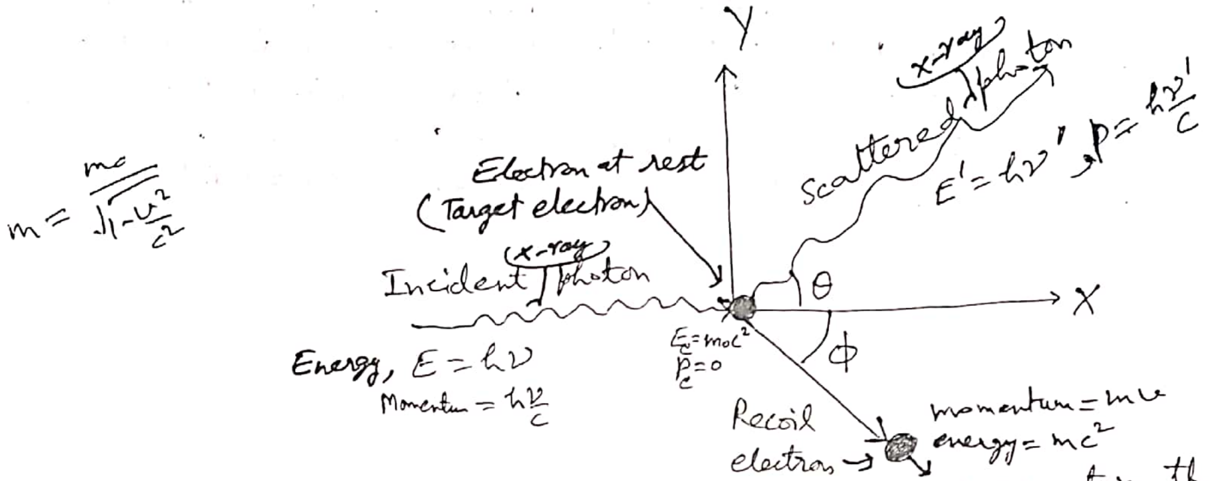
Dr. R. K. SHUKLA.
Department of Physics.
University of Lucknow.

Compton Effect

Chapter 3: COMPTON EFFECT A.H. Compton while making a spectroscopic study of the X-rays scattered by matter, discovered a new phenomenon known as Compton effect. ※ P.T.O. 49

The Compton Effect :- (Further confirmation of the photon model) Compton discovered that when X-rays of a sharply defined frequency were incident on a material of low atomic number like carbon, they suffered a change of frequency on scattering. The scattered beam contains two wavelengths. In addition to the expected incident wavelength, there exists a line of longer wavelength. The change of wavelength is due to loss of energy of the incident X-rays. This elastic interaction is known as Compton effect.



This effect was explained by Compton on the basis of quantum theory of radiation. The whole process is treated as a particle collision event between X-ray photon and a loosely bound electron of the scatterer. In this process, both momentum and energy are conserved. In the photon-electron collision, a portion of the energy of the photon is transferred to the electron. As a result, the X-ray proceeds with less than the original energy (and therefore has a lower frequency or a higher wavelength).

Imp

I

Theoretical Treatment of Compton Effect :- The incident photon with an energy $h\nu$ and momentum $\frac{h\nu}{c}$ strikes on electron at rest. The initial momentum of the electron is zero and its initial energy is only the rest mass energy, m_0c^2. The scattered photon of energy $h\nu'$ and momentum $\frac{h\nu'}{c}$ moves off in a direction inclined at an angle θ to the original direction. The electron acquires a momentum mv and moves at an angle ϕ to the original direction. The energy of the recoil electron is mc^2.

$p = mv$

According to the principle of conservation of energy i.e.
Kinetic energy before collision = Kinetic energy after collision

$$h\nu + m_0c^2 = h\nu' + mc^2 \quad \text{---(1)}$$

Considering the X and Y components of the momentum and applying the principle of conservation of momentum i.e.
Momentum before collision = Momentum after collision

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + mv\cos\phi \quad \text{---(2)}$$

and

$$0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi \quad \text{---(3)}$$

P.T.O.

* X-ray photon may collide with an electron and bounce off with reduced energy in another direction. This is analogous to the collision of two billiard balls. When x-rays were made to fall on a block of carbon (or of any other material of low atomic weight) and the wavelengths of X-rays scattered in different direction were measured with the help of a spectrograph. A comparison of these wavelengths with that of the incident beam showed that while some of the scattered x-rays had the same wavelength as the incident x-rays, others had somewhat greater wavelength than the incident or primary ones. The scattered x-rays photon having the same wavelength as the primary ones are called unmodified x-rays and is known as unmodified or coherent or classical scattering while those having greater wavelength than the primary one are called modified x-rays and this ~~phenomenon~~ is known as modified or incoherent scattering. The incoherent scattering is often called the Compton scattering. The increase in the wavelength of modified x-rays is found to increase with the increase in the angle. In such a case an electron is also ejected with an energy depending upon its direction. The electron is known as compton recoil electron.

From Eq. (2), $mvc\cos\phi = h(\nu - \nu'\cos\theta)$ --- (4)

From Eq. (3), $mvc\sin\phi = h\nu'\sin\theta$ --- (5)

Squaring and adding (4) & (5):

$$m^2v^2c^2 = h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2\cos^2\theta) + h^2\nu'^2\sin^2\theta$$

$$= h^2(\nu^2 - 2\nu\nu'\cos\theta) + h^2\nu'^2 = h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2) \quad \text{--- (6)}$$

From Eq. (1), $mc^2 = h(\nu - \nu') + m_oc^2$

Squaring both sides

$$\therefore m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu-\nu')m_oc^2 + m_o^2c^4 \quad \text{--- (7)}$$

Subtracting (6) from (7),

$$m^2c^2(c^2 - v^2) = -2h^2(1-\cos\theta)\nu\nu' + 2h(\nu-\nu')m_oc^2 + m_o^2c^4 \quad \text{--- (8)}$$

The value of $m^2c^2(c^2-v^2)$ can be obtained ~~from~~ by using the relativistic ~~formula~~ variation of mass with velocity and used the expression

$$m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

, ~~Squaring~~ where m_o is the rest mass of the electron and m is the mass of the recoil electron,

Squaring above relation, we get

$$m^2 = \frac{m_o^2}{1 - v^2/c^2} = \frac{m_o^2c^2}{c^2 - v^2}$$

$$\therefore m^2c^2(c^2 - v^2) = m_o^2c^4 \quad \text{--- (9)}$$

From (8) & (9),

$$m_o^2c^4 = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_oc^2 + m_o^2c^4$$

$$\therefore 2h(\nu-\nu')m_oc^2 = 2h^2\nu\nu'(1-\cos\theta)$$

or $$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_oc^2}(1-\cos\theta)$$

or $$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_oc^2}(1-\cos\theta)$$

or $$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_oc}(1-\cos\theta)$$

or $$\boxed{\lambda' - \lambda = \frac{h}{m_oc}(1-\cos\theta)} \quad \text{--- (10)}$$

$\therefore$ The change in wavelength $= \boxed{d\lambda = \frac{h}{m_oc}(1-\cos\theta)}$

This relation shows that $d\lambda$ is independent of the wavelength of the incident radiation as well as the nature of the scattering substance, but depends upon the angle of scattering.

The quantity $\frac{h}{m_oc}$ is referred to as Compton wavelength and is equal to 0.024 Å.

$d\lambda$ depends upon the angle of scattering only

Case 1 :- when $\theta = 0$, $\cos\theta = 1$ and hence $d\lambda = 0$.

Case 2 :- when $\theta = 90^\circ$, $\cos\theta = 0$ and hence

$$d\lambda = \frac{h}{m_0 c} = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(3 \times 10^{8})} \, m = 0.0243 \, \mathring{A}$$

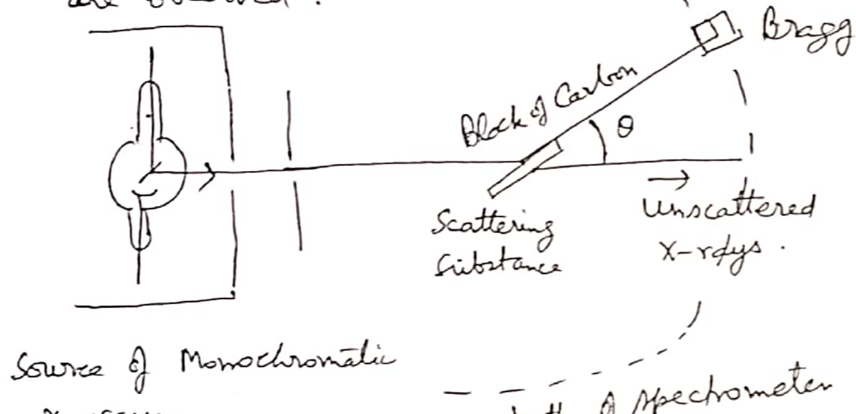
This is known as $\boxed{\text{Compton wavelength}}$ $(= 0.0243\,\mathring{A})$

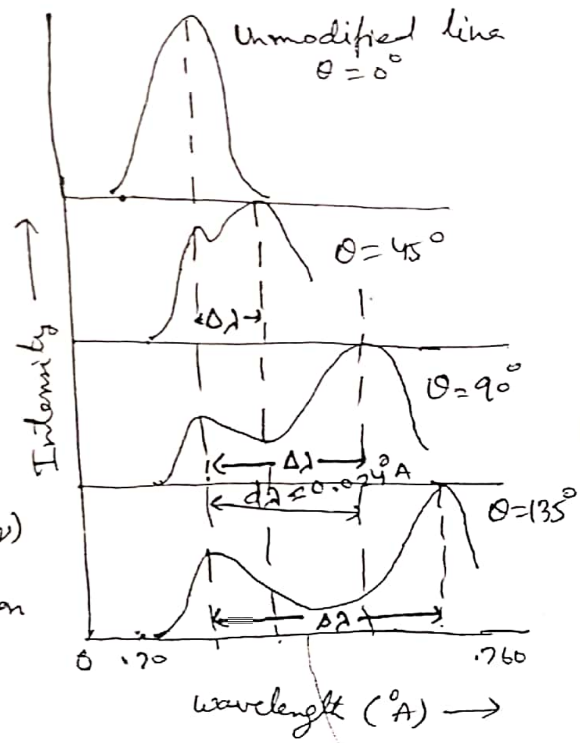
Case 3 :- when $\theta = 180^\circ$, $\cos\theta = -1$ and hence $d\lambda = \frac{2h}{m_0 c} = 0.0485\,\mathring{A}$.

$d\lambda$ has the maximum value at $\theta = 180^\circ$.

* P.T.O.

Experimental verification of Compton Effect :- Monochromatic X-rays of wavelength λ are allowed to fall on a scattering material like a small block of carbon. The scattered X-rays are received by a Bragg spectrometer and their wavelength is determined. The spectrometer can freely swing in an arc about the scatterer. The wavelength of the scattered X-rays is measured for different values of the scattering angle. The experimental results obtained by Compton are shown in figure. In the scattered radiation in addition to the incident wavelength (λ), there exists a line of longer wavelength (λ'). The "Compton shift" $d\lambda$ is found to vary with the angle at which the scattered rays are observed.





It can be shown that

$$h\nu' = \frac{m_0 c^2}{(1-\cos\theta) + (m_0 c^2 / h\nu)}$$

The K.E. of the recoiling electron $= T = \frac{h\nu(1-\cos\theta)}{(1-\cos\theta) + (m_0 c^2/h\nu)}$

The relationship between the scattering angles of the electron and photon is :

$$\cot\phi = \left(1 + \frac{h\nu}{m_0 c^2}\right)\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

* **Distinction between Compton wavelength and Compton Shift :-** Compton wavelength is a constant quantity and its value equal to $0.02426 \mathring{A}$. It is the compton shift for an angle of scattering $\theta = 90^\circ$. compton shift is a variable quantity depending upon the angle of scattering. compton shift $\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$. As $\frac{h}{m_0 c} = \lambda_c$ is the compton wavelength.
Compton shift = Compton wavelength. $(1-\cos\theta)$

or $\quad \Delta\lambda = \lambda_c (1-\cos\theta)$

As the compton shift is proportional to $(1-\cos\theta)$, it increases from zero for $\theta=0$ to $\Delta\lambda = 2\lambda_c$ for $\theta = \pi$.

These curve indicate that

1) At $\theta = 0°$, there is only one spectral line corresponding to wavelength λ of incident x-rays. This belongs to normal scattering.

2) With increase in θ two spectral lines appears of which one lies at the same position as for angle $\theta = 0°$ (this corresponds to normal scattering). The other spectral line lies at a greater wavelength λ, than that of 1st line. This corresponds to compton scattering.

3) At $\theta = 90°$, the separation between the two spectral lines $d\lambda$ is found to be 0.024 °A, which is in agreement with compton formula.

$$d\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

Hence Compton's equation is verified.

Compton found an exact agreement between the wavelength shift $\Delta\lambda$ calculated from the theory and the values measured experimentally. It constitutes a very strong evidence in support of the quantum theory of radiation.

The above theory explains the observed shift in wavelength, but does not account for the presence of scattered x-rays having initial wavelength, at each angle. In deriving the equation it was assumed that the electrons are free. This assumption is quite reasonable as many electrons in matter are only loosely bound to their parent atom and act as free electrons. Some electrons are very tightly bound and when struck by a photon are not detached from the atom but remain bound and the entire atom recoils. In this event the rest mass of the electron m_0 in above equation must be replaced by the mass of the atom which is several thousand times greater than that of an electron. The calculated value of the resulting compton shift $\Delta\lambda$ will then be too small to be detectable.

Imp

Significance of Compton Effect:- The compton effect clearly shows the particle like nature of electro-magnetic radiation. Not only a precise quantum of energy $h\nu$ can be assigned to a photon but also a precise quantum of momentum $\frac{h\nu}{c} = \frac{h}{\lambda}$. The total momentum of a monochromatic radiation cannot have any value but is only an exact multiple of the linear momentum of a single photon. In other words the momentum as well as the energy of the electromagnetic radiation is quantised. The phenomenon of Compton effect is thus due to the elastic collision between two particles, the photon of the incident radiation and electron of the scatterer.

The significance of compton effect lies in the fact that it has put the quantum theory of radiation on a very sound footing.

55 + T

<u>Kinetic Energy of Recoiling Electron during Compton effect</u> :- During Compton effect a part of energy of incident x-ray photon is transferred to electron. Hence kinetic energy of recoiling electron

$$T = h\nu - h\nu' \qquad \text{---(1)}$$

where ν' and ν are frequencies of scattered and incident x-ray photon respectively.

Using $c = \nu\lambda$, we can write

$$T = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = hc\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) \qquad \text{---(2)}$$

$$\boxed{T = \frac{hc\,\Delta\lambda}{\lambda(\lambda+\Delta\lambda)}} \quad ; \lambda'-\lambda = \Delta\lambda \quad ; \therefore \lambda' = \Delta\lambda + \lambda$$

Now Compton shift

$$\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

or

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0 c^2}(1-\cos\theta)$$

$$= \frac{1}{\nu}\left[1 + \frac{h\nu}{m_0 c^2}(1-\cos\theta)\right]$$

$$\nu' = \frac{\nu}{1 + \frac{h\nu}{m_0 c^2}(1-\cos\theta)} \qquad \text{---(3)}$$

With this substitution in Eq.(1), we get K.E. of recoiling electron.

$$T = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1-\cos\theta)}$$

$$= h\nu\left[\frac{h\nu}{m_0 c^2}(1-\cos\theta)\right] \Big/ \left[1 + \frac{h\nu}{m_0 c^2}(1-\cos\theta)\right]$$

The K.E. of recoiling electron.

$$\boxed{T = h\nu\,(1-\cos\theta) \Big/ \left[\frac{m_0 c^2}{h\nu} + (1-\cos\theta)\right]} \qquad \text{---(4)}$$

<u>Relation between Scattering angles of electron and photon</u> :-
~~from Eq(4) and (5) we have~~

$$m v c \cos\phi = h(\nu - \nu'\cos\theta)$$

$$m v c \sin\phi = h\nu' \sin\theta$$

from Conservation of momentum in the expression for Compton shift

we have

$$\cot\phi = \frac{h(\nu - \nu'\cos\theta)}{h\nu'\sin\theta} \qquad \text{-----(5)}$$

Putting ν' from Eq.(3) into Eq.(5) we get

$$\cot\phi = \frac{\nu - \left[\frac{\nu}{1+\frac{h\nu}{m_0c^2}(1-\cos\theta)}\cos\theta\right]}{\frac{\nu\sin\theta}{1+\frac{h\nu}{m_0c^2}(1-\cos\theta)}}$$

$$= \frac{\nu + \frac{h\nu^2}{m_0c^2}(1-\cos\theta) - \nu\cos\theta}{\nu\sin\theta}$$

$$= \frac{\nu(1-\cos\theta) + \frac{h\nu^2}{m_0c^2}(1-\cos\theta)}{\nu\sin\theta}$$

$$= \frac{\nu(1-\cos\theta)\left[1+\frac{h\nu}{m_0c^2}\right]}{\nu\sin\theta}$$

$\sin 2\theta = 2\sin\theta\cos\theta$

or $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ —(a)

$\cos 2\theta = 1-2\sin^2\theta$

or $\cos\theta = 1-2\sin^2\frac{\theta}{2}$

$2\sin^2\frac{\theta}{2} = 1-\cos\theta$ —(b)

$\therefore \frac{1-\cos\theta}{\sin\theta} = \frac{2\sin^2\theta/2}{2\sin\frac{\theta}{2}\cos\theta/2}$

$= \frac{\sin\theta/2}{\cos\theta/2} = \tan\frac{\theta}{2}$

$$\boxed{\cot\phi = \frac{(1-\cos\theta)\left(1+\frac{h\nu}{m_0c^2}\right)}{\sin\theta}}$$

or $\tan\phi = \frac{\sin\theta}{(1-\cos\theta)\left(1+\frac{h\nu}{m_0c^2}\right)}$

$= \frac{\cot\theta/2}{\left(1+\frac{h\nu}{m_0c^2}\right)}$

This is the relation between scattering angles of electron and photon.

$\phi \rightarrow$ scattering angle of electron

$\theta \rightarrow$ " " " photon.

Electron cannot be scattered at greater than 90° :- The angle of scattering for the electron ϕ is related to angle of scattering for the photon θ as under

$$\tan\phi = \frac{\cot\theta/2}{1+\frac{h\nu}{m_0c^2}}$$

Now $1+\frac{h\nu}{m_0c^2}$ = a constant $= k$ (suppose)

$\therefore \; k\tan\phi = \cot\frac{\theta}{2}$

The maximum value of $\cot\frac{\theta}{2} = \infty$

Hence the maximum value of $\tan\phi = \infty$

or $\phi = \frac{\pi}{2}$

Imp || In other words, the electron cannot be scattered at an angle greater than 90°.

Scanned with CamScanner

Compton Effect (not observed) with visible light :- During the Collision, when masses of colliding bodies are equal, maximum energy is transferred from one body to the other. As the moving mass of X-ray photon is nearly equal to rest mass of electron, therefore when they collide there is maximum change in the energy of photon associated with maximum change in the wavelength.

In case of visible light mass of photon is much less than that of electron, therefore during their collision, negligible energy of photon is transferred to electron. Hence change in wavelength is too small to be measured, because even with x-rays maximum change in wavelength is only 0.048 °A.

* P.T.O.

Thus Compton effect cannot be observed with visible radiation.

~~Maximum % change in~~ we know $\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$; $\theta = 180°$

$= 0.048\ °A$

$\lambda = \frac{h}{mv}$

In Compton effect:
Maximum percentage change in λ is ~~Compton effect~~

$$= \frac{\Delta\lambda_{max}}{\lambda} \times 100\ \%$$

$$= \frac{.048}{\lambda} \times 100\ \%$$

$$= \frac{4.8}{\lambda}\ \%.$$

For visible light ($\lambda = 5000\ °A$)

maximum % change in λ $= \frac{4.8}{5000}\ \% = 0.00096\ \%$.

The maximum change in wavelength of visible light is 0.00096 % which is insignificant. Therefore Compton effect can't be observed with visible light. To detect a change in wavelength of this order, the optical instrument should have resolving power $\frac{\lambda}{\Delta\lambda} = \frac{5000\ A°}{1.1 \times 10^{-6}\ A°} = 5 \times 10^{9}$ which is much beyond the limit of resolution of these instruments. Hence Compton effect cannot be observed with visible or ultra violet light.

→ Compton effect with X-ray ($\lambda = 10\ A°$)

Max % change is $= \frac{4.8}{\lambda}\ \% = \frac{4.8}{10}\ \% = .48\ \%$

For X-rays, the maximum % change in λ is 0.48 % which is quite significant compare to the wavelength of X-ray. Therefore Compton effect can be observed with X-rays only.

Mass of photon = (Zero rest mass) ~~Photon rest mass $m_p = 1.672614 \times 10^{-27}$ kg~~

Mass of electron = electronic rest mass $m_e = 9.109558 \times 10^{-31}$ kg.

P.T.O.

	Wavelength (m)	Frequency (Hz)	Energy (J)
Radio	$>1\times10^{-1}$	$<3\times10^{9}$	$<2\times10^{-24}$
Microwave	$1\times10^{-3}-1\times10^{-1}$	$3\times10^{9}-3\times10^{11}$	$2\times10^{-24}-2\times10^{-22}$
Infrared	$7\times10^{-7}-1\times10^{-3}$	$3\times10^{11}-4\times10^{14}$	$2\times10^{-22}-3\times10^{-19}$
Optical (visible)	$4\times10^{-7}-7\times10^{-7}$	$4\times10^{14}-7.5\times10^{14}$	$3\times10^{-19}-5\times10^{-19}$
UV	$1\times10^{-8}-4\times10^{-7}$	$7.5\times10^{14}-3\times10^{16}$	$5\times10^{-19}-2\times10^{-17}$
X-ray	$1\times10^{-11}-1\times10^{-8}$	$3\times10^{16}-3\times10^{19}$	$2\times10^{-17}-2\times10^{-14}$
Gamma Ray	$<1\times10^{-11}$	$>3\times10^{19}$	$>2\times10^{-14}$

* For a measurable Compton's effect, the frequency ν should be in the X-ray or in the γ-ray region (for x-rays $\lambda \leq 1$ Å and $h\nu \geq 10^4$ eV). For such high energy photons, the velocity imparted to the electron is comparable to the speed of light and one must use proper relativistic expression.
* In the derivation of the Compton shift, we have assumed that the electron is free although we know that the electrons are bound to the atoms. The assumption of a free electron is justified because the binding energy ($\approx$ few eV) is usually very much smaller in comparison to the photon energy (≥ 1000 eV).

X-ray wavelength

$1\times10^{-11}\ m = 10\times10^{-10}\ m = 10$ Å

$1\times10^{-8}\ m = .01\times10^{-10}\ m = .01$ Å

---

* The energy of a visible light photon say of wavelength $\lambda = 5000\times10^{-10}$ m is given by

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.625\times10^{-34}\times3\times10^{8}}{5000\times10^{-10}\times1.6\times10^{-19}} = 2.48\ eV$$

whereas the energy of an x-ray photon, say of wavelength 1 Å will be more than 10^3 times the above value. The energy of an ultraviolet photon is of the order of 10 eV.

The binding energy of the electron in the atom is of the order of 10 eV. The binding energy of the electron in the hydrogen atom ($Z=1$) is

$$E_b = \frac{2\pi^2 k^2 Z^2 m e^4}{h^2}$$

where k is a constant the value of which in C.G.S. system is 1 and in S.I. units is $\frac{1}{4\pi\epsilon_0} = 9\times10^{9}\ Nm^2/C^2$.

$$\therefore E_b = \frac{2\pi^2\times(9\times10^{9})^2\times(9.1\times10^{-31})\times(1.6\times10^{-19})^4}{(6.62\times10^{-34})^2\times(1.6\times10^{-19})} = 13.6\ eV$$

Hence these electrons can be treated as free when X-rays or gamma rays are incident but these cannot be treated as free for visible or ultra-violet light. Thus for visible or ultra-violet light the whole of the atom takes part in Compton scattering and not the electron alone.

Presence of Unmodified component in Compton Effect:- In deriving the relation for compton shift it is assumed that the electron is free and stationary. But all the electrons do not satisfy this condition. When compton scattering takes place from an electron tightly bound to the atom, the collision is in fact with the atom to which the electron is tightly bound. As the compton wavelength $\lambda_c = \frac{h}{m_0 c}$ is inversely proportional to the mass of the scattering particle its magnitude for an atom is nearly 10^{-4} times the value for an electron, as calculated below for an atom of carbon of mass $m_0 = 2\times10^{-26}$ kg for which $\lambda_c = \frac{h}{m_0 c} = \frac{6.625\times10^{-34}}{2\times10^{-26}\times3\times10^{8}} = 1.1\times10^{-16}\,m = 1.1\times10^{-6}\,Å$

As compared to the compton wavelength for electron i.e. $2.4\times10^{-2}\,Å$ (.024 Å) this value is nearly 10^{-4} times.

Such a small change in wavelength cannot be resolved by an X-ray spectrometer and hence such photons are recorded without change of wavelength for all values of the scattering angle. This explains the presence of unmodified component of X-ray in the study of compton effect.

Explanation of Unshifted Line in the Compton Effect:- In the theory of Compton effect, it was assumed that electron participating in the collision is free. In actual practice it is bound to the atom more or less tightly and in this way some amount of energy is needed to shake it free. This electron behaves according to the compton theory. But a tightly bound electron remains unaffected when photon strikes it and the photon is simply scattered with no loss of energy (i.e. its wavelength does not change). Thus unshifted line (unchanged wavelength component) is also observed in the scattered radiations.

Imp Comparison of photoelectric effect and Compton effect:- Both of these effects arise on account of interaction of photon (i.e. radiation) with the electron of the matter. But there is an important difference between the two as follows: In photoelectric effect, the electron of target atom under consideration is bound one and the impinging photon transfers its entire energy to the electron and consequently photon completely dies out i.e. disappears. In Compton effect the electron of the target atom is free and after interaction photon still exists, though of lesser energy i.e. larger wavelength. Thus, only a part of energy of the striking (or incident photon) transfers to the electron.

⇒ Compton Scattering is often referred to as incoherent scattering or elastic collision

⇒ Compton effect provides an important research tool in some branches of medicines in molecular chemistry, solid-state physics, high-energy electron accelerators, charged particle storage rings, high-resolution silicon and germanium semiconductor radiation detectors.

Q:- X-rays of wavelength 1.0 Å are scattered by a carbon block. The scattered radiations are observed at 60°, 90° and 180°. Find (1) Compton shift. (2) energy imparted to the recoil electron.

Ans:- Compton shift $\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$

$= \lambda_c (1-\cos\theta)$ where $\lambda_c = \frac{h}{m_0 c} = 0.0243$ Å

λ_c Compton wavelength

$\Delta\lambda_{60°} = \lambda_c(1-\cos 60°) = .5\lambda_c = 0.012$ Å

$\Delta\lambda_{90°} = \lambda_c(1-\cos 90°) = \lambda_c = 0.0243$ Å

$\Delta\lambda_{180°} = \lambda_c(1-\cos 180°) = 2\lambda_c = 0.048$ Å

Kinetic energy imparted to the electron

$$T = E - E' = ch\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{ch}{\lambda}\left(\frac{\Delta\lambda}{\lambda+\Delta\lambda}\right)$$

$\lambda = 1.0$ Å, $\Delta\lambda_{60°} = 0.012$ Å

$$\therefore T_{60} = \frac{12400\ eV}{1\ Å}\left(\frac{0.012}{1+0.012}\right) = 12400\ eV\left(\frac{0.012}{1.012}\right)$$

$= 147\ eV$ Ans

$$T = \hbar\omega - \hbar\omega' = \frac{2\pi\hbar c}{\lambda} - \frac{2\pi\hbar c}{\lambda'} = 2\pi\hbar c\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right) = hc\left[\frac{\Delta\lambda}{\lambda(\lambda+\Delta\lambda)}\right]$$

Q:- An x-ray photon is found to have doubled its wavelength on being scattered by 90°, find the energy and wavelength of incident photon.

A :- ~~Here Compton shift is equal to initial wavelength λ,~~

$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos 90°)$

$2\lambda - \lambda = \lambda = \frac{h}{m_0 c}(1-\cos 90°)$

$$\therefore \lambda = \frac{h}{m_0 c}(1-\cos 90°) = \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}} = 0.024\ Å$$

Energy of incident photon,

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{0.024\times10^{-10}} = 8.275\times10^{-14}\ \text{Joules}.$$