Game Theory

BY

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Introduction

Game theory is the study of mathematical models of strategic interaction among rational decision-makers.[1] It has applications in all fields of social science, as well as in logic, systems science and computer science. Originally, it addressed zero-sum games, in which each participant's gains or losses are exactly balanced by those of the other participants. Today, game theory applies to a wide range of behavioral relations, and is now an umbrella term for the science of logical decision making in humans, animals, and computers.

Strategic Behavior

- Decisions that take into account the predicted reactions of rival firms
  - Interdependence of outcomes
- Game Theory
  - Players
  - Strategies
  - Payoff matrix

- Types of Games
  - Zero-sum games
  - Nonzero-sum games
- Nash Equilibrium
  - Each player chooses a strategy that is optimal given the strategy of the other player
  - A strategy is dominant if it is optimal regardless of what the other player does

Advertising Example

		Firm B	
		Advertise	Don't Advertise
Firm A	Advertise	(4, 3)	(5, 1)
	Don't Advertise	(2, 5)	(3, 2)

What is the optimal strategy for Firm A if Firm B chooses to advertise?

		Firm B	
		Advertise	Don't Advertise
Firm A	Advertise	(4, 3)	(5, 1)
	Don't Advertise	(2, 5)	(3, 2)

What is the optimal strategy for Firm A if Firm B chooses to advertise?

If Firm A chooses to advertise, the payoff is 4. Otherwise, the payoff is 2. The optimal strategy is to advertise.

		Firm B	
		Advertise	Don't Advertise
Firm A	Advertise	(4, 3)	(5, 1)
	Don't Advertise	(2, 5)	(3, 2)

What is the optimal strategy for Firm A if Firm B chooses not to advertise?

		Firm B	
		Advertise	Don't Advertise
Firm A	Advertise	(4, 3)	(5, 1)
	Don't Advertise	(2, 5)	(3, 2)

What is the optimal strategy for Firm A if Firm B chooses not to advertise?

If Firm A chooses to advertise, the payoff is 5. Otherwise, the payoff is 3. Again, the optimal strategy is to advertise.

		Firm B	
		Advertise	Don't Advertise
Firm A	Advertise	(4, 3)	(5, 1)
	Don't Advertise	(2, 5)	(3, 2)

- Regardless of what Firm B decides to do, the optimal strategy for Firm A is to advertise. The dominant strategy for Firm A is to advertise.

		Firm B	
		Advertise	Don't Advertise
Firm A	Advertise	(4, 3)	(5, 1)
	Don't Advertise	(2, 5)	(3, 2)

What is the optimal strategy for Firm B if Firm A chooses to advertise?

		Firm B	
		Advertise	Don't Advertise
Firm A	Advertise	(4, 3)	(5, 1)
	Don't Advertise	(2, 5)	(3, 2)

What is the optimal strategy for Firm B if Firm A chooses to advertise?

If Firm B chooses to advertise, the payoff is 3. Otherwise, the payoff is 1. The optimal strategy is to advertise.

		Firm B	
		Advertise	Don't Advertise
Firm A	Advertise	(4, 3)	(5, 1)
	Don't Advertise	(2, 5)	(3, 2)

- The dominant strategy for Firm A is to advertise and the dominant strategy for Firm B is to advertise. The Nash equilibrium is for both firms to advertise.

		Firm B	
		Advertise	Don't Advertise
Firm A	Advertise	(4, 3)	(5, 1)
	Don't Advertise	(2, 5)	(3, 2)

Prisoners' Dilemma

- Two suspects are arrested for armed robbery. They are immediately separated. If convicted, they will get a term of 10 years in prison. However, the evidence is not sufficient to convict them of more than the crime of possessing stolen goods, which carries a sentence of only 1 year.
- The suspects are told the following: If you confess and your accomplice does not, you will go free. If you do not confess and your accomplice does, you will get 10 years in prison. If you both confess, you will both get 5 years in prison.

TABLE 10-3 Negative Payoff Matrix (Years of Detention) for Suspect A and Suspect B

		Individual B	
		Confess	Don't Confess
Individual A	Confess	(5, 5)	(0, 10)
	Don't Confess	(10, 0)	(1, 1)

Prisoners' Dilemma

- Payoff Matrix (negative values)

		Individual B	
		Confess	Don't Confess
Individual A	Confess	(5, 5)	(0, 10)
	Don't Confess	(10, 0)	(1, 1)

Dominant Strategy
Both Individuals Confess

(Nash Equilibrium)

		Individual B	
		Confess	Don't Confess
Individual A	Confess	(5, 5)	(0, 10)
	Don't Confess	(10, 0)	(1, 1)

TABLE 10-4 Payoff Matrix for a Pricing Game

		Firm B	
		Low Price	High Price
Firm A	Low Price	(2, 2)	(5, 1)
	High Price	(1, 5)	(3, 3)

- Application: Price Competition

		Firm B	
		Low Price	High Price
Firm A	Low Price	(2, 2)	(5, 1)
	High Price	(1, 5)	(3, 3)

Application: Price Competition

Dominant Strategy: Low Price

		Firm B	
		Low Price	High Price
Firm A	Low Price	(2, 2)	(5, 1)
	High Price	(1, 5)	(3, 3)

Prisoners' Dilemma

Application: Cartel Cheating

Dominant Strategy: Cheat

		Firm B	
		Cheat	Don't Cheat
Firm A	Cheat	(2, 2)	(5, 1)
	Don't Cheat	(1, 5)	(3, 3)

Extensions of Game Theory

- Repeated Games
  - Many consecutive moves and countermoves by each player
- Tit-for-Tat Strategy
  - Do to your opponent what your opponent has just done to you
- Tit-for-Tat Strategy
  - Stable set of players
  - Small number of players
  - Easy detection of cheating
  - Stable demand and cost conditions
  - Game repeated a large and uncertain number of times
- Threat Strategies
  - Credibility
  - Reputation
  - Commitment
  - Example: Entry deterrence

Sequential Games

- Sequence of moves by rivals
- Payoffs depend on entire sequence
- Decision trees
  - Decision nodes
  - Branches (alternatives)
- Solution by reverse induction
  - From final decision to first decision

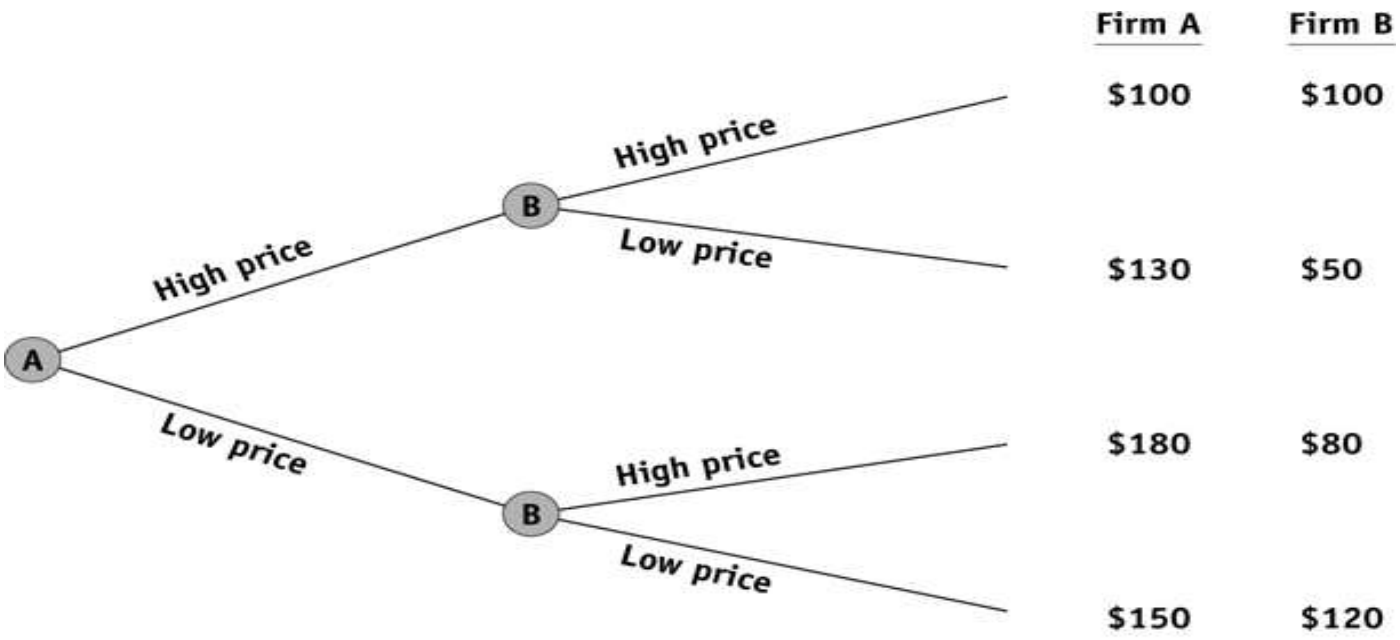
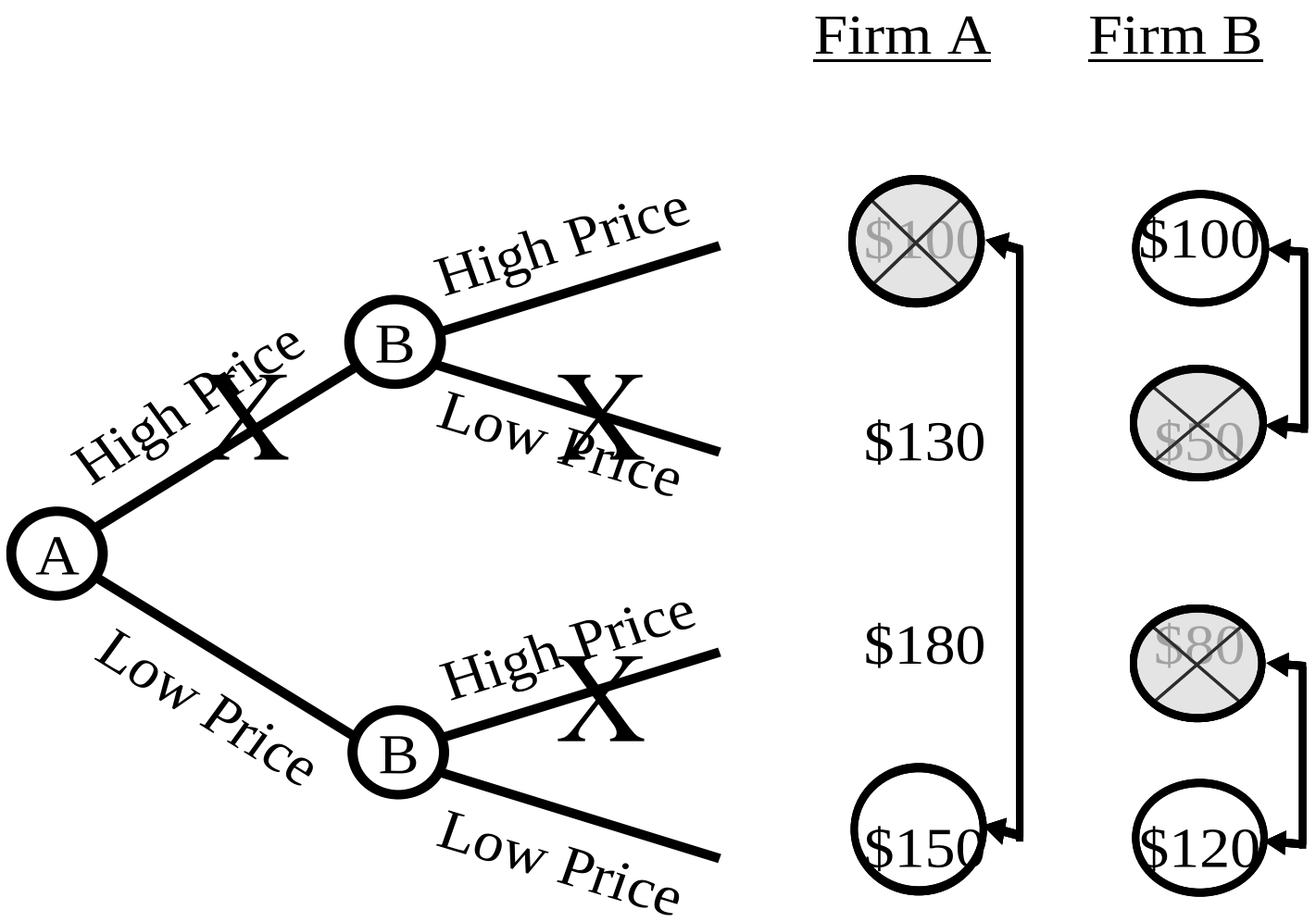


FIGURE 10-1 High-Price, Low-Price Strategy Game The strategy or highest payoff for firm A is to adopt a low-price strategy (the bottom branch node) rather than a high-price strategy (the top branch node). Given firm A's decision, firm B's best payoff is to also adopt a low-price strategy.

High-Price, Low-Price: Strategy Game



Solution: Both firms choose low price.