Machine Design note by Er. Prashant Kumar Singh

Spur Gear. (unit. 1)

Gears are defined as toothed wheels or multilobed cams, which transmit power & motion from one shaft to another by means of teeth

Advantages:-

- → It is a positive drive and velocity ratio remains Constant.
- → The Centre distance between the shaft is relatively small, which results in compact ~~Construction~~ Construction.
- → It can transmit very large power, which is beyond the range of belt or chain drives.
- → It can transmit motion at very low velocity.
- → Efficiency is very high even upto 99% in case of spur gears.
- → The provision can be made in the gearbox for gear shifting, thus changing the velocity ratio over a wide range.

Classification of Gears

- → ~~Spure gear~~. spur gear
- → Helical gear.
- → Bevel gear.
- → Worm gear.

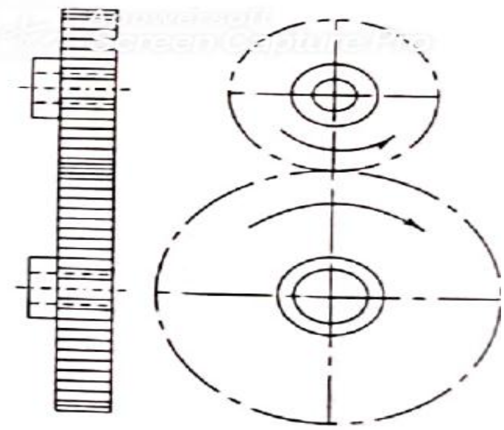


Fig. 17.1 *Spur gears*

Gears are broadly classified into four groups, viz. *spur, helical, bevel and worm* gears. A pair of spur gears is shown in Fig. 17.1. In case of spur gears, the teeth are cut parallel to the axis of the shaft. As the teeth are parallel to the axis of the shaft, spur gears are used only when the shafts are parallel. The profile of the gear tooth is in the shape of an involute curve and it remains identical along the entire width of the gear wheel. Spur gears impose radial loads on the shafts.

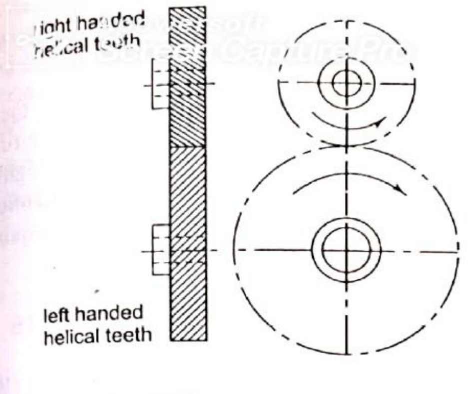


Fig. 17.2 *Helical gears*

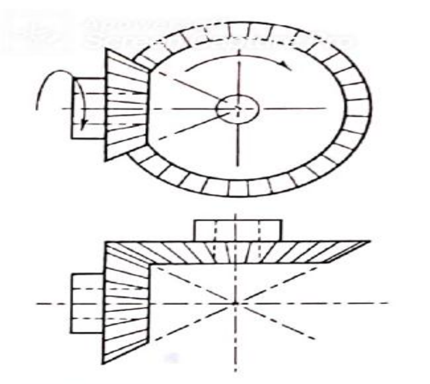


Fig. 17.4 *Bevel gears*

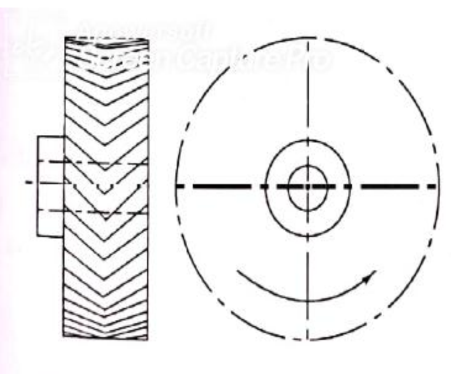


Fig. 17.3 *Herringbone gear*

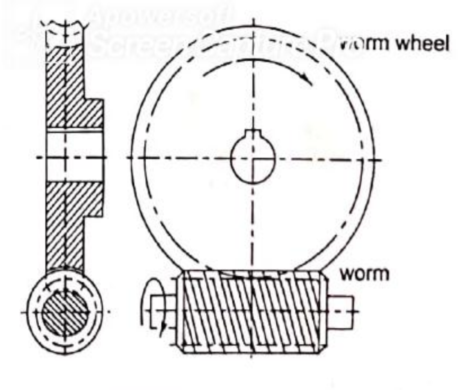
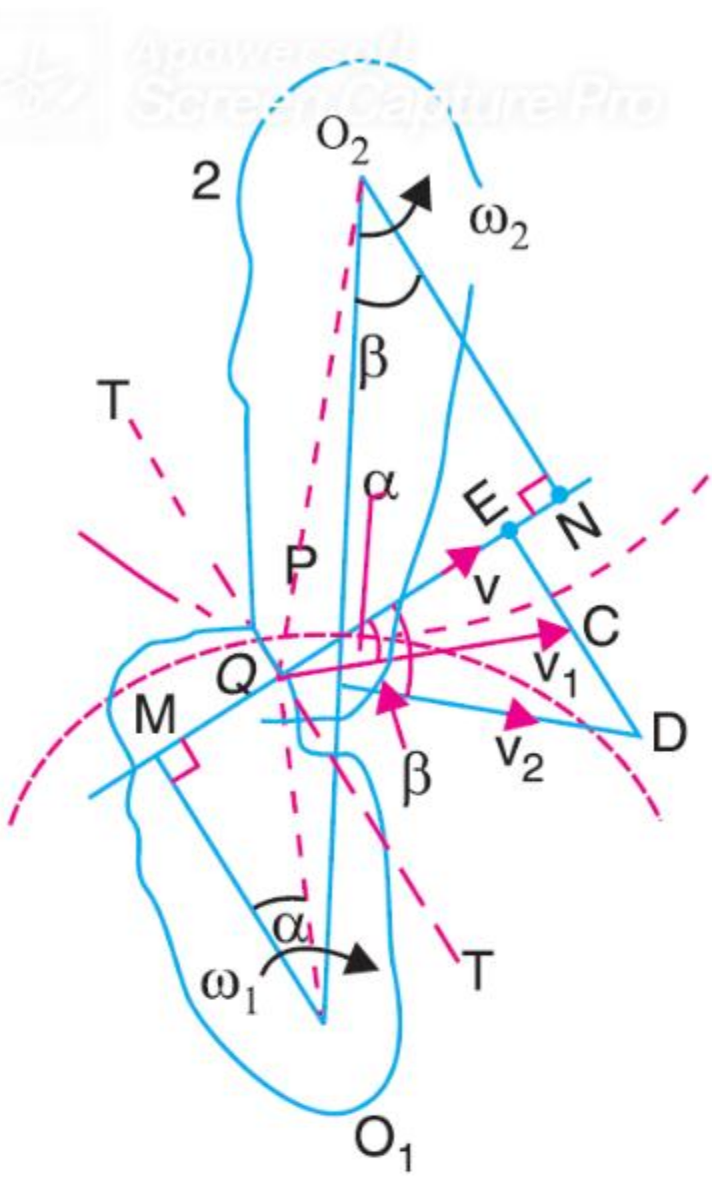


Fig. 17.5 *Warm gears*

LAW OF GEARING

The **law of gearing** states that the common normal at the point of contact between a pair of teeth must always pass through the pitch point. ... In other words, the angular velocity of a **gear** in mesh is inversely proportional to distance between the centre of **gear** and the pitch point.

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2. Let the two teeth come in contact at point *Q*, and the wheels rotate in the directions as shown in the figure.
Let *T T* be the common tangent and *MN* be the common normal to the curves at the point of contact *Q*. From the centres O_1 and O_2, draw O_1M and O_2N perpendicular to *MN*. A little consideration will show that the point *Q* moves in the direction *QC*, when considered as a point on wheel 1, and in the direction *QD* when considered as a point on wheel 2.
Let v_1 and v_2 be the velocities of the point *Q* on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal *MN* must be equal.



$$v_1 \cos\alpha = v_2 \cos\beta (\omega_1 \times O_1Q)\cos\alpha = (\omega_2 \times O_2Q)\cos\beta$$
$$(\omega_1 \times O_1Q)\frac{O_1M}{O_2Q} = (\omega_2 \times O_2Q)\frac{O_2N}{O_2Q}\omega_1 \times O_1M = \omega_2 \times O_2N\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}$$

Also from similar triangles O_1MP and O_2NP

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

Combining equations we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point *P* from the centers O_1 and O_2, or the common normal to the two surfaces at the point of contact *Q* intersects the line of centers at point *P* which divides the center distance inversely as the ratio of angular velocities.

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point *P* must be the fixed point (called pitch point) for the two wheels. In other words, ***the common normal at the point of contact between a pair of teeth must always pass through***

the pitch point. This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as ***law of gearing.***

Notes:

1. The above condition is fulfilled by teeth of involute form, provided that the root circles from which the profiles are generated are tangential to the common normal.
2. If the shape of one tooth profile is arbitrarily chosen and another tooth is designed to satisfy the above condition, then the second tooth is said to be conjugate to the first. The conjugate teeth are not in common use because of difficulty in manufacture, and cost of production.
3. If D_1 and D_2 are pitch circle diameters of wheels 1 and 2 having teeth T_1 and T_2 respectively, then velocity ratio,

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

Velocity of Sliding of Teeth

The sliding between a pair of teeth in contact at Q occurs along the common tangent T T to the tooth curves. ***The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact***.

The velocity of point Q, considered as a point on wheel 1, along the common tangent T T is represented by EC. From similar triangles QEC and O_1MQ,

$$\frac{EC}{MQ} = \frac{v_1}{O_1Q} = \omega_1 \quad \text{or} \quad EC = \omega_1.MQ$$

Similarly, the velocity of point Q, considered as a point on wheel 2, along the common tangent T T is represented by ED. From similar triangles QCD and O_2NQ,

$$\frac{ED}{QN} = \frac{v_2}{O_2Q} = \omega_2 \quad \text{or} \quad ED = \omega_2.QN$$

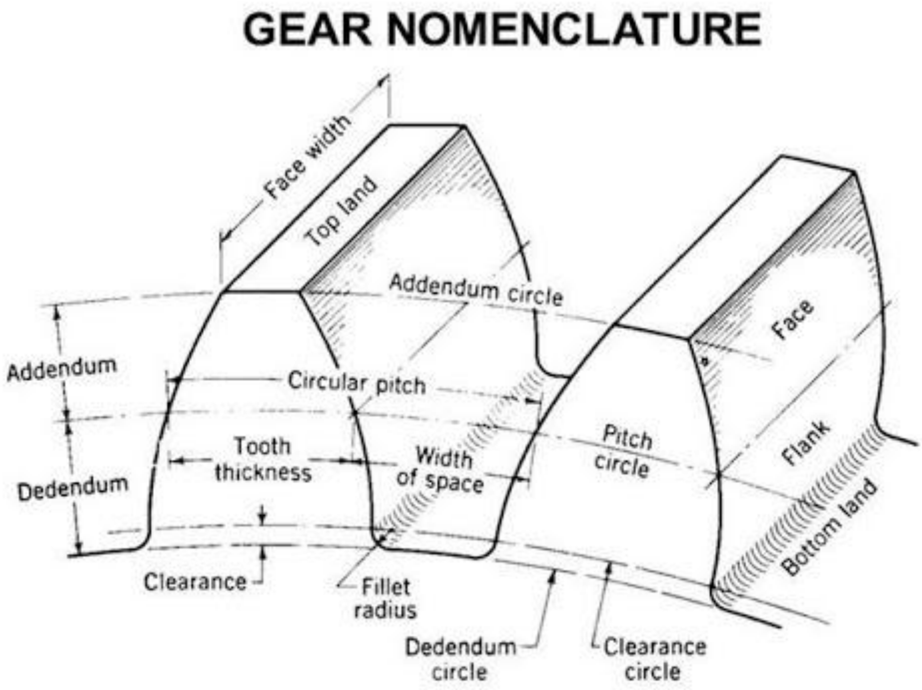
Let v_s = Velocity of sliding at Q.

$$v_s = ED - EC = \omega_2.QN - \omega_1.MQ v_s = \omega_2(QP + PN) - \omega_1(MP - QP)$$

$$v_s = (\omega_1 + \omega_2)QP + \omega_2.PN - \omega_1 MP \quad ..\text{(i)}$$

Since $\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{PN}{NP}$ or $\omega_1.MP = \omega_2.PN$

Therefore the equation becomes $v_s = (\omega_1 + \omega_2)QP$ ..(ii)



Face Width

Length of the Gear teeth in axial direction.

Base Circle

It is the circle from where involute teeth profile is generated.

Dedendum / Root Diameter

Root Diameter is the diameter of the gear at root circle or bottom teeth.

Pitch Circle

It is the imaginary circle on a gear where mating gears meet with each other.

Pitch Circle Diameter

Diameter of pitch circle is known as pitch circle diameter.

Addendum / Outside Diameter

It is the diameter of gear at the top tip of gear teeth.

Addendum

It is the height of gear teeth above pitch circle.

Dedendum

Height of gear teeth below pitch circle is known as Dedendum.

Diametral Pitch (p)

Diametral pitch of the gear is the number of teeth per inch of pitch circle diameter. It defines the spacing of teeth along the pitch circle.

p = number of teeth / pitch circle diameter

Circular Pitch

Circular pitch is the distance between two gear teeth at pitch circle.

Module

Module of a gear is inverse of diametric pitch. Mathematically it is equal to the ratio of pitch circle diameter and number of teeth. Gears with equal module can only mate with each other.

Module (m) = Pitch circle diameter / Number of teeth

Tooth Thickness

It is the width of the tooth at pitch circle diameter.

Tooth Space

It is the distance between two adjacent teeth at the pitch circle.

Tooth Face

Surface of the tooth above the pitch surface.

Flank of Tooth

It is the surface of the tooth below the pitch surface.

Top Land

Top tooth surface is known as gear top land .

Pitch Point

In mating gears, pitch point is point of contact between two pitch circles.

Pressure Angle

Pressure angle is the angle between gear center line and a line perpendicular to line of action. It defines tooth profile. Generally pressure angle value is kept 20°, 25°, 22.5°or 14.5°.

Clearance

Mathematically clearance is equal to the difference between the dedendum of one gear and the addendum of the mating gear.

Clearance = Dedendum of Gear2 – Addendum of Gear1

Backlash

It is the tangential space between gear mating teeth at pitch circles.

Backlash = Teeth Space – Teeth Width

STANDARD SYSTEM OF GEAR TOOTH

There are three standard systems for the shape of gear teeth. They are as follows:

(i) 14.5° Full depth involute system;

(ii) 20° Full depth involute systems; and

(iii) 20° Stub involute system.

1) **14.5^0 full depth involute system**

The basic rack for this system is composed of straight sides except for the fillet arcs. In this system interference occurs when number of teeth on pinion is less than 23.

2) **20^0 full depth involute system.**

The basic rack for the system is also composed of straight side except for the fillet arcs. In this system the interference occurs when the number of teeth are less than 17

3) **20^0 sub involute system.**

The gears in this system have shorter addendum and shorter dedendum. In this system, the minimum number of teeth are 14.

Table 17.1 Proportions of standard involute teeth (in terms of module m)

	14.5° Full depth system	*20° Full depth system*	*20° Stub system*
Pressure angle	14.5°	20°	20°
Addendum	m	m	0.8 m
Dedendum	1.157 m	1.25 m	m
Clearance	0.157 m	0.25 m	0.2 m
Working depth	2 m	2 m	1.6 m
Whole depth	2.157 m	2.25 m	1.8 m
Tooth thickness	1.5708 m	1.5708 m	1.5708 m

BACKLASH IN GEARS

For smooth rotation of meshed gears, backlash is necessary. Backlash is the amount by which a tooth space exceeds the thickness of a gear tooth engaged in mesh. Backlashes are classified in the following ways.

Types of Backlashes

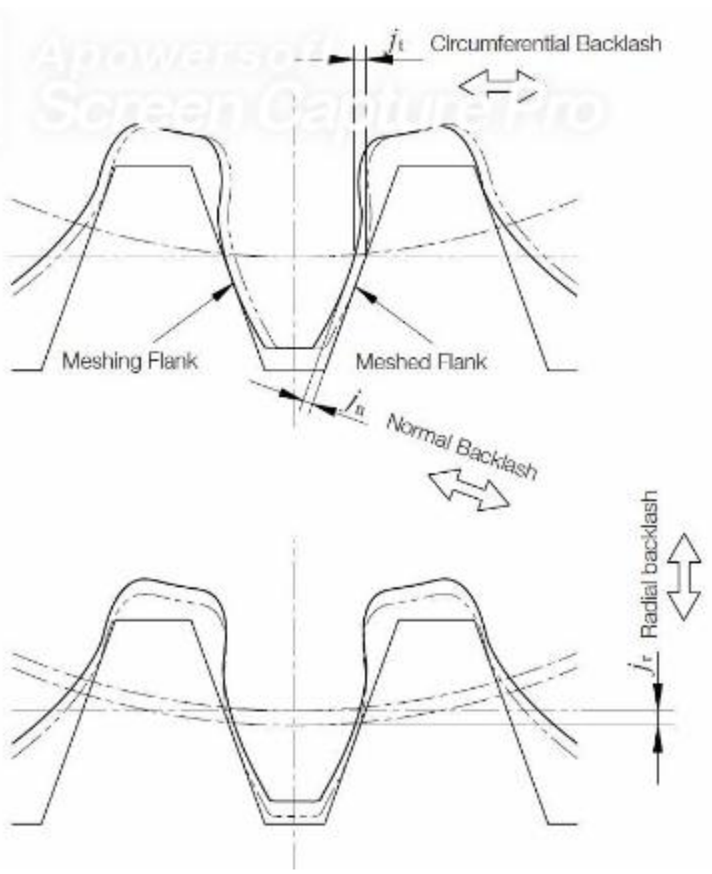
Circumferential Backlash (j t)
Circumferential Backlash is the length of arc on the pitch circle. The length is the distance the gear is rotated until the meshed tooth flank makes contacts while the other mating gear is held stationary.

Normal Backlash (j n)
The minimum distance between each meshed tooth flank in a pair of gears, when it is set so the tooth surfaces are in contact.

Angular Backlash (j θ)
The maximum angle that allows the gear to move when the other mating gear is held stationary.

Radial backlash (j r)
The radial Backlash is the shrinkage (displacement) in the stated center distance when it is set so the meshed tooth flanks of the paired gears get contact each other.

Axial Backlash (j x)
The axial backlash is the shrinkage (displacement) in the stated center distance when a pair of bevel gears is set so the meshed tooth flanks of the paired gears contact each other.


j_t Circumferential Backlash
Meshing Flank
Meshed Flank
j_n Normal Backlash
Radial backlash
j_r

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