Financial Mathematics
and
Time Value of Money

By:
Dr. Vijay Shankar Pandey
Asst. Professor
IMS, New Campus
University of Lucknow, Lucknow

Introduction

There are two basic question in front of every individual when we try to find the answer for the question: What is financial mathematics?

Question No. 1: What is mathematics?

Question No. 2: What is finance?

Answer for the question what is mathematics?

The tools applied with certain numbers to get quantitative value which is used to analyze and interpret according to fulfill individual needs and objectives.

And the answer to second question what is finance? In broader term it is analysis and evaluation of giving and getting money or in another term outflow and inflow of cash.

Now we are capable to get the answer that what one will do when they use mathematical tool to analyze finance (or outflow and inflow of cash).

The interpretation from both the answer is that one is using mathematical tools to analyze outflow and inflow of cash which support him in various type of decision making. Such as

What is rate of return?

What will be future value?

If some money one get in future what is its present value?

What is the concept of discounting?

And in last what is the value of money?

Is the value of money change in respect to time?

Why?

In all above questions two words are common ***time and value of money***.

Time? Means when the outflow and inflow will take place?

Value? Means the purchasing power of money is constant in respect to time or changing.

Means is it possible to buy a good or services by what amount of money today will buy same thing and quantity by same amount of money?

Means Is it possible for you to buy a pen from Rs. 10 today may buy same pen from Rs. 10 in future also?

Definitely not. Because prices are always increasing continuously in respect to time. Means you will pay more money to get same pen.

If some asking why this happened peoples are answering that this happened due to inflation.

Therefore we may say that due to inflation prices are increasing or value of money is decreasing.

As one may give (or lend)money to someone today and get return in future or get (or borrow) today and return it in future.

From this we see that outflow and inflow of money is at different times and value of money is changing in respect to time become necessary for one to analyze what S/he is paying and what they are getting in future (vice versa) have same value or not? The amount one will get as compensation to be remain indifference in respect of time have same level of satisfaction or not?

Answer to all these questions through quantitative calculation is called financial mathematics and time value of money.

So in the concept of financial mathematics outflow and inflow are at two different points.

We have two segment of valuation tools;

1. What will be future value of certain amount if you pay today?
2. And what is present value of certain amount you get in future?

These two questions formally called concept of

1. Future value of money, and
2. Present value of money

Before going to get the future or present value and decision to which of the mathematical tool (or formula) we use it become compulsory for us to know;

What is mode of outflow or inflow? And

How much time involved in the process?

Mode means number of times inflow and outflow take place.

One time or many time?

And time involved in the process means number of years to complete the transactions (or year to maturity).

And one another question how many times inflow or outflow take place within the year? The inflow or outflow is in the beginning or in the ending of the year.

The answer to all these questions are given under the concept of future value of money and present value of money.

Future Value of Money

Means one wanted to know how much amount one will get in future if S/he has a outflow at present?

Before going to answer this question one must first decide.

Mode: one time outflow or in many times

If may times it is equal amount on constant interval or different amount.

Or even different amount at same interval Or different amount at different interval.

Such as:

Mr. A have invested Rs. 10,000 one time for 10 year, or

Same Mr. A invested same amount (Rs. 10,000) equally in 10 instalments (Rs. 1000 annually). Or

Same Mr. A invested same amount (Rs. 10000) but in 1^{st} year Rs. 2000 and in 2^{nd} year Rs.1000, in3^{rd} year nothing, in 4^{th} year Rs.4000 in 6^{th} year Rs. 1000 and and in 10^{th} year Rs. 2000.

And the compounding is once in a year.

Here I want to clear that compounding is a factor which may also called rate of interest/ return on investment or the percentage of money given as a reward for lending money.

Now you may see all these illustration in table Value at the end of tenth year assuming rate of interest is 10 percent

Concept of future value assuming r=10% or 0.10 and time period(n)=10

Year	0	1	2	3	4	5	6	7	8	9	10
Plan 1	10000										
Plan 2		1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
Plan 3		2000	1000		4000		1000				2000

For Plan 1: We may write a formula

F= P(1+r)^n

For Plan 2: We may derived the formula

F= P1(1+r)^n-1+P2(1+r)^n-2+…..+Pn(1+r)^n-n…(1)

Multiply both the side of equ.(1)by (1+r) gives:

F(1+r)=P1(1+r)^n+P2(1+r)^n-1+….+Pn(1+r)…..(2)

As P1, P2, …….Pn have same value may we denote by A… We may rewrite equation (1) and (2) as..

F= A(1+r)^n-1+A(1+r)^n-2+…..+A(1+r)^n-n…(1)

F(1+r)=A(1+r)^n+A(1+r)^n-1+….+A(1+r)…..(2)

Fr=A[(1+r)^n-1]

F=A[(1+r)^n-1]/r

Concept of Due

When in any of the problem the word due come is simply denoting that one more year is added in your time period we may rewrite the formulae such as

For Plan 1: We may write a formula for due

F= (P(1+r)^n)(1+r)

For Plan 2: We may derive the formula for due

F=(A[(1+r)^n-1]/r)(1+r)

Present Value

Concept of present value is used to evaluate the inflow of cash (or get money) in future how much it is in today?

Before going to calculate the present value it become important for one to know what is mode of payment?

Is one get Only single amount? Or

Multiple?

If multiple it is equal amount on constant interval or variable (unequal) amount at constant interval or at different intervals.. means

Such as:

Mr. A have get Rs. 10,000 one time after 10 year, or

Same Mr. A get same amount (Rs. 10,000) equally in 10 instalments (Rs. 1000 annually). Or

Same Mr. A get same amount (Rs. 10000) but in 1^{st} year Rs. 2000 and in 2^{nd} year Rs.1000, in3^{rd} year nothing, in 4^{th} year Rs.4000 in 6^{th} year Rs. 1000 and and in 10^{th} year Rs. 2000.

And the discounting is once in a year.

Here I want to clear that discounting is a factor which may also called rate of interest/ return on investment or the percentage used for compensating to accept money in future date instead of today.

Concept of present value assuming r=10% or 0.10 and time period(n)=10

Year	0	1	2	3	4	5	6	7	8	9	10
Plan 1											10000
Plan 2		1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
Plan 3		2000	1000		4000		1000				2000

Plan1: Formula

P= F(1+r)^-n or P = F (1/(1+r)^n

Plan2: Formula

P= F1(1+r)^-1+ F2(1+r)^-2 +…….+Fn(1+r)^-n….(1)

F1, F2….Fn are common value we may replace it by A (called Annuity)

Rewrite equation (1) such as

P= A(1+r)^-1+ A(1+r)^-2 +…….+A(1+r)^-n….(1)

Multiplying both the side of equation (1) gives:

P(1+r)= A+ A(1+r)^-1+….+A(1+r)^-n+1…..(2)

Subtracting equ. (1) from equ. (2)

Pr= A[1-(1+r)^n] = A[1-1/(1+r)^n]

P = A[1-1/(1+r)^n]/r Or

P = A [(1+r)^n-1)/r(1+r)^n]

Concept of Due

When in any of the problem the word due come is simply denoting that one more year is added in your time period we may rewrite the formulae such as

For Plan 1: We may write a formula for due

P= F(1+r)^-n(1+r) or P = F (1/(1+r)^n (1+r)

For Plan 2: We may derived the formula for due

P = A [(1+r)^n-1)/r(1+r)^n](1+r)

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