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Wave Particle Duality

Wave – Particle Duality

The old quantum theory has explained some of the phenomena like photo-electric effect, black body spectrum, Compton effect, emission of spectral lines etc., by considering particle nature of a wave. However, the difficulty with quantum theory was how to explain the optical effects i.e. phenomena like interference, diffraction, polarization etc, which are exhibited by the same light. For this, Einstein suggested the light waves merely serves as guide for photons, each carrying energy $h\nu$. This idea was difficult to accept at that time because particle nature and wave nature are apparently different for the reason the particle has a definite mass, velocity, momentum etc. in the space while a wave is associated with wavelength, frequency, phase, amplitude, intensity etc.

To resolve this, Victor de-Broglie, in 1924 suggested the duality of matter for which he was awarded Nobel Prize in 1929. He proposed that the matter possess 'wave' as well as 'particle' characteristics. The motion of electrons in atom is guided by peculiar kind of waves similar to the electromagnetic waves, which serves to guide photons. According to him, electrons as well as protons which ordinarily behave like particle under certain conditions behave like a train a waves. The wavelength of these waves depends upon the momentum which in turn depends upon the mass and velocity of the particles. This idea was immediately accepted and latter it was extended by Heisenberg, Schrodinger, Dirac, Max Born and others. This led to the foundation of new branch called 'wave mechanics' or 'New quantum mechanics'. The waves associated with a moving particle are called matter waves.

Matter waves of de-Broglie :-

The Wave Particle Duality :- In explaining phenomena like reflection, refraction, interference, polarisation and diffraction etc, one has to consider wave nature of light. But in explaining P.E. effect, compton effect, zeeman effect etc, one has to consider particle nature of (light) e.m. radiations. This means light as well as other e.m. radiation have dual nature. To overcome this controversy deBroglie proposed that light has dual nature. ~~Light particles and~~ Light is constituted by photons which can be treated like particles and waves, ~~are associated with photons~~. ~~He~~ de-Broglie (later extended his idea and) suggested that whenever any material particle moves, waves are associated with it called as "matter waves" or "deBroglie waves".

* **Wavelength of Matter waves of de-Broglie waves :-** For photon, we know, Energy is given by

$$E = h\nu = mc^2 \qquad ; c = \lambda\nu$$

$$h\frac{c}{\lambda} = mc^2$$

$$\text{or} \quad \lambda = \frac{h}{mc}$$

$$\boxed{\lambda = \frac{h}{p}} \quad ; p \text{ is the momentum of photon}$$

For any particle (micro or macro) moving with velocity v, the wavelength of matter waves associated with it is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

This equation shows that the greater the particle momentum, the smaller is its wavelength.

In this equation relativistic mass $\frac{m_0}{\sqrt{1-v^2/c^2}}$ must be used in place of m.

Here it ~~must be made~~ is clear that just as light sometimes behaves as of wave nature and at other times as of particle nature but never as waves and particle both, the wave and particle behaviour of moving bodies also never appear together in the same experiment. ~~All that~~ we can say ~~is~~ that at certain times a moving body acts as a particle and at other times as a wave.

<u>De-Broglie Hypothesis for matter waves</u>:– According to quantum theory, radiation of frequency ν consists of quanta or photons, each of energy $E = h\nu$, where h is plank's constant the value of which is 6.62×10^{-34} Joule-sec.

The equivalent mass of the photon, $m = \frac{E}{c^2} = \frac{h\nu}{c^2}$

Since the speed of the photon in free space is c, the equivalent momentum of photon. $p = mc = \frac{E}{c^2} \cdot c = \frac{h\nu}{c} = \frac{h}{\lambda}$ where λ is the wavelength of the radiation frequency ν.

By analogy with this, de-Broglie suggested that a moving particle is associated with a wave. The frequency of the wave is taken to be $\boxed{\nu = \frac{E}{h} = \frac{mc^2}{h}}$ where m is the mass of the particle. The wavelength of the wave

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

where v is the velocity of the particle.

Thus, a particle of mass 'm' moving with a velocity v has an associated wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

This is known as de-Broglie wave equation and λ is called de-Broglie wavelength. The associated wave is termed as matter, guide, pilot or de-Broglie wave. <u>de-Broglie wave velocity</u> or <u>phase velocity of the wave</u> is given by

$$v_p = \nu\lambda = \frac{mc^2}{h} \cdot \frac{h}{mv} = \frac{c^2}{v}$$

Since the particle velocity v, must be less than the velocity of light c, de-Broglie waves travel faster than velocity of light.

The wave vector $\vec{k}$ for de-Broglie wave is given by

$$|\vec{k}| = k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/p} = \frac{p}{h/2\pi} = \frac{p}{\hbar}$$

where $$\hbar = \frac{h}{2\pi}$$

Absence of Matter waves in macroscopic world :- In our general observation we usually come across macroscopic objects. Let us calculate the de Broglie wavelength of a fast moving automobile whose speed is 100 km/hr and of an electron moving with a speed of 10^7 m/sec. This would give us an idea of the relative magnitude of the wavelength of these waves under different circumstances and would help us to know that the wave aspects cannot always be expected in the behaviour of a material body.

Suppose the automobile weights 2000 kgm.

$$v = \frac{100 \times 1000}{3600} \text{ m/sec} = 27.8 \text{ m/sec}$$

Now $v = 100 \text{ km/hr} = 27.8 \text{ m/sec}$

$$\lambda = \frac{h}{mv} = \frac{6.625 \times 10^{-34}}{2000 \times 27.8} = 119.15 \times 10^{-40} \text{ m}$$

This is so small as compared to the dimensions of the automobile that ~~we can expect~~ so λ no wave aspects in its behaviour.

For the electron, ~~[illegible]~~,

$$\lambda = \frac{h}{mv} = \frac{6.625 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{7}}$$

$$= 7.28 \times 10^{-11} \text{ m}$$

which is of the order of the magnitude of atomic dimensions. ~~It is~~ ~~therefore not surprising that~~ So the electrons with speed of about 10^7 m/sec exhibit wave characteristic in as much as they are diffracted when incident upon crystals.

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Derivation an equation for wavelength of matter waves :- In matter waves, a particle in motion is treated as a source of stationary waves. Let ψ be a variable quantity known as wave function gives matter waves. So we can write at any instant t_0.

$$\psi = \psi_0 \sin 2\pi \nu_0 t_0 \quad \text{---(1)}$$

$y = a \sin \omega t$

where ψ_0 is amplitude of vibration and ν_0 is the frequency of vibration for any observer at rest at point (x_0, y_0, z_0).

Let particle moves along +ve x-axis with velocity v. Then at any time t at point x

$$\psi = \psi_0 \sin 2\pi \nu_0 t$$

where t_0 and t are related as, from Lorentz transformation

$$t_0 = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}} \quad \text{---(2)}$$

putting this value in Eq.(1), we get

$$\psi = \psi_0 \sin 2\pi \nu_0 \left(\frac{t - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}} \right) \quad \text{---(3)}$$

The standard equation of wave is

$$\psi = A \sin \frac{2\pi}{\lambda} (ut - x)$$

$$= A \sin 2\pi \left(\frac{u}{\lambda}\right)\left(t - \frac{x}{u}\right)$$

$$= A \sin 2\pi \nu \left(t - \frac{x}{u}\right) \quad \text{---(4)}$$

$v = \nu \lambda$

$\nu = \frac{v}{\lambda}$

Comparing equation (3) & (4), we get

$$\nu = \frac{\nu_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad u = \frac{c^2}{v}$$

v = velocity of particle

u = velocity of wave

$v = n\lambda$

Also

$$E = h\nu_0 = m_0 c^2$$

$$\nu_0 = \frac{m_0 c^2}{h}$$

So

$$\nu = \frac{m_0 c^2}{h\sqrt{1 - v^2/c^2}} = \frac{mc^2}{h} \quad \text{where } m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Wavelength of matter wave

$$\lambda = \frac{u}{\nu} = \frac{c^2/v}{mc^2/h} = \frac{h}{mv}$$

$u = \nu \lambda$

$\lambda = \frac{u}{\nu}$

$$\boxed{\lambda = \frac{h}{p}}$$

Proof.

p = momentum of particle

de Broglie Wavelength of Charged particles / Uncharged particles:

When a particle or object moves, matter waves are associated. There may be two types of motion.

(i) Non-relativistic motion:- when velocity of object is small compared to speed of light or a charged particle is put under potential difference very less than million volts (MV) or 10^6 volts its motion is non relativistic.

(ii) Relativistic motion:- when velocity of an object is of the order of speed of light or energy is of the order of MeV or a charged particle is subjected to a potential difference of the order of M.V.

For Non-relativistic motion:-

(a) Uncharged particle (non-relativistic motion):- let E_k be the K.E. of a particle of mass 'm' moving with non relativistic speed v, then

$$E_k = \frac{1}{2}mv^2$$

or $$m^2v^2 = 2mE_k$$

or $$p = mv = \sqrt{2mE_k}$$

de Broglie wavelength $$\lambda = \frac{h}{p}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE_k}}}$$

(b) Charged particle accelerated through a p.d. of V volts:-

$$\frac{1}{2}mv^2 = qV$$

$$\frac{1}{2}\frac{m^2v^2}{m} = qV$$

$$\frac{1}{2}\frac{p^2}{m} = qV$$

$$p = \sqrt{2mqV}$$

de Broglie wavelength
$$\lambda = \frac{h}{p}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mqV}}}$$

For electron :- $m_0 = 9.1\times10^{-31}$ kg, $q = 1.6\times10^{-19}$ C

$$\lambda = \frac{h}{\sqrt{2mqV}} = \frac{6.67\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\,V}}$$

$$\boxed{\lambda = \sqrt{\frac{150}{V}}\ \text{Å}}$$

$$\boxed{\lambda = \frac{12.27}{\sqrt{V}}}\ \text{Å}$$

This is the direct formula to calculate de Broglie wavelength of an electron accelerated.

(c) Gas molecules at temperature T :- The K.E. per molecule of a gas at temperature T is given by $\frac{3}{2}kT$. So.

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$\frac{1}{2}\frac{m^2v^2}{m} = \frac{3}{2}kT$$

$$m^2v^2 = 3mkT$$

$$p^2 = 3mkT$$

de Broglie wavelength

$$\lambda = \frac{h}{p}$$

$$\boxed{\lambda = \frac{h}{\sqrt{3mkT}}}$$

where T is temperature in ^0K.

m is mass of a mole. of gas $= \frac{\text{molecular weight}}{N \text{ (avogadro number)}}$

(2) For Relativistic motion :-

(a) Uncharged particle :- K.E., $T = (m - m_o)c^2$ --- (1)

Also we know

$$m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

or

$$m^2 = \frac{m_o^2}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$m^2c^2 - m^2v^2 = m_o^2c^2$$

$$-p^2 = m_o^2c^2 - m^2c^2$$

$$p^2 = m^2c^2 - m_o^2c^2$$

$$p^2 = c^2(m^2 - m_o^2)$$

$$p = c\sqrt{m^2 - m_o^2} \quad \text{--- (2)}$$

$$\sqrt{m^2 - m_o^2}$$

$$\sqrt{\left(m_o + \frac{T}{c^2}\right)^2 - m_o^2}$$

$$= \sqrt{m_o^2 + \frac{T^2}{c^4} + \frac{2m_oT}{c^2} - m_o^2}$$

$$= \sqrt{\frac{T^2}{c^4} + \frac{2m_oT}{c^2}}$$

$$= \sqrt{\frac{T}{c^2}\left(\frac{T}{c^2} + 2m_o\right)}$$

$$= \sqrt{\frac{T}{c^4}(T + 2m_oc^2)}$$

$$= \frac{1}{c^2}\sqrt{T(2m_oc^2 + T)}$$

From Eq. (1)

$$m = m_o + \frac{T}{c^2}$$

$$p = c\sqrt{\left(m_o + \frac{T}{c^2}\right)^2 - m_o^2} = c\sqrt{\frac{T}{c^2}\left(2m_o + \frac{T}{c^2}\right)} = \frac{1}{c}\sqrt{T(2m_oc^2 + T)}$$

de Broglie wavelength $\boxed{\lambda = \frac{h}{p} = \frac{hc}{\sqrt{T(2m_oc^2 + T)}}}$

P.T.O.

De-Broglie wavelength in terms of Energy & Temperature :-

De-Broglie wavelength ~~according~~ is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

If E is the kinetic energy of the moving particle, then

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

Hence de-Broglie wavelength

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}}$$

According to kinetic theory of gases, the average kinetic energy of the material particle is given by

$$E = \frac{1}{2}mv^2 = \frac{3}{2}kT$$

where k is Boltzmann's constant $= 1.38\times10^{-23}\ JK^{-1}$ and T is the absolute temperature.

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{3mkT}}$$

Thus, wavelength associated with a moving particle is proportional to $1/\sqrt{T}$.

P.T.O.

Q :- Calculate the wavelength of thermal neutrons at $27°C$, given mass of neutron $= 1.67\times10^{-27}$ kg, $h = 6.6\times10^{-34}$ Js and $K = 1.376\times10^{-23}\ JK^{-1}$.

A :- $$\lambda = \frac{6.6\times10^{-34}}{[3\times1.67\times10^{-27}\times1.376\times10^{-23}\times(273+27)]^{1/2}} = 1.452\times10^{-10}\ m = 1.452\ Å.$$

Q :- Calculate the de-Broglie wavelength of 1 MeV proton. Do we require relativistic calculation.

A :- If we calculate the velocity of proton using the relation $E = \frac{1}{2}mv^2$, we get

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times10^6\times1.6\times10^{-19}}{1.67\times10^{-27}}} = 1.38\times10^7\ m/s$$

which is nearly less than one twentieth of the velocity of light. So we do not require relativistic calculation.

Hence de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.6\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.6\times10^{-19}\times10^6}} = 2.86\times10^{-14}\ m$ Ans

{Energy of the photon $= 1\,MeV = 10^6\times1.6\times10^{-19}\,J$}

P.T.O.

De – Broglie wavelength of Electron in terms of potential difference :- de-Broglie wavelength of an electron in terms of potential difference (V) through which it is accelerated is calculated as under.

Let v be the velocity acquired by the electron when it is accelerated through potential difference V, then

work done on the electron $= eV$

Kinetic energy gained by the electron $= \frac{1}{2}mv^2$

$$\therefore \quad \frac{1}{2}mv^2 = Ve$$

$$v = \sqrt{\frac{2eV}{m}}$$

The wavelength associated with the electron (for a non-relativistic case)

$$\lambda = \frac{h}{mv}$$

$$= \frac{h}{m\sqrt{\frac{2eV}{m}}} = \frac{h}{\sqrt{2eVm}}$$

In S.I. units. : V is in Volts, e in Coulomb, m in kg, h in JS.

Various values are

$$e = 1.6\times10^{-19}\,C, \quad m = 9.11\times10^{-31}\,kg, \quad h = 6.6\times10^{-34}\,JS$$

$$\therefore \quad \lambda = \frac{6.6\times10^{-34}}{\sqrt{V}\sqrt{2\times1.6\times10^{-19}\times9.11\times10^{-31}}} = \frac{12.26\times10^{-10}}{\sqrt{V}} \text{ metre}$$

$$= \frac{12.26}{\sqrt{V}}\,\text{Å} = \sqrt{\frac{150}{V}}\,\text{Å}.$$

Wavelength of Electron moving with relativistic velocity :- According to the theory of relativity, the mass m of a particle moving with velocity v is given by

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

where m_0 is the rest mass of the particle and c the velocity of light

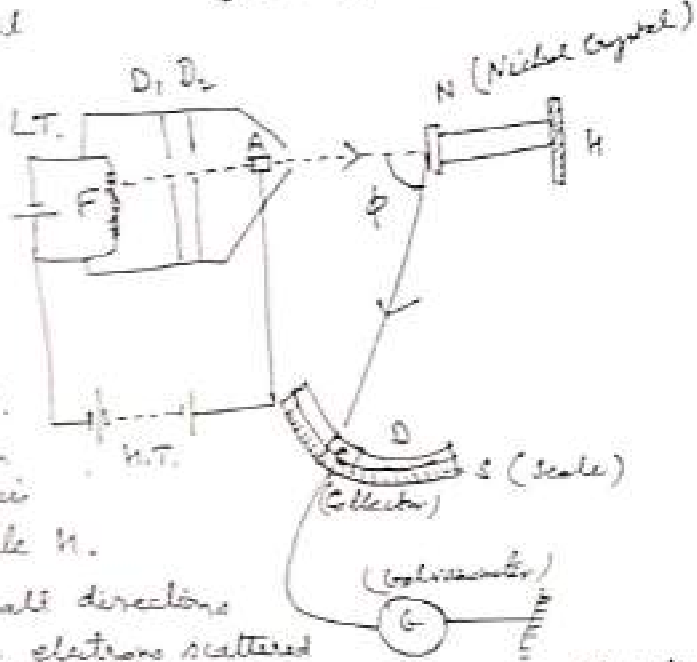
The momentum of the particle $p = mv = \frac{m_0 v}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}$

According to de-Broglie wave equation the wavelength λ associated with the moving particle is given by

$$\lambda = \frac{h}{p} = \frac{h}{\frac{m_0 v}{(1-v^2/c^2)^{\frac{1}{2}}}} = \frac{h}{m_0 v}\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}} = \frac{h}{m_0 c}\,\frac{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}{v/c}$$

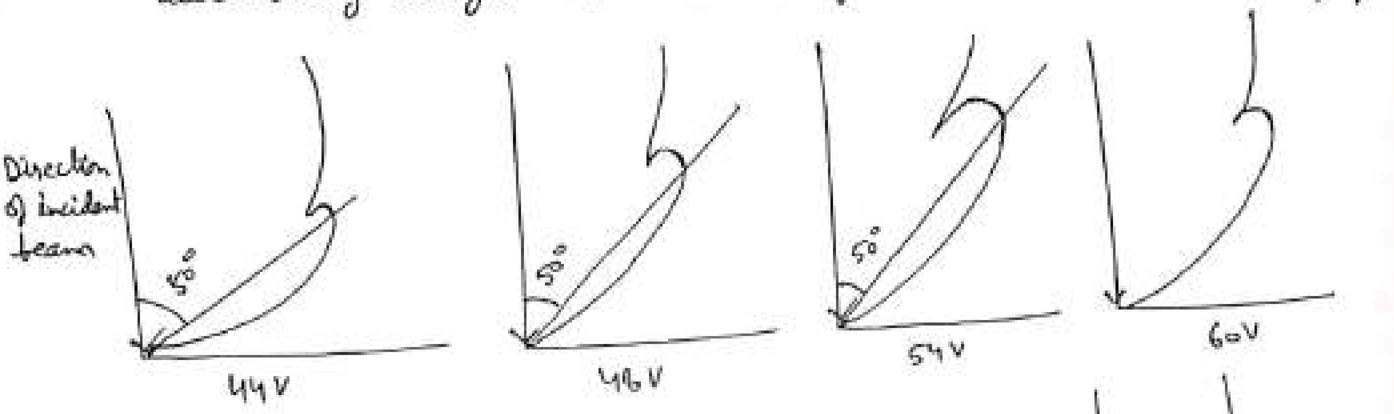
Experimental verification of matter waves In 1927 Davisson and Germer in the United States and G.P. Thomson in England independently demonstrated that the electrons exhibit diffraction when they are scattered from crystals whose atoms are spaced appropriately thus confirming the de Broglie's hypothesis of matter waves.

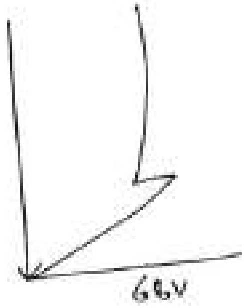
Davisson and Germer's Experiment :- The experimental arrangement is shown in figure. Electrons are produced by heating a filament (F) by a low tension battery (L.T.). These electrons are restricted to a fine parallel pencil by thin aluminium diaphragms D_1 and D_2. The electrons are then accelerated by passing them through an aluminium cylinder A to which required high potentials (H.T.) can be applied. This electron beam falls on a large single crystal of nickel N. The crystal is capable of rotation about an axis parallel to the axis of the incident beam by a handle H.



The electrons are scattered in all directions by the atoms in the crystal. The electrons scattered in different directions are collected by a Faraday cylinder C, called the collector. The collector is connected to a sensitive galvanometer G and can be moved along a graduated circular scale S, so that it is able to receive the reflected electrons at all angles between 20° and 90°. The collector has two walls C and D, insulated from each other. A retarding potential is applied between the two walls of collector so that only the electrons with nearly the incident velocity and not the secondary slow electrons, excited by collision with atoms, may enter the collector. The nickel crystal belongs to the face centred cubic type and it is so cut as to present a smooth reflecting surface parallel to the lattice plane (1,1,1). By turning the handle, any azimuth of the crystal can be presented to the plane defined by the incident beam and the reflected beam entering the collector.

Experimental procedure :- Normal incidence. Here, the beam of electrons falls normally on the surface of the crystal. A diffraction effect from the surface layer of the crystal acting as a plane grating is produced. For each azimuth of the crystal, a beam of electrons is made to fall normally on the crystal. The collector is moved to various positions on the scale S and the galvanometer deflection at each position is noted. This deflection gives a measure of the intensity of the diffracted beam of electrons. The galvanometer deflection is plotted against the angle between the incident beam and the beam entering the collector (Colatitude i.e. angle between incident and scattered beam). The observations are repeated for different accelerating voltages and a number of curves are drawn. The graph



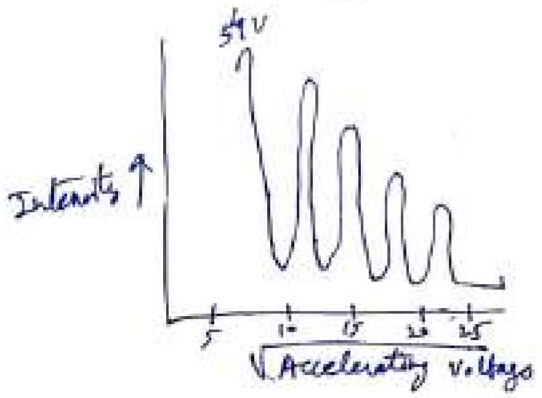


remains fairly smooth, till the accelerating voltage becomes 44V when a spur appears on the curve. As the accelerating voltage is increased, the length of the spur increases, till it reaches a maximum at 54V at an angle of ~~ϕ~~ 50°. With further increase in the accelerating voltage, the spur decreases in length and finally disappears at 68V.

The occurrence of a pronounced spur (bump) at 50° with the electrons accelerated through 54V can be explained, as due to the constructive interference of the electron waves, scattered in this direction, from the regularly spaced parallel planes in the crystal, which are rich in atoms. According to de Broglie's theory, the wavelength of 54V electrons is given by

$$\lambda = \frac{12.25}{\sqrt{V}}\,°A = \frac{12.25}{\sqrt{54}}\,°A = 1.66\,°A$$

According to experiment, we have a diffracted beam at colatitude of 50°. For nickel, for the (111) reflecting plane, $d = 2.15°A$. Applying the equation for a plane reflecting grating $n\lambda = d\sin\theta$, n referring here to the first order, we get $\lambda = 2.15 \sin 50° = 1.65°A$. Thus the experimental value is in close agreement with the theoretical value. This shows that the beam of electrons behaves like X-rays, suffers diffraction at reflecting surfaces and thus has wavelike characteristic.

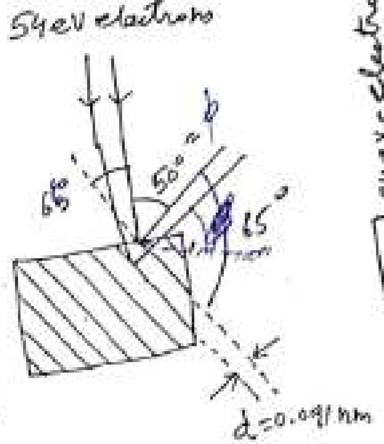


Calculation of Observed Wavelength :- For nickel, the spacing of the atomic planes which can be measured by x-ray diffraction is $d = 0.091\,nm$.

There is an intense reflection of the beam at the angle $\phi = 50°$.

The angle of incidence relative to the family of Bragg planes shown in figure is

$$\theta = \frac{180 - 50}{2} = 65°. \quad \left| 2\theta + \phi = 180° \right. \quad 2\theta = 180° - \phi, \quad \theta = \frac{180 - \phi}{2}$$



Single crystal of nickel

The Bragg equation for maxima in the diffraction pattern is

$$n\lambda = 2d\sin\theta$$

For $n=1$ the de-Broglie wavelength λ of the diffracted electrons is

$$\lambda = 2d\sin\theta = 2 \times 0.091 \times \sin 65° = 0.165\,nm$$

Calculation of the expected wavelength of the electrons using de-Broglie's formula. The electron kinetic energy KE is 54eV. The electron wavelength is therefore

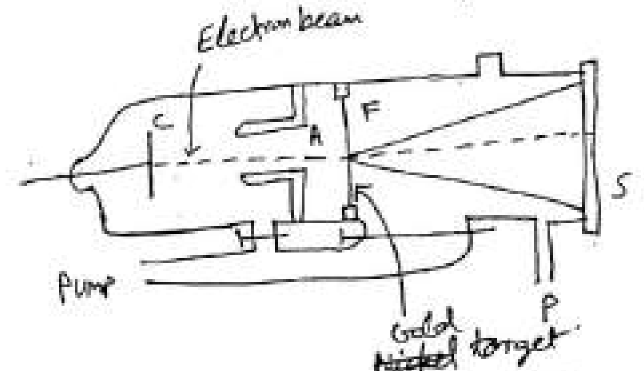
$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK.E.}} = \frac{6.63 \times 10^{-34}\,J.s}{\sqrt{2 \times 9.1 \times 10^{-31} \times 54\,eV \times 1.6 \times 10^{-19}\,J/eV}}$$

$$= 0.166\,nm$$

The observed wavelength agrees well with the expected wavelength. The Davisson-Germer experiment thus directly verifies de-Broglie's hypothesis of the wave nature of moving bodies.

G.P. Thomson's Experiment :- The experimental arrangement is shown in figure. A beam of cathode rays is produced in a discharge tube AC by means of an induction coil. The electrons passing through a fine hole A, are incident on a thin gold foil F. The thickness of the foil is about $10^{-8}m$. The emergent beam of electrons is received on a photographic plate P. Visual examination of the pattern is made possible by the fluorescent screen S. A very high vacuum is maintained in the camera part FP of the apparatus while air is allowed to leak into the discharge tube section through a needle valve. Thus a beam of required voltage can be continuously produced, the discharge tube being sufficiently soft.

~~Experimental~~ procedure.



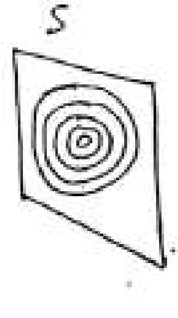
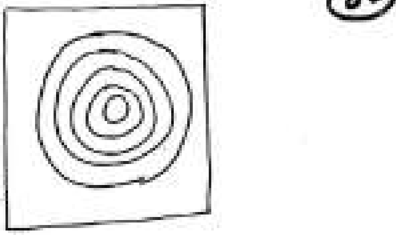


Figure: Apparatus of G.P. Thomson

P.T.O.

Experimental procedure :- A beam of electrons of known velocity is made to fall on the photographic plate, after traversing the thin gold foil. When the plate is developed a symmetrical pattern consisting of concentric rings about a central spot, is obtained as shown in figure. The pattern is similar to the pattern produced by x-rays in the powdered crystal method. When the cathode rays in the discharge tube are deflected by a magnetic field, the entire pattern on the screen S is found to shift correspondingly. Thus the pattern is confirmed as due to diffracted electrons and not due to secondary x-rays, generated by the electrons going through the foil. Further, on removing the film F the pattern disappear, showing that the presence of the film is essential. If the electrons behaved as corpuscles, the electrons passing through the foil should have been scattered through a wide angle. Clearly, this experiment demonstrates that the electron beam behaves as waves, since diffraction patterns can be produced only by waves.



Verification of the de Broglie equation :- G.P. Thomson employed very high voltages of the order of 50,000 volts to accelerate the electrons. With very high speed electrons, relativistic correction for the mass of the electron has to be applied. It can be shown that

$$\lambda = \frac{12.27}{\sqrt{V}} = \frac{1}{\sqrt{1+9.836\times10^{-7}V}}\ \text{Å}$$

The correction factor is very small, except for very large values of V.

To calculate λ from the radii of the rings :- In the polycrystalline film there will be some crystals set at the correct angle to give a Bragg reflection. If there are enough crystals distributed at random, the result of such reflections will be a series of rings, arising from the intersection of the cones of diffraction with the photographic plate. Let AB be the incident beam passing through the film at B. BP is the direction of the beam which has suffered a Bragg reflection in some crystal in the film at B. This reflected beam falls at the point P on the photographic plate at a distance R from the central point O. as shown in figure. Let the distance BO from the film to the plate, be L. ∠PBO = 2θ where θ is the glancing angle given by Bragg relation,

$$n\lambda = 2d\sin\theta$$

$$\frac{R}{L} = \tan 2\theta = 2\theta \quad \text{since } \theta \text{ is small}$$

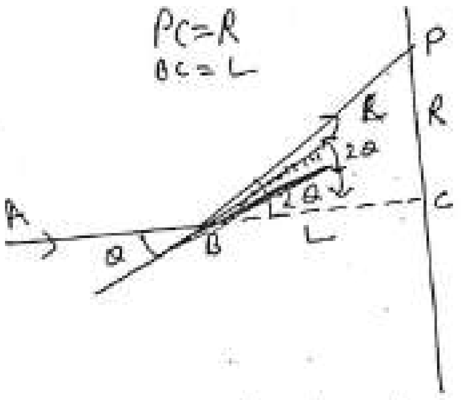
$$\therefore\ \theta = \frac{R}{2L}$$

But $2d\sin\theta = 2d\theta = n\lambda$.

or $\theta = \frac{n\lambda}{2d}$

$\therefore\ \frac{n\lambda}{2d} = \frac{R}{2L}$

or $\lambda = \frac{Rd}{n\lambda}$



From this the wavelength is calculated. This agrees with the calculated value given by Eq(..). This provides ultimate confirmation for the wave nature of the electron.

$2d\sin\theta = n\lambda$

Two slit experiment for demonstrating interference of material particles :- The interference of electron waves can be exhibited by a double slit experiment. The interference pattern so obtained resembles the pattern in case of visible light and prove the association of wave-packets with electrons. The experimental arrangement is shown in figure.

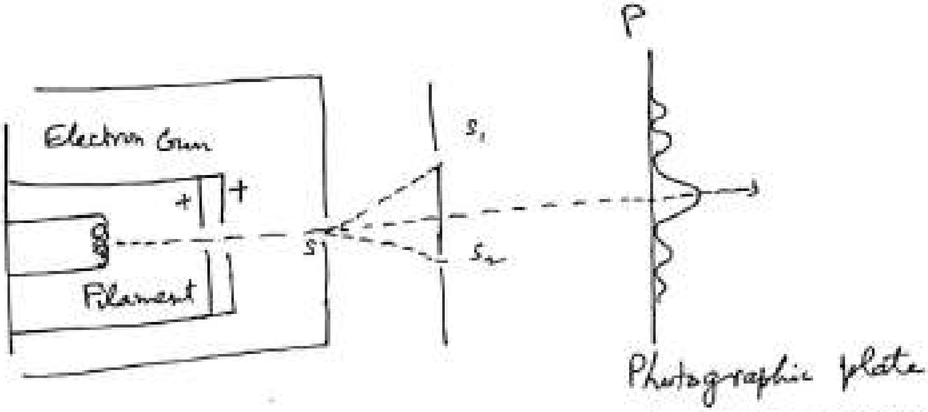


Fig :- Two slit experiments to show interference by electron waves.

Electrons of known velocity are supplied by an electron-gun in the form of a thin beam. These electrons after passing through two slits S_1 and S_2 equidistant from the slit S strike the photographic plate on which the interference pattern is observed. The whole arrangement is kept in a highly evacuated chamber so that the electrons do not collide with air particles and lose energy. The photographic plate, on examination under an electron microscope, shows the formation of interference pattern which proves the wave nature of electrons. The intensity distribution curve obtained for the electrons striking the screen is shown on the right of figure. The pattern is formed by the interference of electrons diffracted by the two slits S_1 and S_2. This can be proved by closing one of the two slits which causes the two slit pattern to disappear.

The fringe width can be measured by obtaining a magnified copy of the photograph, knowing the separation between S_1 and S_2 and the distance between S_1S_2 and the photographic plate, the wavelength of the electron waves can be calculated. This comes out to be the same as given by the de-Broglie relation which proves the existence of electron waves which behave like light waves in forming interference pattern.

Wave velocity & Phase velocity :- A wave is a disturbance from equilibrium condition that propagates with time from one region to another in space. This disturbance gives rise to an elastic force in the material adjacent to it, then the next particle is displaced and then the next and so on. Hence the motion is handed over from one particle to the other particle. Therefore, every particle begins its vibration a little later than its predecessor. Thus, there is a progressive change of phase from one particle to another. The velocity with which planes of constant phase (wavefront) propagates through the medium is known as wave velocity or phase velocity.

For this let us consider a string stretched along the x-axis whose vibrations are in the y-direction. The displacement of the string at a distance x from the origin and at a time t is given by

$$y = a \sin(\omega t - kx) \quad \text{——— (1)}$$

where 'a' is the amplitude, ω is the angular frequency and k is the propagation constant of the wave.

The term $(\omega t - kx)$ represent phase of the wave motion.

Planes of constant phase are given by

$$\omega t - kx = \text{constant} \quad \text{——— (2)}$$

Differentiating Eq.(2) w.r.t. time

$$\omega - k\frac{dx}{dt} = 0$$

or $$k\frac{dx}{dt} = \omega$$

or $$\frac{dx}{dt} = \frac{\omega}{k}$$

But $\frac{dx}{dt} = v_p$, phase or wave velocity

$\therefore$ $$\boxed{v_p = \frac{\omega}{k}}$$

Thus, wave velocity or phase velocity is the ratio of angular frequency (ω) and the propagation constant (k).

Motion of Wave Packets :- Schrodinger assumed that a material particle is equivalent to a wave packet rather than a single wave.

A wave packet comprises a group of waves, each with a slightly different velocity and wavelength superimposed upon each other. The mutual interference between component waves result in the variation of amplitude that defines the shape of the wave packet. The component waves interfere constructively over only a small region of space outside of which they interfere destructively and hence the amplitude reduces to zero rapidly.

Imp

The observed velocity is the velocity with which the maximum amplitude of the group advances in the medium. Thus, the group velocity is the velocity with which the energy in the group is transmitted. The individual waves travel inside the group with their phase velocities.

Imp

For calculating the group velocity, let us suppose that a group is formed due to the combination of two waves of the same amplitude but differing in angular frequency and propagation constant. The two waves can therefore be written as

$$y_1 = a\sin(\omega_1 t - k_1 x) \quad \text{——(1)}$$

and

$$y_2 = a\sin(\omega_2 t - k_2 x) \quad \text{——(2)}$$

The resultant displacement 'y' at any position x at any time t is the sum of y_1 and y_2 i.e.

$$y = y_1 + y_2$$

$$= a\sin(\omega_1 t - k_1 x) + a\sin(\omega_2 t - k_2 x)$$

$$= 2a\sin\left[\frac{(\omega_1+\omega_2)t}{2} - \frac{(k_1+k_2)x}{2}\right]\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t - \left(\frac{k_1-k_2}{2}\right)x\right]$$

$$= 2a\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t - \left(\frac{k_1-k_2}{2}\right)x\right]\sin\left[\left(\frac{\omega_1+\omega_2}{2}\right)t - \left(\frac{k_1+k_2}{2}\right)x\right]$$

This equation represent a wave system of amplitude

$$A = 2a\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t - \left(\frac{k_1-k_2}{2}\right)x\right] \quad \text{——(3)}$$

The maximum value of amplitude is 2a. The velocity with which this envelope moves is the velocity of the maximum amplitude of the group and is given by

$$\boxed{\text{Group velocity } (v_g) = \frac{\omega_1-\omega_2}{k_1-k_2} = \frac{\Delta\omega}{\Delta k}} \quad \text{——(4)}$$

If a group contains a number of frequency components in a very small frequency interval then Eq.(4) can be written as

$$\boxed{v_g = \frac{d\omega}{dk}}$$

This is the expression for group velocity.

Relation between Phase Velocity and Group Velocity :- The phase or wave velocity is given by

$$v_p = \frac{\omega}{k}$$

or $$\omega = v_p k \qquad \text{--- (1)}$$

The Group velocity is given by
$$v_g = \frac{d\omega}{dk} \qquad \text{--- (2)}$$

Comparing Eq.(1) & (2) we get
$$v_g = \frac{d}{dk}(v_p k)$$

or
$$= v_p + k\frac{dv_p}{dk} \qquad \text{--- (3)}$$

But $k = \frac{2\pi}{\lambda}$; $\therefore$ $dk = -\frac{1}{\lambda^2} 2\pi d\lambda$ --- (4)

Eq.(3) can be written as
$$v_g = v_p + \left(\frac{2\pi}{\lambda}\right)\frac{dv_p}{\left(-\frac{2\pi}{\lambda^2}\right)d\lambda}$$

$$\boxed{v_g = v_p - \lambda\frac{dv_p}{d\lambda}} \qquad \text{--- (5)}$$

This is the required expression.

Imp | This relation shows that group velocity, v_g, is less than the phase velocity v_p in a dispersive medium i.e., where v_p is function of λ.

Imp | In a non-dispersive medium or free space, waves of all wavelength travel with the same speed i.e.
$$\frac{dv_p}{d\lambda} = 0$$

or $$\boxed{v_g = v_p}$$

Imp | So, group velocity is same as the wave velocity for light waves in free space.

Relation between Group velocity and velocity of moving body :- 87

~~Group Velocity and Particle Velocity~~ :- In quantum mechanics a wave packet surrounds the position of the classical particle and C.G. of wave packet always follows the path of classical particle.

A wave packet comprises a group of waves each with different λ. Inside this region of space these waves interfere constructively, outside this region these waves interfere distructively, so that amplitude reduces to zero.

→ Group velocity (v_g) is the velocity with which (a wave of maximum) amplitude of wave packet move forward.

→ Phase velocity (v_p) is the velocity of individual waves in a group of waves.

From mass transformation formula, moving mass of a body is given by

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} \quad \text{---(1)}$$

where m_0 is the rest mass of the body and v is velocity of the body.

From Einstein's mass-energy relation and Plank's theory.

$$E = mc^2 = h\nu$$

$$\Rightarrow \quad \nu = \frac{mc^2}{h}$$

$\therefore$ angular frequency $\omega = 2\pi\nu = \frac{2\pi\, mc^2}{h}$

$$= \frac{2\pi c^2}{h} \cdot \frac{m_0}{\sqrt{1-v^2/c^2}}$$

$$\therefore \frac{d\omega}{dv} = \frac{2\pi}{h} m_0 \cancel{c^2} \cdot \left(-\frac{1}{2}\right)\left(-\frac{2v}{\cancel{c^2}}\right) \cdot \frac{1}{(1-v^2/c^2)^{3/2}}$$

$$= \frac{2\pi}{h} \frac{m_0 v}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} \quad \text{----(2)}$$

Again, propagation constant $k = \frac{2\pi}{\lambda} = 2\pi \frac{mv}{h}$ --- (3) where $\frac{1}{\lambda} = \frac{1}{\frac{h}{mv}}$

$\because$ de-Broglie wavelength, $\lambda = \frac{h}{mv}$

$$v_p = \frac{\omega}{k}, \quad \frac{d\omega}{dk} = v_g$$

Using Eq.(1), Eq. (3) can be written as.

$$k = \frac{2\pi v}{h} \cdot \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Differentiating w.r.t. v we get

$$\frac{dk}{dv} = \frac{2\pi m_0}{h} \frac{d}{dv}\left[v.\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \right]$$

P.T.O.

$$= \frac{2\pi}{h} \cdot \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} \quad \text{---(4)}$$

Now group velocity

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}$$

Substituting values of $\frac{d\omega}{dv}$ and $\frac{dk}{dv}$ from Eq.(2) and (4) we get

$$v_g = \frac{\frac{2\pi}{h} \cdot \frac{m_0 v}{(1-v^2/c^2)^{3/2}}}{\frac{2\pi}{h} \frac{m_0}{(1-v^2/c^2)^{3/2}}}$$

$$\boxed{v_g = v}$$

i.e. the group velocity associated with body moves with velocity of the body.

Q:- Find the de Broglie wavelength associated with
(i) A 46 gm golf ball with velocity 36 m/sec.
(ii) an electron with a velocity 10^7 m/sec.
which of these two show wave character and why.

Ans:- (i) Since $v \ll c$, we can take $m = m_0 \rightarrow$ the rest mass, Hence

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ JS}{(0.046\ kg)(36\ m/sec)} = 4.0 \times 10^{-34}\ m$$

Thus the wavelength associated with golf ball is much smaller as compared to its dimensions. Hence no wave aspects can be expected in its behaviour.

(ii) Again $v \ll c$, so $m = m_0 = 9.1 \times 10^{-31}$ kg

$$\therefore \quad \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(9.1 \times 10^{-31}) \times 10^7} = .72857 \times 10^{-10} = 7.3 \times 10^{-11}\ m$$

This wavelength is comparable with the atomic dimensions. Hence moving electron exhibits a wave character.

Q:- Show that the de Broglie wavelength associated with an electron of energy V electron-volts is approximately $(1.227/\sqrt{V})$ nm.

Ans:- The de Broglie wavelength λ associated with an electron of mass m and energy E is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

Here, K.E, $E_k = V\,eV = 1.6 \times 10^{-19}\ V\ J$

$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{(2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}\ V)}} = \frac{6.62 \times 10^{-34}}{5.396 \times 10^{-25} \sqrt{V}}$$

$$= \frac{1.227 \times 10^{-9}}{\sqrt{V}}\ m$$

$$= \frac{1.227}{\sqrt{V}}\ nm$$

Q:- Find the K.E. of a proton whose de Broglie wavelength is 1 fm.

Ans:- $pc = (mv)c = \frac{hc}{\lambda} = \frac{(4.136 \times 10^{-15}\ eV.s)(3 \times 10^8\ m/sec)}{1 \times 10^{-15}\ m}$

$$= 1.241\ GeV$$

The rest mass energy of the proton $= E_0 = 0.938$ GeV

$pc > E_0$. Hence a relativistic calculation is needed

The total energy of the proton is $E = \sqrt{E_0^2 + p^2c^2} = \sqrt{(0.938\ GeV)^2 + (1.241\ GeV)^2}$
$= 1.556$ GeV

The K.E. of the proton is

$$K.E. = E - E_0 = (1.556 - 0.938)\ GeV = 0.618\ GeV$$

Q. The first order Bragg's maximum of electron diffraction in a nickel crystal ($d = 0.9086°A$) occurred at a glancing angle of $65°$. Calculate the deBroglie's wavelength and their velocity.

Ans:- Bragg's law is

$$2d\sin\theta = n\lambda$$

for $n=1$, $\theta = 65°$, $d = .9086 \times 10^{-10}$ m

$$2 \times 0.9086 \times 10^{-10} \times \sin 65° = \lambda$$

or $$\lambda = 1.8172 \times 0.9063 \times 10^{-10}\ m = 1.65°A.$$

$$\lambda = \frac{h}{mv}.$$

or $$v = \frac{h}{m\lambda} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.65 \times 10^{-10}} = 4.39 \times 10^{6}\ m/s \quad \text{Ans}$$

Q. Calculate the energy of a proton in eV with a deBroglie wavelength of $0.8°A$. (mass of proton $= 1.6 \times 10^{-27}$ kg)

Ans:- From deBroglie relation

$$p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{0.8 \times 10^{-10}}$$

So energy $$E = \frac{p^2}{2m} = \left(\frac{6.6 \times 10^{-34}}{0.8 \times 10^{-10}}\right)^2 \times \frac{1}{2 \times 1.6 \times 10^{-27}} = \cancel{0.13\,eV} \quad \text{Ans}$$

mass of proton

$$= 21.269 \times 10^{-48+27}$$

$$= 21.269 \times 10^{-21} \text{ Joule}$$

$$= \frac{21.269 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 13.293 \times 10^{-21+19} \text{ eV}$$

$$= .13 \text{ eV} \quad \text{Ans}$$

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Q. A proton and an electron have equal kinetic energies. Compare their de-Broglie wavelengths.

A.- Let M be the mass of the proton, V its velocity and m the mass of electron and v its velocity, then

Kinetic energy of the proton $= \frac{1}{2}MV^2$

and Kinetic energy of the electron $= \frac{1}{2}mv^2$

$$\therefore \frac{1}{2}MV^2 = \frac{1}{2}mv^2 \text{ or } \frac{v^2}{V^2} = \frac{M}{m} \text{ or } \frac{v}{V} = \sqrt{\frac{M}{m}}$$

de-Broglie wavelength of proton $\lambda_p = \frac{h}{MV}$

de-Broglie wavelength of electron $\lambda_e = \frac{h}{mv}$

$$\therefore \frac{\lambda_p}{\lambda_e} = \frac{h}{MV}\cdot\frac{mv}{h} = \frac{m}{M}\frac{v}{V} = \frac{m}{M}\left(\frac{M}{m}\right)^{\frac{1}{2}} = \left(\frac{m}{M}\right)^{\frac{1}{2}} = \left(\frac{9.1\times10^{-31}}{1.67\times10^{-27}}\right)^{\frac{1}{2}} = 0.0233$$

Q:- Calculate de-Broglie wavelength of an α-particle accelerated through a potential difference of 4KV.

Ans:- mass of α-particle, $m = 4a.m.u. = 4\times1.66\times10^{-27}\,kg = 6.64\times10^{-27}\,kg$.

Charge on the α-particle, $q = 2e = 2\times1.6\times10^{-19}\,C = 3.2\times10^{-19}\,C$

let v be the velocity of the α-particle when it is accelerated through a potential difference V, then work done on the α-particle $= qV$

Kinetic energy gained by the α-particle $= \frac{1}{2}mv^2 = qV$

$$v = \sqrt{\frac{2qV}{m}}$$

The wavelength associated with α-particle (non-relativistic case)

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2qV}{m}}} = \frac{h}{\sqrt{2qmV}}$$

Here $V = 4KV = 4000$ Volts

$$\therefore \lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times3.2\times10^{-19}\times6.64\times10^{-27}\times4000}} = 0.16\times10^{-12}\,m$$

Q. Calculate de-Broglie wavelength of an electron moving with velocity $\frac{3}{5}c$.

A

$$\lambda = \frac{h}{mc}\frac{(1-v^2/c^2)^{1/2}}{v/c} = \frac{h}{mc}\frac{\left(1-\frac{9}{25}\right)^{1/2}}{3/5} = \frac{h}{mc}\times\frac{4}{5}\times\frac{5}{3} = \frac{4}{3}\frac{h}{mc} = \frac{4}{3}\times\frac{6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}$$

$= 0.032\times10^{-10}\,m$

$= 0.032\,\mathring{A}$

Q:- Calculate the de-Broglie wavelength of 1MeV electron. mass of the electron $= 9.1\times10^{-31}$ kg.

A:- If we calculate the velocity of the electron using the relation $E = \frac{1}{2}mv^2$, we get

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times10^{6}\times1.6\times10^{-19}}{9.1\times10^{-31}}} = 5.927\times10^{8}\,ms^{-1}$$

which is not possible as it is greater than the velocity of light. We should, therefore, use relativistic form of the equation.

Kinetic energy of the electron $= 10^6\,eV = 10^6\times1.6\times10^{-19}\,J = 1.6\times10^{-13}\,J$.

Q:- Calculate the energy in eV corresponding to a wavelength of 1Å for electron and neutron. Given $h = 6.6\times10^{-34}$ Js^{-1}, mass of electron $= 9.1\times10^{-31}$ kg, mass of neutron $= 1.7\times10^{-27}$ kg.

Ans:- According to de-Broglie, wavelength $\lambda = \frac{h}{mv}$

For an electron, $v = \frac{h}{m\lambda} = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times10^{-10}} = 7.2\times10^{6}\ ms^{-1}$

For a neutron, $v = \frac{h}{m\lambda} = \frac{6.6\times10^{-34}}{1.7\times10^{-27}\times10^{-10}} = 3.88\times10^{3}\ ms^{-1}$

As the velocity in both the cases is much less than the velocity of light, it is a non-relativistic case. Hence

$$\lambda = \frac{h}{\sqrt{2mE}} \text{ or } \lambda^2 = \frac{h^2}{2mE} \text{ or } E = \frac{h^2}{2m\lambda^2}$$

For an electron, $E = \frac{6.6\times6.6\times10^{-34}\times10^{-34}}{2\times9.1\times10^{-31}\times10^{-10}\times10^{-10}} = 1.49\times10^{2}\ eV = 149\ eV$

For a neutron, $E = \frac{6.6\times6.6\times10^{-34}\times10^{-34}}{2\times1.7\times10^{-27}\times10^{-10}\times10^{-10}} = 8\times10^{-2}\ eV = 0.08\ eV$

(5)- Find the de-Broglie wavelength of an electron in the first Bohr orbit of hydrogen.

A:- Energy of electron in the n^{th} Bohr orbit of hydrogen atom $E_n = \frac{-13.6}{n^2}$ eV

For the first orbit $n=1$

$\therefore E_1 = \frac{-13.6}{1} = -13.6\ eV = -13.6\times1.6\times10^{-19}\ J$

$\therefore$ de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times13.6\times10^{-19}}} = 3.3\times10^{-10}\ m = 3.3\ Å$

(7)- Ultra violet light of wavelength 1500Å is incident on Molybdenum whose work function is 4.15 eV. Find the de-Broglie wavelength associated with the photo-electrons.

A:- Energy of ultra-violet light of wavelength $\lambda = 1500$Å will be

$E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{1500\times10^{-10}} = 1.324\times10^{-18}\ J = \frac{1.324\times10^{-18}}{1.6\times10^{-19}} = 8.275\ eV$

$E = h\nu$
$= \frac{hc}{\lambda}$

work function of molybdenum $W = 4.150\ eV$

$\therefore$ Energy of photo-electron emitted, $E = 8.275 - 4.150 = 4.125\ eV$

de-Broglie wavelength associated with photo-electron

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times4.125\times1.6\times10^{-19}}} = 6\times10^{-10}\ m = 6\ Å$$

(8) An electron and a proton have the same de-Broglie wavelength. Prove that the kinetic energy of the electron is greater than that of proton.

A:- de-Broglie wavelength of electron $\lambda_e = \frac{h}{\sqrt{2m_eE_e}}$ and de-Broglie wavelength of proton, $\lambda_p = \frac{h}{\sqrt{2m_pE_p}}$

But $\lambda_e = \lambda_p$ or $\frac{h}{\sqrt{2m_eE_e}} = \frac{h}{\sqrt{2m_pE_p}}$ or $m_eE_e = m_pE_p$

or $\frac{E_e}{E_p} = \frac{m_p}{m_e}$

As $m_p > m_e \therefore E_e > E_p$ i.e. the K.E. of the electron is greater than that of proton.

P.T.O.

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Let m_0 be the rest mass of the electron and m the mass of the moving electron.

$\therefore$ Total energy of the moving electron

$$mc^2 = m_0c^2 + K.E.$$

$$= 9.1\times10^{-31} \times 9\times10^{16} + 1.6\times10^{-13}$$

$$m = \frac{9.1\times10^{-31}\times9\times10^{16} + 1.6\times10^{-13}}{9\times10^{16}} = 26.88\times10^{-31}\,kg$$

Now

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{or} \quad 1-\frac{v^2}{c^2} = \left(\frac{m_0}{m}\right)^2 = \left(\frac{9.1\times10^{-31}}{26.88\times10^{-31}}\right)^2 = (0.3389)^2$$

$$= 0.1149$$

$$\cancel{\frac{v}{c} = 0.940} \quad \frac{v^2}{c^2} = 1-0.1149 = 0.8851$$

$$\frac{v}{c} = 0.9407$$

$$v = 3\times10^{8}\times0.9407 = 2.822\times10^{8}\,ms^{-1}$$

Hence de-Broglie wavelength

$$\lambda = \frac{h}{mv} = \frac{6.6\times10^{-34}}{26.88\times10^{-31}\times2.822\times10^{8}} = 0.0087\times10^{-14} = 0.0087\,\text{Å}$$

Q. Show that the expression for de-Broglie wavelength of a particle with rest mass energy E_0, and kinetic energy K both in eV can also be written as

$$\lambda = \frac{hc}{[K(2m_0c^2+K)]^{\frac{1}{2}}} = \frac{1.241\times10^{-6}}{[K(2E_0+K)]^{\frac{1}{2}}}\ \text{metre}.$$

Ans:- Rest mass energy $E_0 = m_0c^2$ where m_0 is the rest mass of the particle and c the velocity of light. If m is the mass of the particle in motion then

$$E_0 + K = mc^2$$

If v is the velocity of the particle, then

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{m}{m_0} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_0+K}{E_0}$$

$$\frac{1}{1-\frac{v^2}{c^2}} = \frac{(E_0+K)^2}{E_0^2}$$

$$E_0^2 = (E_0+K)^2 - \frac{v^2}{c^2}(E_0+K)^2 = (E_0+K)^2 - m^2c^2v^2$$

$$m^2c^2v^2 = 2KE_0 + K^2 = K(2E_0+K)$$

$$mv = \frac{\sqrt{K(2E_0+K)}}{c}$$

$$\because E_0 = m_0c^2$$

Hence

$$\lambda = \frac{h}{mv} = \frac{hc}{[K(2E_0+K)]^{\frac{1}{2}}} = \frac{hc}{[K(2m_0c^2+K)]^{\frac{1}{2}}}$$

As E_0 and K are in eV, we have $h = 6.62\times10^{-34}\,J\text{-}s = 4.136\times10^{-15}\,eVs$, $c = 3\times10^{8}\,ms^{-1}$

$$\therefore hc = 4.136\times10^{-15}\times3\times10^{8} = 1.241\times10^{-6}\,eVm$$

Hence

$$\lambda = \frac{[1.241\times10^{-6}]}{K(2E_0+K)^{\frac{1}{2}}}$$

Q:- A beam of mono-energetic neutrons corresponding to 27°c is allowed to fall on a crystal. A first order reflection is observed at a glancing angle 30°. Calculate the interplanar spacing of crystal.

Ans:- Bragg's law is given by

$$2d\sin\theta = n\lambda$$

Given $\theta = 30°$, $n=1$, then

$$2d\sin 30° = \lambda$$

So, $d = \lambda$.

Again neutron energy $E = kT$.

$$T = 27+273 = 300\,°K$$

So, $E = 1.38\times10^{-23}\times 300 = 4.1\times10^{-21}\,J$

So $p = \sqrt{2mE} = \sqrt{2\times1.67\times10^{-27}\times4.1\times10^{-21}}$

$\therefore$ $d = \lambda = \dfrac{h}{p} = \dfrac{6.6\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times4.1\times10^{-21}}}$

$$= 1.775\times10^{-10}\,m$$ Ans.

Q:- X-rays of wavelength 0.62 Å fall on a metal plate. Find the wavelength associated with the photoelectrons emitted. (neglect the work function of metal).

Ans :- Einstein photoelectric equation gives

$$h\nu = W + \tfrac{1}{2}mv^2$$

or $h\nu = \tfrac{1}{2}mv^2$; $W=0$

or velocity of photo $\bar{e}$, $v = \sqrt{\dfrac{2h\nu}{m}} = \sqrt{\dfrac{2hc}{\lambda m}}$

The momentum of photo electron, $p = mv = m\sqrt{\dfrac{2hc}{\lambda m}}$.

then deBroglie wavelength, $\lambda = \dfrac{h}{p} = \dfrac{h}{m}\sqrt{\dfrac{\lambda m}{2hc}} = \sqrt{\dfrac{h\lambda}{2mc}}$

$$= \sqrt{\frac{6.6\times10^{-34}\times .62\times10^{-10}}{2\times9.1\times10^{-31}\times3\times10^{8}}}$$

$$= 0.99\times10^{-11}\,m$$ Ans

Q:- If the electron beam in a television picture tube is accelerated by 10,000 Volts, what is the deBroglie wavelength,

Ans:- If v is the velocity of electron, then we have,

$$\tfrac{1}{2}mv^2 = eV = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2meV}$$

Then deBroglie wavelength $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2meV}} = \dfrac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times10{,}000}}$

$$= 1.224\times10^{-11}\,m$$ Ans

Q:- The first order Bragg's maximum of electron diffraction in a nickel crystal ($d = 0.9086\ °A$) occures at a glancing angle of $65°$. Calculate the deBroglie wavelength and their velocity.

Ans:- Braggs law is

$$2d\sin\theta = n\lambda$$

for $n=1,\ \theta = 65°,\ d = 0.9086\ °A$

$$2\times .9086\times10^{-10}\sin 65° = \lambda$$

or $\lambda = 1.8172\times0.9063\times10^{-10}\,m = 1.65\ °A$

$$\lambda = \frac{h}{mv}$$

$$v = \frac{h}{m\lambda} = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times1.65\times10^{-10}}$$

$$= 4.29\times10^{5}\ m/sec$$

Ans.

Q:- Calculate the energy of a proton in eV with a deBroglie wavelength of $0.6°A$ (mass of proton $= 1.6\times10^{-27}\,kg$).

Ans:- From deBroglie relation

$$p = \frac{h}{\lambda} = \frac{6.6\times10^{-34}}{0.6\times10^{-10}}$$

& energy $E = \frac{p^2}{2m} = \left(\frac{6.6\times10^{-34}}{0.6\times10^{-10}}\right)^2\frac{1}{2\times1.6\times10^{-27}} = \frac{21.26\times10^{-21}}{1.6\times10^{-19}} = 0.13\ eV.$

Q:- Show that if the uncertainty in the location of a particle is equal to its deBroglie wavelength, the uncertainty in its velocity is equal to its velocity.

Ans:- Heisenberg uncertainty principle is $\Delta x\,\Delta p = h$

Given $\Delta x \approx \lambda$, But according to deBroglie, $\lambda = \frac{h}{mv}$

then $\Delta p_x = \frac{h}{\Delta x} \approx \frac{h}{\lambda} \approx \frac{h}{h}mv \approx mv$ —(1)

Now as $p_x = mv$ —

$\Delta p_x = m\Delta v$ —(2)

Comparing Eq.(1) & (2) we get

$$mv = m\Delta v$$

or $\boxed{v = \Delta v}$ Ans

Q:- When 400 volt electrons are diffracted by a Crystal, the angular diffraction pattern is quite same as that given out by X-rays having wavelength 0.61 °A. Calculate planck's constant.

Ans:- Energy of electron $= E = \frac{1}{2}mv^2 \overset{= eV}{=} 400 \times 1.6 \times 10^{-19}$ J

or $m^2v^2 = 2mE = 2 \times 9.1 \times 10^{-31} \times 400 \times 1.6 \times 10^{-19}$

or $mv = \sqrt{2 \times 9.1 \times 10^{-31} \times 400 \times 1.6 \times 10^{-19}}$

$= 10.79 \times 10^{-24}$

So, $\lambda = \frac{h}{mv}$.

or $h = \lambda mv = 0.61 \times 10^{-10} \times 10.79 \times 10^{-24}$

$= 6.5819 \times 10^{-34}$ kg m²/sec Ans.

Q:- An electron moves with a speed of 10^3 m/sec accurate to 0.006%. Find the accuracy with which the position of the electron can be located.

Ans:- Electronic mass $= 9.1 \times 10^{-31}$ kg, velocity $= 10^3$ m/sec.

So, momentum of electron $p = mv = 9.1 \times 10^{-31} \times 10^3$ kg m/sec

The uncertainty in momentum = 0.006 % of p

i.e. $\Delta p = .00006 \times 9.1 \times 10^{-28}$ kg m/sec

Then from Heisenberg uncertainity relation

$$\Delta x = \frac{\hbar}{\Delta p} = \frac{6.6 \times 10^{-34}}{2 \times 3.14 \times .00006 \times 9.1 \times 10^{-28}} = 0.04 \times 10^{-2} \text{ m}$$

Q:- Life time of a nucleus in excited state is 10^{-12} sec. Calculate the probable uncertainty in energy of γ-photon emitted ~~p~~ by it.

Ans:- Uncertainity in time $\Delta t \approx 10^{-12}$

So uncertainity in Energy, $\Delta E \approx \frac{\hbar}{\Delta t}$

$$= \frac{6.6 \times 10^{-34}}{2 \times 3.14 \times 10^{-12}} \text{ J}$$

$= 6.6 \times 10^{-22}$ Joule Ans

Q:- A ball of mass 10 gm has velocity 100 cm/sec. Calculate the wavelength associated with it. Why do we not observe this wave nature in our daily observations

Ans:- The de-Broglie wavelength 'λ' is given by

$$\lambda = \frac{h}{mv} = \frac{6.62\times10^{-34}}{(10\times10^{-3})(1)} = 6.62\times10^{-32}\,m = 6.62\times10^{-22}\,\mathring{A}.$$

The wavelength is much smaller than the dimensions of the ball, therefore in such cases (e.g.; ball) wave like properties cannot be observed in our daily observations.

Q:- What voltage must be applied to an electron microscope to produce electrons of wavelength 0.5 °A.

Ans:- The de-Broglie wavelength associated with an electron is given by

$$\lambda = \sqrt{\frac{150}{V}}\ \mathring{A}$$

But $\lambda = 0.5\,\mathring{A}$.

$$\therefore\quad 0.5 = \sqrt{\frac{150}{V}}$$

$$(0.5)^2 = \frac{150}{V}$$

$$V = \frac{150}{(0.5)^2} = 600\ \text{Volt}.$$

Ans.

Q:- A proton and an electron have equal kinetic energies. Compare their de-broglie wavelengths.

Ans. De-Broglie wavelength is given by $\lambda = \frac{h}{\sqrt{2mK}}$

If m_e and m_p are the masses of electron and proton

$$\lambda_e = \frac{h}{\sqrt{2m_eK}} \quad \text{and} \quad \lambda_p = \frac{h}{\sqrt{2m_pK}}$$

$$\frac{\lambda_p}{\lambda_e} = \frac{h/\sqrt{2m_pK}}{h/\sqrt{2m_eK}} = \sqrt{\frac{m_e}{m_p}}$$

Ans.

$$\boxed{\frac{\lambda_p}{\lambda_e} = \sqrt{\frac{m_e}{m_p}}}$$

Q:- Calculate de-Broglie wavelength of neutron of energy 12.8 MeV. Given mass of neutron $= 1.675 \times 10^{-27}$ kg, $h = 6.62 \times 10^{-34}$ J.s, $c = 3\times10^{8}$ m/sec, $1\,eV = 1.6\times10^{-19}$ Joule

Ans:. The rest mass energy of the neutron $= m_0c^2$

$$= (1.675\times10^{-27})(3\times10^{8})^2$$
$$= 15.075 \times 10^{-11}\ \text{Joule}$$
$$= \frac{15.075\times10^{-11}}{1.6\times10^{-19}} = 9.42\times10^{8}\ eV = 942\ MeV.$$

The given energy 12.8 MeV is much smaller than rest mass energy, so we can neglect relativistic effect. Using non relativistic relation

$$\lambda = \frac{h}{\sqrt{2mK}}$$

where $K = 12.8\ MeV = 12.8\times10^{6}\times1.6\times10^{-19}$ Joule.

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times12.8\times10^{6}\times1.6\times10^{-19}}} = 0.8\times10^{-14}\,m = 8\times10^{-5}\,\mathring{A}$$

Q:- Determine the velocity and kinetic energy of a neutron having de-Broglie wavelength $1\,\mathring{A}$ (mass of neutron is 1.67×10^{-27} kg, $h = 6.63\times10^{-34}$ Js).

Ans:- We have
$$\lambda = \frac{h}{mv}$$
$$v = \frac{h}{m\lambda}$$

Given that $\lambda = 1\,\mathring{A} = 1\times10^{-10}\,m$, $m = 1.67\times10^{-27}$ kg, $h = 6.63\times10^{-34}$ Js

$$\therefore\quad v = \frac{6.63\times10^{-34}}{1.67\times10^{-27}\times1\times10^{-10}} = 3.97\times10^{3}\ m/s$$

The kinetic energy is given by
$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\times1.67\times10^{-27}\times(3.97\times10^{3})^2$$
$$= 13.16\times10^{-21}\,J = 8.2\times10^{-2}\,eV$$
$$= 0.082\ eV$$
Ans

Q:- Calculate the de-Broglie wavelength of an proton of energy 10 MeV.

Ans:- Given Energy, $E = 10\,MeV$.

This energy is very much smaller that of rest mass energy so this energy is K.E. of the proton. We have
$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times10\times10^{6}\times1.6\times10^{-19}}}$$
$$= \frac{6.63\times10^{-34}}{7.31\times10^{-20}} = 0.906\times10^{-14}\,m = 9\times10^{-5}\,\mathring{A}$$
Ans

Q - A 60 watt light source is emitting light of wavelength 5400 °A, calculate the rate of emission of photon's from the source.

Ans :- Given $\lambda = 5400$ °A

Photon energy, $E = h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34} \times 3\times10^{8}}{5400\times10^{-10}} = 3.68\times10^{-19}$ Joule

The rate of emission of energy from the source is 60 joule/sec.

Hence, the number of photons emitted per second

$$= \frac{60}{3.68\times10^{-19}} = 1.63\times10^{20}$$ Ans.

Q :- Energy of a thermal neutron at absolute temperature T is of the order of kT. Calculate the wavelength of neutrons at 27°C. Given $k = 8.6\times10^{-5}$ ev/deg.

Ans :- By using relation $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

But $E = kT$

$\therefore$ $\lambda = \frac{h}{\sqrt{2mkT}}$; $m = 1.67\times10^{-27}$ kg, $T = 27+273 = 300$°K

$k = 8.6\times10^{-5}\times1.6\times10^{-19}$ Joule/deg

$= 1.376\times10^{-23}$ Joule/degree

$$= \frac{6.6\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times300\times1.376\times10^{-23}}}$$

$$= \frac{6.6\times10^{-34}}{3.71\times10^{-24}}$$

$= 1.779\times10^{-10}$ m

$= 1.779$ °A Ans

Q:- Calculate the de-Broglie wavelength of an electron whose kinetic energy is 50eV. Given $h = 6.63 \times 10^{-34}$ Joule-sec, $m_0 = 9.1 \times 10^{-31}$ kg, $1eV = 1.6 \times 10^{-19}$ Joule.

Ans. The de-Broglie wavelength of an electron moving with velocity v is given by

$$\lambda = \frac{h}{mv}$$

where m = relativistic mass of the electron.

The given energy 50eV is much smaller than rest mass energy (0.51MeV) of the electron, so we can neglect relativistic effect or we can write

$$m = m_0$$

So $$\lambda = \frac{h}{m_0 v}$$

If K is the K.E. of the electron, $K = \frac{1}{2} m_0 v^2$

$$v = \sqrt{\frac{2K}{m_0}}$$

Given $K = 50eV = 50 \times 1.6 \times 10^{-19}$ Joule

de-Broglie wavelength, $$\lambda = \frac{h}{\sqrt{2m_0K}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 50 \times 1.6 \times 10^{-19}}} = 1.735 \times 10^{-10} m$$
$$= 1.73 \mathring{A}$$

Q:- Each of an electron and a photon has an energy 2eV. Calculate their corresponding de-Broglie wavelengths.

Ans; The rest mass energy of the electron $(m_0c^2) = 9.1 \times 10^{-31} \times (3 \times 10^8)^2 = 8.19 \times 10^{-14}$ Joule

$$= \frac{8.19 \times 10^{-14}}{1.6 \times 10^{-19}} = 0.51 \times 10^6 eV$$
$$= 0.51 MeV$$

The given energy 2eV of the electron is much less than its rest mass energy (0.51 MeV). So the relativistic effect can be ignored. We can use no relativistic relation for de-Broglie wavelength

Wavelength of electron $$\lambda = \frac{h}{\sqrt{2mK}}, \text{ here } m = m_0$$

$$\lambda = \frac{6.63 \times 10^{-34}}{(2 \times 9.1 \times 10^{-31})(2 \times 1.6 \times 10^{-19})} = \frac{6.63 \times 10^{-34}}{7.63 \times 10^{-25}} = 8.6 \,\mathring{A}.$$

wavelength of the photon, In case of photon rest mass = 0, so the rest mass energy = 0
Hence, the energy of photon is wholly kinetic, which is given by

$$E = h\nu = \frac{hc}{\lambda}.$$

So $$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{2 \times 1.6 \times 10^{-19}} = 6.216 \times 10^{-7} m$$
$$= 6216 \,\mathring{A}$$ Ans

Q:- Using the relativistic relation for de-Broglie wavelength, calculate the wavelength of an electron of kinetic energy 1 MeV. What will be the value of λ if $K \ll m_0c^2$.

Ans :- The relativistic relation for de-Broglie wavelength is given by

$$\lambda = \frac{hc}{\sqrt{K(K+2m_0c^2)}}$$

For an electron, $m_0c^2 = 0.51\,MeV$

Given, $K.E. = 1\,MeV$ or $K \gg m_0c^2$

$$\therefore \lambda = \frac{(6.63\times10^{-34})\times(3\times10^{8})}{1.6\times10^{-19}\times\sqrt{[1\times10^{6}\times(1\times10^{6}+2\times.51\times10^{6})]}}$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{\sqrt{2}\times1.6\times10^{-13}}.$$

$$= 8.8\times10^{-3}\,Å.$$

If $K \ll m_0c^2$, then $K+2m_0c^2 \simeq 2m_0c^2$

$$\therefore \lambda = \frac{hc}{\sqrt{2m_0Kc^2}} = \frac{h}{\sqrt{2m_0K}}$$

Q:- Particle with rest mass m_0 is moving with speed c. Calculate de-Broglie wavelength associated with it.

Ans:- we know that
$$\lambda = \frac{h}{mv}$$
In the present case $\lambda = \frac{h}{mc}$

we know
$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Putting $v=c$, we get $m=\infty$

so de-Broglie wavelength $\lambda = \frac{h}{\infty} = 0$

Q:- Calculate de-Broglie wavelength associated with proton moving with a velocity equal to one twentieth of the velocity of light.

Ans:- Given that
$$v_p = \frac{c}{20}$$
de-Broglie wavelength, $\lambda = \frac{h}{m_p v_p}$
$$= \frac{h\times 20}{m_p \times c}$$
$$= \frac{6.6\times10^{-34}\times 20}{1.67\times10^{-27}\times 3\times10^{8}} = 2.63\times10^{-14}\,m$$
$$= 2.63\times10^{-4}\,\mathring{A}$$

Q:- Show that the group velocity of de-Broglie waves is equal to the velocity of the particle with which the waves are associated.

Ans:- The phase velocity of a wave associated with a particle comes out to be greater than the velocity of light which is impossible. This problem can be removed by considering a moving particle associated with a wave group or wave packet rather than a single wave.

Suppose a particle of rest mass m_0 is moving with velocity v. The angular frequency and propagation constant is given by $\omega = 2\pi\nu = \frac{2\pi E}{h} = \frac{2\pi mc^2}{h}$ —(1) $\because E = mc^2$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h} \quad \text{—(2)}$$

From theory of relativity $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$ —(3)

Putting the value of m in Eq.(1) & (2), we get
$$\omega = \frac{2\pi m_0 c^2}{h\sqrt{1-v^2/c^2}} \; ; \; k = \frac{2\pi m_0 v}{h\sqrt{1-v^2/c^2}}$$

Differentiating above eq^n, we get
$$\frac{d\omega}{dv} = \frac{2\pi m_0 v}{h(1-v^2/c^2)^{3/2}} \; , \; \frac{dk}{dv} = \frac{2\pi m_0}{h(1-v^2/c^2)^{3/2}}$$

but group velocity is given by $v_g = d\omega/dk = v$

Hence wave packet associated with a moving particle travels with the same velocity as the particle velocity.

Q:- Show that in relativistic case the ratio of compton wavelength and de-Broglie wavelength of an electron is

$$\frac{\lambda_c}{\lambda} = \frac{v}{c}\sqrt{1-\frac{v^2}{c^2}} = \sqrt{(E/m_0c^2)-1}$$

Ans:- We know compton wavelength is given by

$$\lambda_c = \frac{h}{m_0 c}$$

de-Broglie wavelength $\lambda = \frac{h}{mv} = \frac{h}{p}$

$$\frac{\lambda_c}{\lambda} = \frac{p}{m_0 c} = \frac{mv}{m_0 c} = \frac{v\, m_0/\sqrt{1-v^2/c^2}}{m_0 c} = \boxed{\frac{v}{c}\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \frac{\lambda_c}{\lambda}}$$

But $\quad E^2 = p^2c^2 + m_0^2c^4$

or $\quad p^2c^2 = E^2 - m_0^2c^4$

$$pc = \sqrt{E^2 - m_0^2c^4}$$

$$\frac{\lambda_c}{\lambda} = \frac{p}{m_0 c} = \frac{pc}{m_0c^2} = \frac{\sqrt{E^2 - m_0^2c^4}}{m_0c^2}$$

$$\boxed{\frac{\lambda_c}{\lambda} = \sqrt{\left(\frac{E}{m_0c^2}\right)^2 - 1}}$$

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Q:- A beam of monoenergetic neutrons corresponding to 27°C is allowed to fall on a crystal. A first order reflection is observed at a glancing angle 30°; calculate the interplanar spacing of the crystal. Given mass of neutron $= 1.67\times10^{-27}$ kg, Boltzmann constant $k = 1.38\times10^{-23}$ Joule/deg.).

Ans:- According to Bragg's law

$$2d\sin\theta = n\lambda$$

Given $\theta = 30°$, $n=1$

$$\therefore \quad 2d\sin 30° = \lambda$$

$$\text{or} \quad \lambda = d$$

The energy of neutron $E = kT$

$$T = 27+273 = 300\,°K$$

$$E = 1.38\times10^{-23}\times300 = 4.14\times10^{-21}\ \text{Joule}$$

Momentum $p = \sqrt{2mE}$

$$= \sqrt{2\times1.67\times10^{-27}\times4.14\times10^{-21}}$$

de-Broglie wavelength $\lambda = \frac{h}{p}$

$$\text{or} \quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2\times1.67\times10^{-27}\times4.14\times10^{-21}}}$$

$$= \frac{6.6\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times4.14\times10^{-21}}}$$

$$= 1.77\,°A$$

Q:- Davisson and Germer studied the electron diffraction with a nickel crystal for which the inter atomic distance 'd' was found to be 0.91 °A using x-rays. When the electrons with a kinetic energy of 54 eV were scattered the principal maximum occurs at $\theta = 65°$. Show that the experiment verified the de-Broglie relation. Given $\sin 65° = 0.9063$

Ans:- According to Bragg's law $2d\sin\theta = n\lambda$

Given $\theta = 65°$, $n=1$, $d = 0.91\,°A = 0.91\times10^{-10}$ m,

$$\therefore \quad \lambda = 2\times0.91\times10^{-10}\times\sin 65° = 2\times0.91\times10^{-10}\times0.9063 = 1.65\,°A$$

The momentum of each electron is $p = \sqrt{2mE} = \sqrt{2\times9.1\times10^{-31}\times54\times1.6\times10^{-19}} = 4.0\times10^{-24}$ kg m/sec

de-Broglie wavelength is given by $\lambda = \frac{h}{p} = \frac{6.67\times10^{-34}}{4.0\times10^{-24}} = 1.65\,°A$

we observe that this wavelength agrees with that obtained in the diffraction experiment.