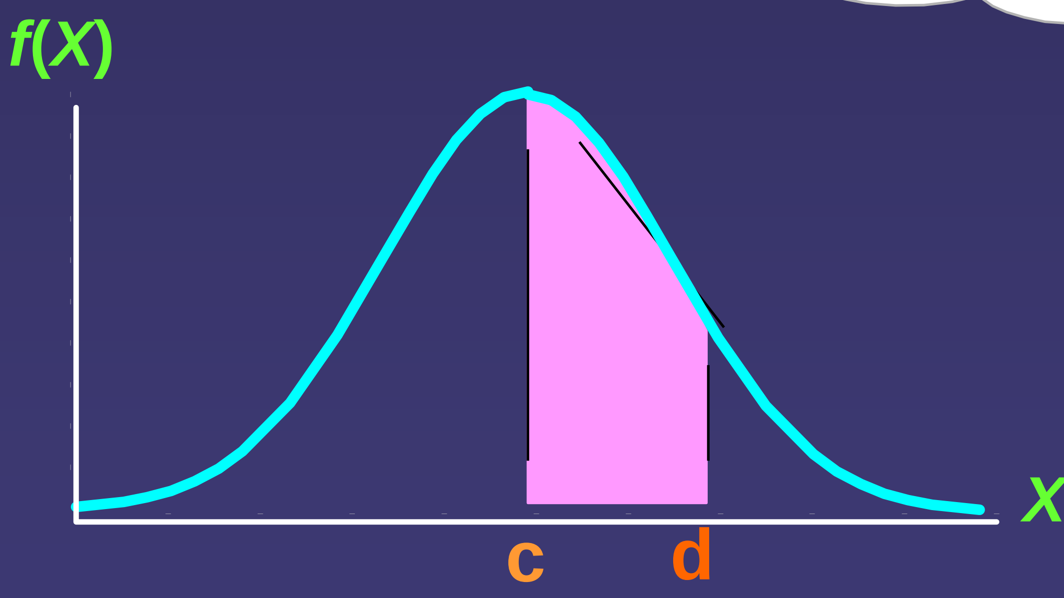


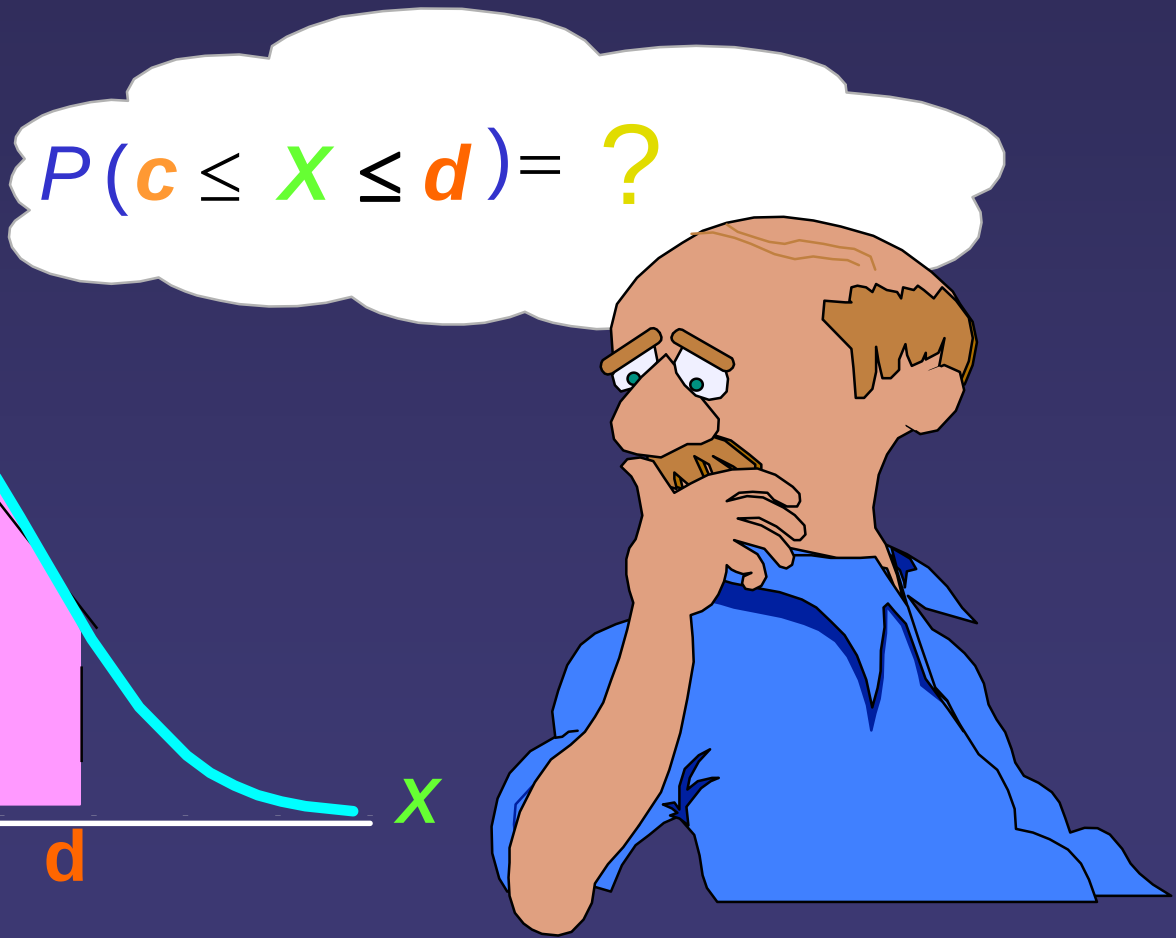
There are an Infinite Number

Varying the Parameters σ and μ, we obtain **Different Normal Distributions**.

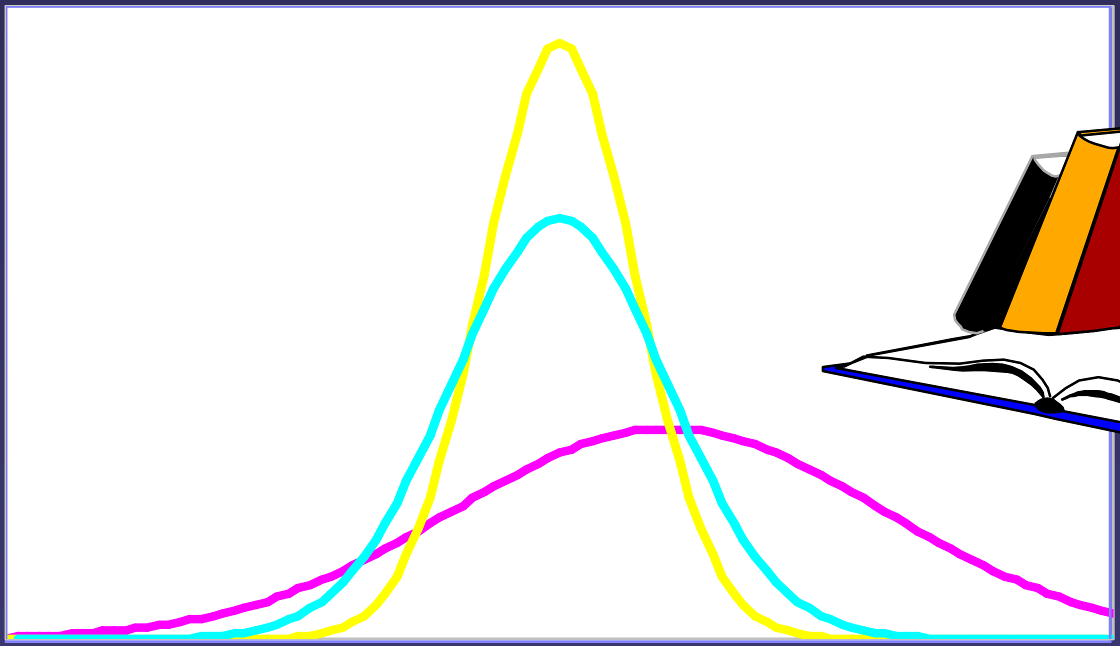
Normal Distribution: Finding Probabilities

Probability is the area under the curve!





Which Table?



Each distribution has its own table?

Infinitely Many Normal Distributions Means Infinitely Many Tables to Look Up!

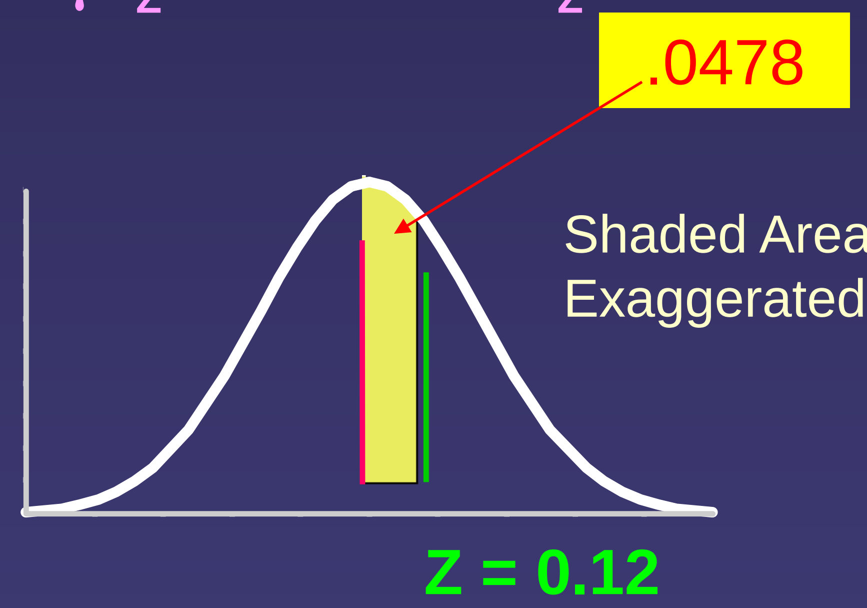
Solution (I): The Standardized Normal Distribution

Standardized Normal Distribution Table (Portion)

$\mu_Z = 0$ and $\sigma_Z = 1$

Z	.00	.01	.02
0.0	.0000	.0040	.0080
0.1	.0398	.0438	.0478
0.2	.0793	.0832	.0871
0.3	.0179	.0217	.0255

Probabilities



Only One Table is Needed

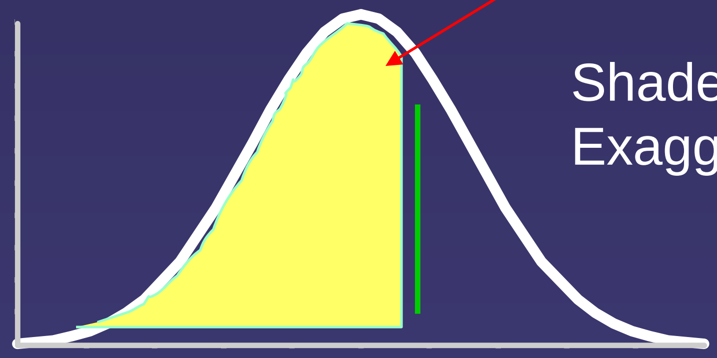
Solution (II): The Cumulative Standardized Normal Distribution

Cumulative Standardized Normal Distribution Table (Portion)

Z	.00	.01	**.02**
0.0	.5000	.5040	.5080
0.1	.5398	.5438	**.5478**
0.2	.5793	.5832	.5871
0.3	.5179	.5217	.5255

Probabilities

$\mu_Z = 0$ and $\sigma_Z = 1$ .5478



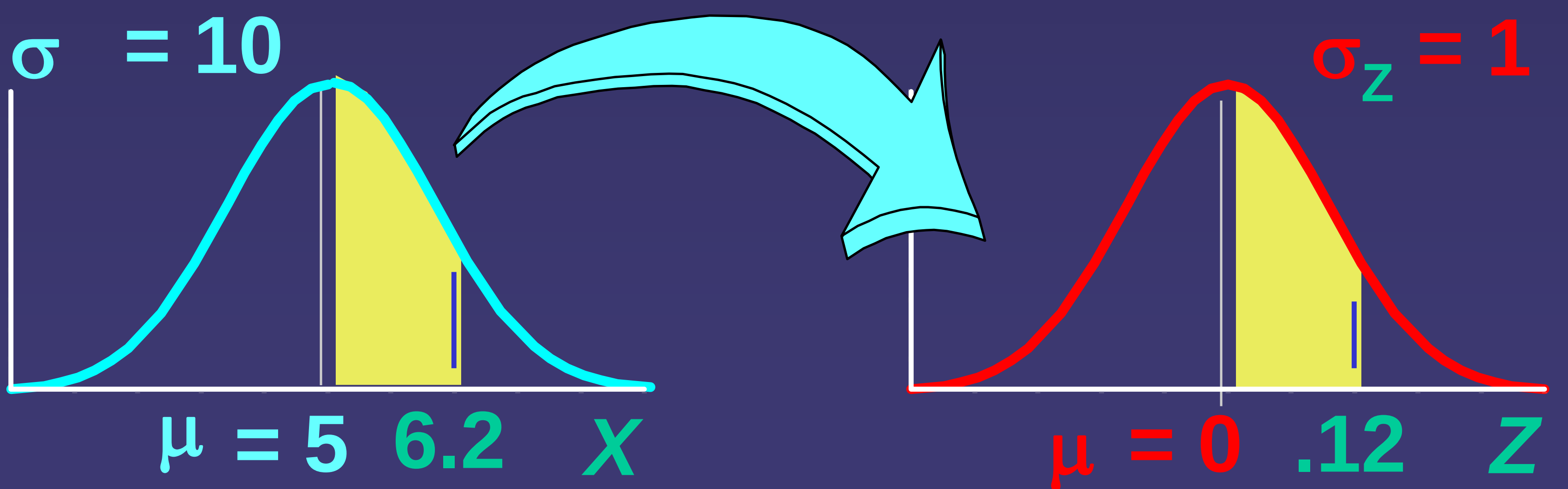
Only One Table is Needed

Standardizing Example

$$Z = \frac{X - \mu}{\sigma} = \frac{6.2 - 5}{10} = 0.12$$

Normal Distribution

Standardized Normal Distribution

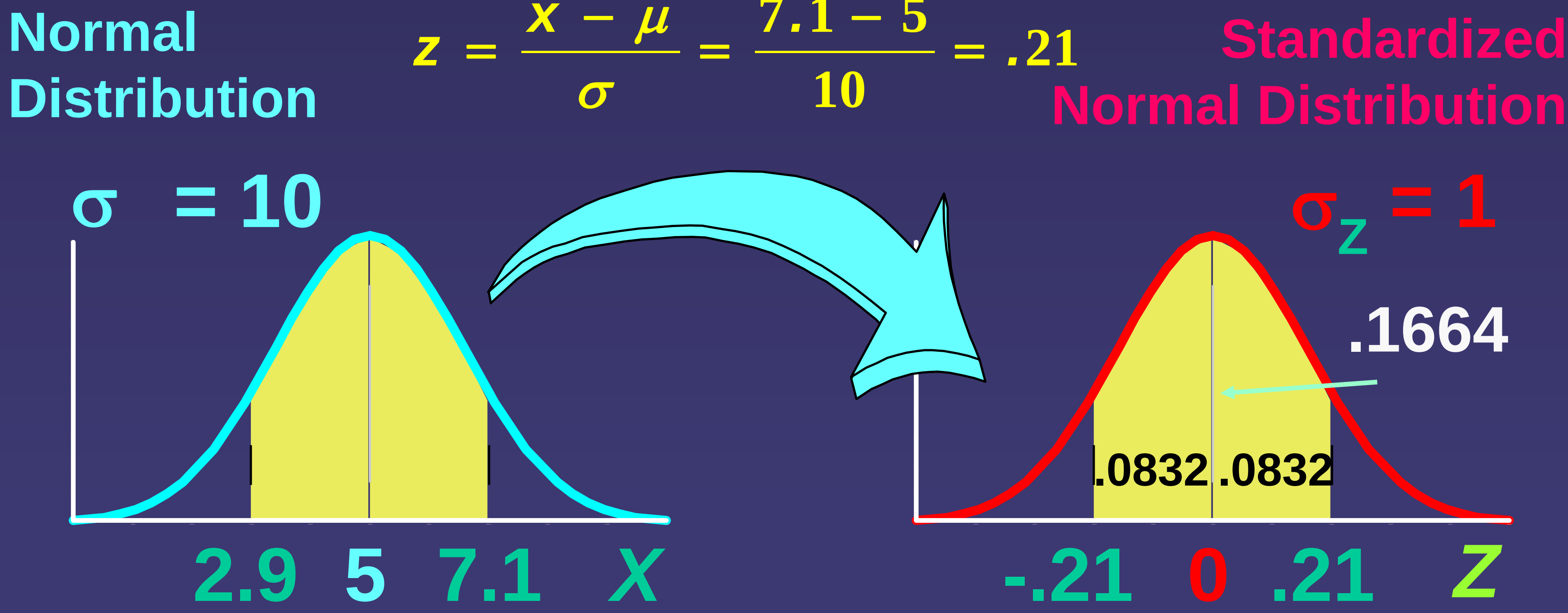


Shaded Area Exaggerated

Example:
$P(2.9 < X < 7.1) = .1664$

$$z = \frac{x - \mu}{\sigma} = \frac{2.9 - 5}{10} = -.21$$

$$z = \frac{x - \mu}{\sigma} = \frac{7.1 - 5}{10} = .21$$



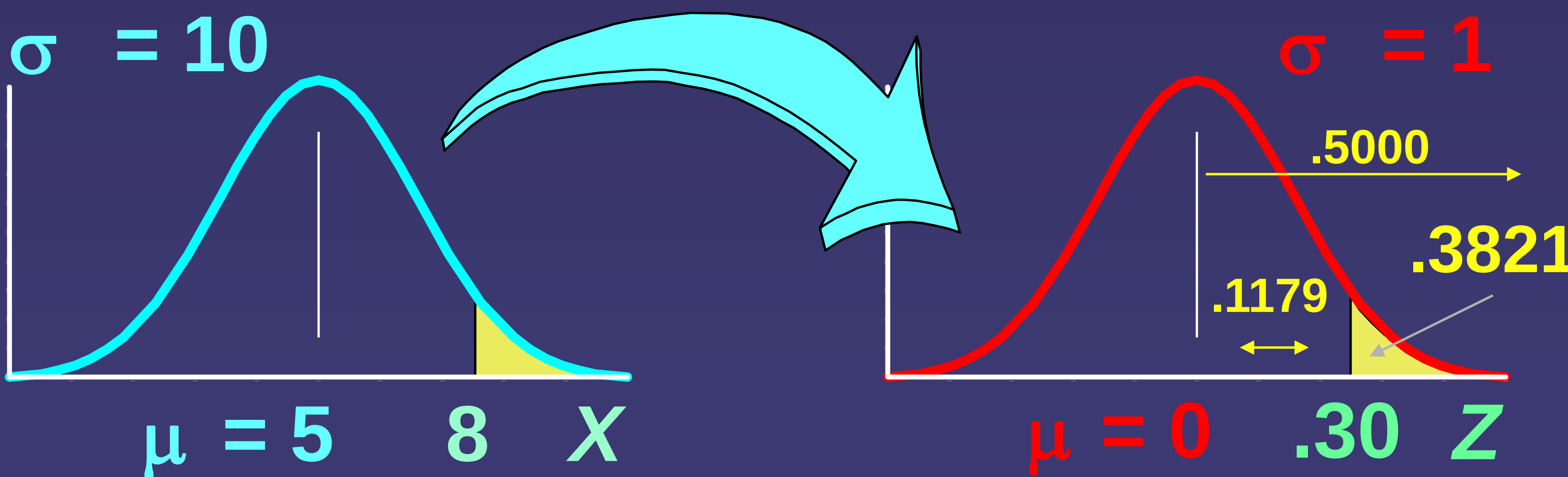
Shaded Area Exaggerated

Example: $P(X \geq 8) = .3821$

$$z = \frac{x - \mu}{\sigma} = \frac{8 - 5}{10} = .30$$

Normal Distribution

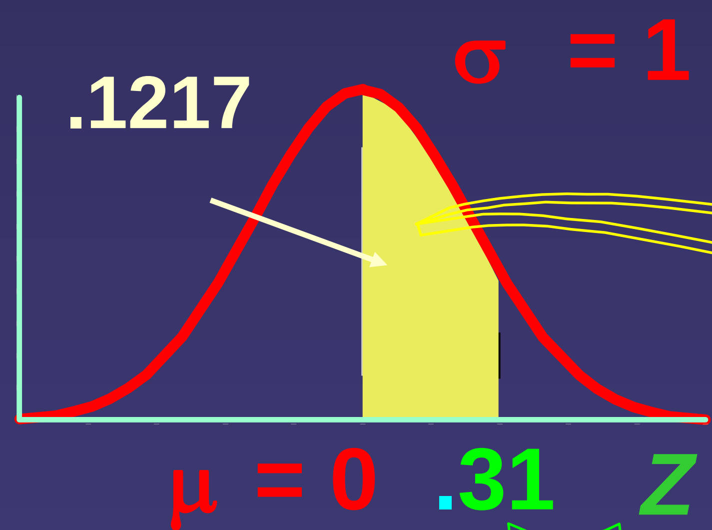
Standardized Normal Distribution



Shaded Area Exaggerated

Finding Z Values for Known Probabilities

What Is *Z* Given Probability = 0.1217?

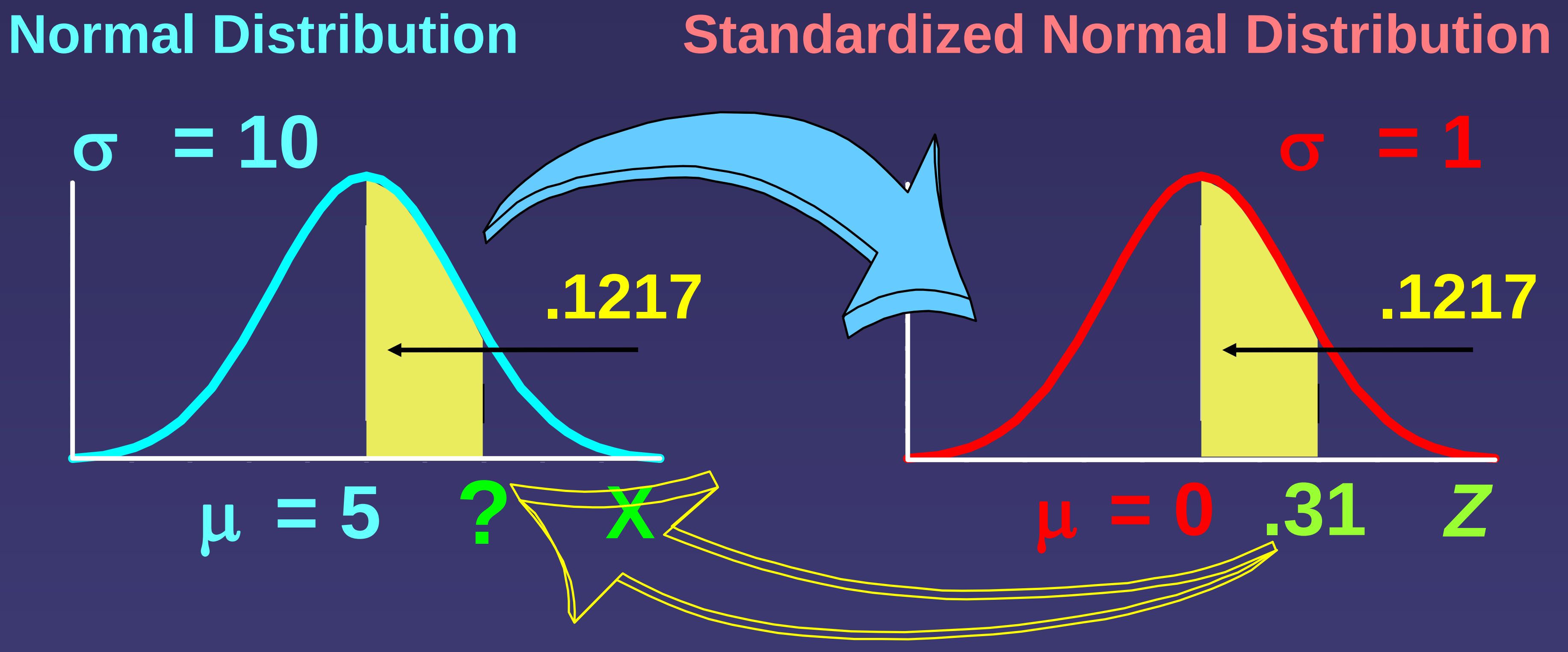


Shaded Area Exaggerated

Standardized Normal Probability Table (Portion)

Z	.00	.01	0.2
0.0	.0000	.0040	.0080
0.1	.0398	.0438	.0478
0.2	.0793	.0832	.0871
0.3	.1179	.1217	.1255

Recovering *X* Values for Known Probabilities



$$X = \mu + Z\sigma = 5 + (0.31)(10) = 8.1$$

Shaded Area Exaggerated

Normal Distribution

Applied to single variable continuous data
e.g. heights of plants, weights of lambs, lengths of time

- *Used to calculate the probability of occurrences less than, more than, between given values*

e.g. “the probability that the plants will be less than 70mm”,
“the probability that the lambs will be heavier than 70kg”,
“the probability that the time taken will be between 10 and 12 minutes”

Standard Normal tables give probabilities - you will need to be familiar with the Normal table and know how to use it.

First need to calculate how many standard deviations above (or below) the mean a particular value is, i.e., calculate the value of the "standard score" or "Z-score".

Use the following formula to convert a raw data value, , to a standard score, : X to a standard score:

Z = $\frac{x-\mu}{\sigma}$

eg. Suppose a particular population has m= 4 and σ = 2. Find the probability of a randomly selected value being greater than 6.

The Z score corresponding to X = 6 is $\frac{6-4}{2} = 1$. (Z=1 means that the value X = 6 is 1 standard deviation above the mean.) Now use standard normal tables to find P(Z>1) = 0.6587 *(more about this later)*.

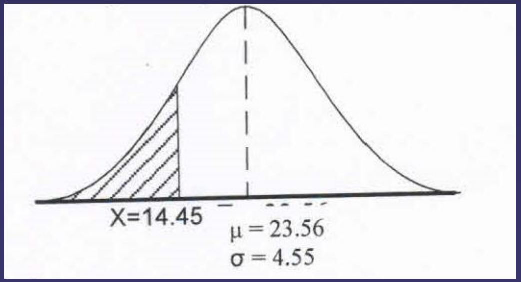
Process:

- Draw a diagram and label with given values i.e. μ(population mean), σ(pop. s.d.) and *X(raw score)*
- Shade area required as per question
- Convert raw score(*X*) to standard score (Z) using formula
- Use tables to find probability: eg. $p(0<Z<z)$
- Adjust this result to required probability

EXAMPLE

Wool fibre breaking strengths are normally distributed with mean μ = 23.56 Newtons and standard deviation σ = 4.55.

What proportion of fibres would have a breaking strength of 14.45 or less?



- Draw a diagram, lebel and shade area required:
- Convert raw score (***X***) to standard score (***Z***):

$$Z = \frac{14.45 - 23.56}{4.55} = -2.0$$

That is, the raw score of 14.45 is equivalent to a standard score of -2.0. It is negative because it is on the left hand side of the curve.

- **Use tables** to find the probability and adjust the result to required probability:

$p(X < 14.45) = p(Z<-2.0) = 0.5 – p(0<Z<2) = 0.5\text{-}0.4772 =$ **0.0228**

Binomial Distribution

- Applied to single variable discrete data where results are the numbers of “successful outcomes” in a given scenario.

e.g.: no. of times the lights are red in 20 sets of traffic lights, no. of students with green eyes in a class of 40, no. of plants with diseased leaves from a sample of 50 plants.

- Used to calculate the probability of occurrences *exactly, less than, more than, between* given values:

e.g. the “probability that the number of red lights will be exactly 5” . “probability that the number of green eyed students will be less than 7” . “probability that the no. of diseased plants will be more than 10”

Binomial Distribution:
Parameters, statistics and symbols involved are:

	Population Parameter symbol	Sample statistic symbol
Probability of Success	**π**	**P**
Sample size	**N**	**n**

Other symbols:

X , the number of successful outcomes wanted X ,

C_x^n: *the number of ways in which x successes can be chosen from sample size n.*

- Formula used:

$$P(\underbrace{X}_{\text{random variable }X} = \overbrace{x}^{\text{No. of successes}}) = \overbrace{{}^{n}C_{x}}^{\text{Combination of }x\text{ successes from }n\text{ trials}} . \underbrace{p}_{\text{probability of success}}{}^{x} . \underbrace{(1-p)}_{\text{probability of failure}}{}^{\overbrace{(n-x)}^{\text{number of failures}}}$$

Example

An automatic camera records the number of cars running a red light at an intersection (that is, the cars were going through when the red light was against the car). Analysis of the data shows that on average 15% of light changes record a car running a red light. Assume that the data has a binomial distribution. What is the probability that in 20 light changes there will be exactly three (3) cars running a red light?

Write out the key statistics from the information given:

$p = 0.15, n = 20, X = 3$

Apply the formula, substituting these values:

$P(X = 3) = 20\, C_3 \, x \, 0.15^3 \text{ x } 0.85^{17} = 0.243$

That is, the probability that in 20 light changes there will be three (3) cars running a red light is 0.24 (24%).

Poisson Distribution

This is often known as the *distribution of rare events*. Firstly, a Poisson process is where DISCRETE events occur in a CONTINUOUS, but finite interval of time or space. The following conditions must apply:

- For a small interval the probability of the event occurring is proportional to the size of the interval.
- The probability of more than one occurrence in the small interval is negligible (i.e. they are rare events). Events must not occur simultaneously
- Each occurrence must be independent of others and must be at random.
- The events are often defects, accidents or unusual natural happenings, such as earthquakes, where in theory there is no upper limit on the number of events. The interval is on some continuous measurement such as time, length or area.

The parameter for the Poisson distribution is λ (lambda). It is the average or mean number of occurrences over a given interval.

The probability function is:
Use e^x on calculator

$$p(x) = \frac{e^{-\lambda} . \lambda^x}{x!} \text{ for } x = 0, 1. 2, 3\ldots$$

For example: The average number of accidents at a level-crossing every year is 5. Calculate the probability that there are exactly 3 accidents there this year.

Solution: Here, $\lambda = 5$ and $x = 3$

$$P(X = 3) = \frac{e^{-\lambda}}{x!} \lambda^x = \frac{e^{-5}}{3!} 5^3 = 0.1404$$

That is, there is 14% chance that there will be exactly 3 accidents there this year.

Practice (Normal Distribution)

1. Potassium blood levels in healthy humans are normally distributed with a mean of 17.0 mg/100 ml, and standard deviation of 1.0 mg/100 ml. Elevated levels of potassium indicate an electrolyte balance problem, such as may be caused by Addison's disease. However, a test for potassium level should not cause too many "false positives".
What level of potassium should we use so that only 2.5 % of healthy individuals are classified as "abnormally high"?
2. For a particular type of wool the number of 'crimps per 10cm' follows a normal distribution with mean 15.1 and standard deviation 4.79. (a) What proportion of wool would have a 'crimp per 10 cm' measurement of 6 or less? (b) If more than 7% of the wool has a 'crimp per 10 cm' measurement of 6 or less, then the wool is unsatisfactory for a particular processing. Is the wool satisfactory for this processing?

3. The finish times for marathon runners during a race are normally distributed with a mean of 195 minutes and a standard deviation of 25 minutes.

a) What is the probability that a runner will complete the marathon within 3 hours?

b) Calculate to the nearest minute, the time by which the first 8% runners have completed the marathon.

c) What proportion of the runners will complete the marathon between 3 hours and 4 hours?

4. The download time of a resource web page is normally distributed with a mean of 6.5 seconds and a standard deviation of 2.3 seconds.

a) What proportion of page downloads take less than 5 seconds?

b) What is the probability that the download time will be between 4 and 10
seconds?

c) How many seconds will it take for 35% of the downloads to be completed?

Practice (Binomial Distribution)

1 *Executives in the New Zealand Forestry Industry claim that only 5% of all old sawmills sites contain soil residuals of dioxin (an additive previously used for anti-sap-stain treatment in wood) higher than the recommended level. If Environment Canterbury randomly selects 20 old saw mill sites for inspection, assuming that the executive claim is correct*:

a) Calculate the probability that less than 1 site exceeds the recommended level of dioxin.

b) Calculate the probability that less than or equal to 1 site exceed the recommended level of dioxin.

c) Calculate the probability that at most (i.e., maximum of) 2 sites exceed the recommended level of dioxin.

2 *Inland Revenue audits 5% of all companies every year. The companies selected for auditing in any one year are independent of the previous year's selection*.

a) What is the probability that the company ‘Ross Waste Disposal’ will be selected for auditing exactly twice in the next 5 years?

b) What is the probability that the company will be audited exactly twice in the next 2 years?

c) What is the exact probability that this company will be audited at least once in the next 4 years?

3 The probability that a driver must stop at any one traffic light coming to Lincoln University is 0.2. There are 15 sets of traffic lights on the journey.

a) What is the probability that a student must stop at exactly 2 of the 15 sets of traffic lights?

b) What is the probability that a student will be stopped at 1 or more of the 15 sets of traffic lights?

Practice (Poisson Distribution)

1. A radioactive source emits 4 particles on average during a five-second period.

a) Calculate the probability that it emits 3 particles during a 5-second period.

b) Calculate the probability that it emits at least one particle during a 5- second period.

c) During a *ten*-second period, what is the probability that 6 particles are emitted?

2. The number of typing mistakes made by a secretary has a Poisson distribution. The mistakes are made independently at an average rate of 1.65 per page. Find the probability that a *three*-page letter contains no mistakes.

3. A 5-litre bucket of water is taken from a swamp. The water contains 75 mosquito larvae. A 200mL flask of water is taken form the bucket for further analysis. What is (a) the expected number of larvae in the flask? and (b) the probability that the flask contains at least one mosquito lava?

Thank you