<u>Business Research Methodology</u>

Chapter- Hypothesis Testing

(M. Com- IV Semester)

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What is Hypothesis Testing?

Hypothesis is usually considered as the principal instrument in research. The main goal in many research studies is to check whether the data collected support certain statements or predictions. A statistical hypothesis is an assertion or conjecture concerning one or more populations. Test of hypothesis is a process of testing of the significance regarding the parameters of the population on the basis of sample drawn from it. Thus, it is also termed as "Test of Significance'.

In short, hypothesis testing enables us to make probability statements about population parameter. The hypothesis may not be proved absolutely, but in practice it is accepted if it has withstood a critical testing.

Points to be considered while formulating Hypothesis

- Hypothesis should be clear and precise.
- Hypothesis should be capable of being tested.
- Hypothesis should state relationship between variables.
- Hypothesis should be limited in scope and must be specific.
- Hypothesis should be stated as far as possible in most simple terms so that the same is easily understandable by all concerned.
- Hypothesis should be amenable to testing within a reasonable time.
- Hypothesis must explain empirical reference.

Types of Hypothesis:

There are two types of hypothesis, i.e., Research Hypothesis and Statistical Hypothesis

1. **Research Hypothesis:** A research hypothesis is a tentative solution for the problem being investigated. It is the supposition that motivates the researcher to accomplish future course of action. In research, the researcher determines whether or not their supposition can be supported through scientific investigation.
2. **Statistical Hypothesis:** Statistical hypothesis is a statement about the population which we want to verify on the basis of sample taken from population. Statistical hypothesis is stated in such a way that they may be evaluated by appropriate statistical techniques.

Types of Statistical Hypotheses

There are two types of statistical hypotheses:

1. **Null Hypothesis (H_0)** – A statistical hypothesis that states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters.

2. **Alternative Hypothesis (H_1 or Ha)** – A statistical hypothesis that states the existence of a difference between a parameter and a specific value, or states that there is a difference between two parameters. Alternative hypothesis is created in a negative meaning of the null hypothesis.
   Suppose we want to test the hypothesis that the population mean (μ) is equal to the hypothesised mean (μ_{H0}) = 100. Then we would say that the null hypothesis is that the population mean is equal to the hypothesised mean 100 and symbolically we can express as:

$$H_0: \mu = \mu_{H0} = 100$$

   If our sample results do not support this null hypothesis, we should conclude that something else is true. What we conclude rejecting the null hypothesis is known as alternative hypothesis. In other words, the set of alternatives to the null hypothesis is referred to as the alternative hypothesis. If we accept H_0, then we are rejecting H_1 and if we reject H_0, then we are accepting H_1. For H_0: $\mu = \mu_{Ho} = 100$, we may consider three possible alternative hypotheses as follows:

Alternative hypothesis	To be read as follows
$H_1: \mu \neq \mu_{Ho}$	(The alternative hypothesis is that the population mean is not equal to 100 i.e., it may be more or less than 100)
$H_1: \mu > \mu_{Ho}$	(The alternative hypothesis is that the population mean is greater than 100)
$H_1: \mu < \mu_{Ho}$	(The alternative hypothesis is that the population mean is less than 100)

   The null hypothesis and the alternative hypothesis are chosen before the sample is drawn (the researcher must avoid the error of deriving hypotheses from the data that he/she collects and then testing the hypotheses from the same data). In the choice of null hypothesis, the following considerations are usually kept in view:
   1. Alternative hypothesis is usually the one which one wishes to prove and the null hypothesis is the one which one wishes to disprove. Thus, a null hypothesis represents the hypothesis we are trying to reject, and alternative hypothesis represents all other possibilities.
   2. Null hypotheses should always be specific hypothesis i.e., it should not state about or approximately a certain value.
   3. In testing hypothesis**,** there are two possible outcomes:
      - Reject H_0 and accept H_1 because of sufficient evidence in the sample in favour of H_1;
      - Do not reject H_0 because of insufficient evidence to support H_1.

BASIC CONCEPTS CONCERNING TESTING OF HYPOTHESES

1. **The level of significance:** This is a very important concept in the context of hypothesis testing. It is always some percentage (usually 5%) which should be chosen with great care, thought and reason. In case we take the significance level at 5 per cent, then this implies that H_0 will be rejected when the sampling result (i.e., observed evidence) has a less than 0.05 probability of occurring if H_0 is true. In other words, the 5 per cent level of significance means that researcher is willing to take as much as a 5 per cent risk of rejecting the null hypothesis when it (H_0) happens to be true. Thus, the significance

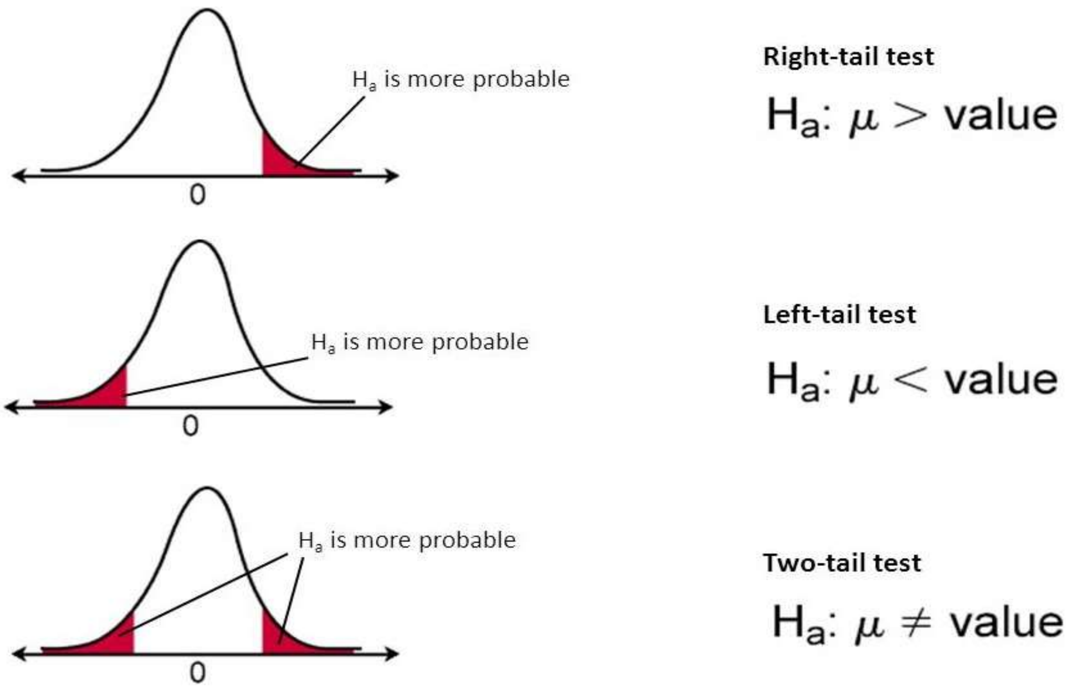
level is the maximum value of the probability of rejecting H_0 when it is true and is usually determined in advance before testing the hypothesis.

2. **Decision rule or Test of Hypothesis:** A decision rule is a procedure that the researcher uses to decide whether to accept or reject the null hypothesis. The decision rule is a statement that tells under what circumstances to reject the null hypothesis. The decision rule is based on specific values of the test statistic (e.g., reject H_0 if Calculated value > table value at the same level of significance)
3. **Types of Error:** In the context of testing of hypotheses, there are basically two types of errors we can make.
   a. **Type 1 error:** To reject the null hypothesis when it is true is to make what is known as a type I error. The level at which a result is declared significant is known as the type I error rate, often denoted by α.
   b. **Type II error:** If we do not reject the null hypothesis when in fact there is a difference between the groups, we make what is known as a type II error. The type II error rate is often denoted as β.

   In a tabular form the said two errors can be presented as follows:

Particulars	**Decision**	
	Accept H_0	**Reject H_0**
H_0 (True)	Correct Decision	Type I error (α error)
H_0 (False)	Type II error (β error)	Correct decision

4. **One- tailed and Two-tailed Tests:** A test of statistical hypothesis, where the region of rejection is on only one side of the sampling distribution, is called a one tailed test. For example, suppose the null hypothesis states that the mean is less than or equal to 10. The alternative hypothesis would be that the mean is greater than 10. The region of rejection would consist of a range of numbers located on the right side of sampling distribution i.e., a set of numbers greater than 10.
   A test of statistical hypothesis, where the region of rejection is on both sides of the sampling distribution, is called a two-tailed test. For example, suppose the null hypothesis states that the mean is equal to 10. The alternative hypothesis would be that the mean is less than 10 or greater than 10. The region of rejection would consist of a range of numbers located on both sides of sampling distribution; i.e., the region of rejection would consist partly of numbers that were less than 10 and partly of numbers that were greater than 10.

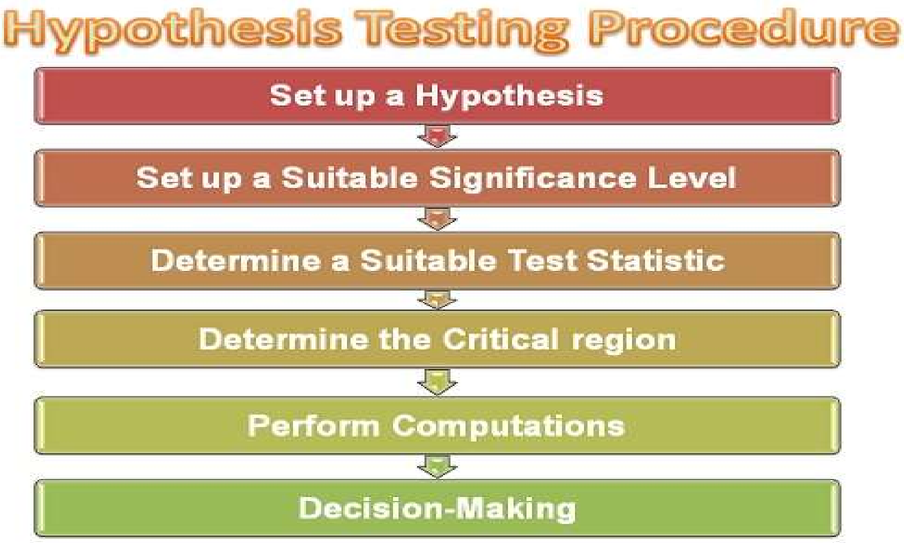


Procedure of Hypothesis Testing

Procedure for hypothesis testing refers to all those steps that we undertake for making a choice between the two actions i.e., rejection and acceptance of a null hypothesis. The various steps involved in hypothesis testing are stated below:

1. **Making a formal statement:** The step consists in making a formal statement of the null hypothesis (H_0) and also of the alternative hypothesis (Ha or H_1). This means that hypotheses should be clearly stated, considering the nature of the research problem.
2. **Selecting a significance level:** The hypotheses are tested on a pre-determined level of significance and as such the same should be specified. Generally, in practice, either 5% level or 1% level is adopted for the purpose.
3. **Deciding the distribution to use:** After deciding the level of significance, the next step in hypothesis testing is to determine the appropriate sampling distribution. The choice generally remains between normal distribution and the t-distribution.
4. **Selecting a random sample and computing an appropriate value:** Another step is to select a random sample(s) and compute an appropriate value from the sample data concerning the test statistic utilizing the relevant distribution. In other words, draw a sample to furnish empirical data.
5. **Calculation of the probability:** One has then to calculate the probability that the sample result would diverge as widely as it has from expectations, if the null hypothesis were in fact true.
6. **Comparing the probability and Decision making:** Yet another step consists in comparing the probability thus calculated with the specified value for α, the significance level. If the calculated probability is equal to or smaller than the α value in case of one-tailed test (and α /2 in case of two-tailed test), then reject the null hypothesis (i.e., accept the alternative hypothesis), but if the calculated probability is greater, then accept the null hypothesis.

The above stated general procedure for hypothesis testing can also be depicted in the form of a cart flow-



Tests of Hypotheses

Hypothesis testing determines the validity of the assumption (technically described as null hypothesis) with a view to choose between two conflicting hypotheses about the value of a population parameter. Hypothesis testing helps to decide on the basis of a sample data, whether a hypothesis about the population is likely to be true or false. Statisticians have developed several tests of hypotheses (also known as the tests of significance) for the purpose of testing of hypotheses which can be classified as:

a) Parametric tests or standard tests of hypotheses; and
b) Non-parametric tests or distribution-free test of hypotheses.

Parametric tests usually assume certain properties of the parent population from which we draw samples. Assumptions like observations come from a normal population, sample size is large, assumptions about the population parameters like mean, variance, etc., must hold good before parametric tests can be used. But there are situations when the researcher cannot or does not want to make such assumptions. In such situations we use statistical methods for testing hypotheses which are called non-parametric tests because such tests do not depend on any assumption about the parameters of the parent population. Besides, most non-parametric tests assume only nominal or ordinal data, whereas parametric tests require measurement equivalent to at least an interval scale. As a result, non-parametric tests need more observations than parametric tests to achieve the same size of Type I and Type II errors.

IMPORTANT PARAMETRIC TESTS

The important parametric tests are: (1) *z*-test; (2) *t*-test; and (3) *F*-test. All these tests are based on the assumption of normality i.e., the source of data is considered to be normally distributed.

1. **z- test**: It is based on the normal probability distribution and is used for judging the significance of several statistical measures, particularly the mean. This is a most frequently used test in research studies. This test is used even when binomial distribution or *t*-distribution is applicable on the presumption that such a distribution tends to approximate normal distribution as '*n*' becomes larger. *z*-test is generally used for comparing the mean of a sample to some hypothesised mean for the population in case of large sample, or when population variance is known. *z*-test is also used for judging he significance of difference between means of two independent samples in

case of large samples, or when population variance is known. z-test is also used for comparing the sample proportion to a theoretical value of population proportion or for judging the difference in proportions of two independent samples when n happens to be large. Besides, this test may be used for judging the significance of median, mode, coefficient of correlation and several other measures.

2. **t- test:** It is based on t-distribution and is considered an appropriate test for judging the significance of a sample mean or for judging the significance of difference between the means of two samples in case of small sample(s) when population variance is not known (in which case we use variance of the sample as an estimate of the population variance). In case two samples are related, we use paired t-test (or what is known as difference test) for judging the significance of the mean of difference between the two related samples. It can also be used for judging the significance of the coefficients of simple and partial correlations.
3. **F-test**: It is based on F-distribution and is used to compare the variance of the two-independent samples. This test is also used in the context of analysis of variance (ANOVA) for judging the significance of more than two sample means at one and the same time. It is also used for judging the significance of multiple correlation coefficients.

Non parametric Tests

Non parametric tests are used when the data isn't normal. Therefore, the key is to figure out if you have normally distributed data. The only non-parametric test you are likely to come across in elementary stats is the chi-square test. However, there are several others. For example: the Kruskal Willis test is the non-parametric alternative to the One-way ANOVA and the Mann Whitney is the non- parametric alternative to the two-sample t test.

Illustration1:

A sample of 400 male students is found to have a mean height 67.47 inches. Can it be reasonably regarded as a sample from a large population with mean height 67.39 inches and standard deviation 1.30 inches? Test at 5% level of significance.

Solution: Taking the null hypothesis that the mean height of the population is equal to 67.39 inches, we can write:

$$H_0: \mu_{H0} = 67.39''$$

$$H_1: \mu_{H0} \neq 67.39''$$

and the given information as $\bar{X}$ = 67.47'', $\sigma p = 1.30''$, n = 400. Assuming the population to be normal, we can work out the test statistic z as under:

Z-TEST

- Formula to find the value of Z (z-test) Is:

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

- $\bar{x}$ = mean of sample
- μ_0 = mean of population
- σ = standard deviation of population
- n = no. of observations

$$\text{Hence, } z = \frac{67.47-6\ \ .39}{1.30/\sqrt{400}} = \frac{0.08}{0.065} = 1.231$$

As Ha is two-sided in the given question, we shall be applying a two-tailed test for determining the rejection regions at 5% level of significance which comes to as under, using normal curve area table:

$$R: |z| > 1.96$$

The observed value of z is 1.231 which is in the acceptance region since R: | z | > 1.96 and thus, H0 is accepted. We may conclude that the given sample (with mean height = 67.47") can be regarded to have been taken from a population with mean height 67.39" and standard deviation 1.30" at 5% level of significance.

Illustration 2.

Suppose we are interested in a population of 20 industrial units of the same size, all of which are experiencing excessive labour turnover problems. The past records show that the mean of the distribution of annual turnover is 320 employees, with a standard deviation of 75 employees. A sample of 5 of these industrial units is taken at random which gives a mean of annual turnover as 300 employees. Is the sample mean consistent with the population mean? Test at 5% level.

Solution: Taking the null hypothesis that the population mean is 320 employees, we can write:

$$H_0: \mu_{H_0} = 320 \text{ employees}$$
$$H_a: \mu_{H_0} \neq 320 \text{ employees}$$

and the given information as under:

$$\bar{X} = 300 \text{ employees}, \ \sigma_p = 75 \text{ employees}$$
$$n = 5;\ N = 20$$

Assuming the population to be normal, we can work out the test statistic z as under:

$$z^* = \frac{\bar{X} - \mu_{H_0}}{\sigma_p/\sqrt{n} \times \sqrt{(N-n)/(N-1)}}$$

$$= \frac{300 - 320}{75/\sqrt{5} \times \sqrt{(20-5)/(20-1)}} = -\frac{20}{(33.54)\ (.888)}$$

$$= -0.67$$

As H_a is two-sided in the given question, we shall apply a two-tailed test for determining the rejection regions at 5% level of significance which comes to as under, using normal curve area table:

$$R: |z| > 1.96$$

The observed value of z is –0.67 which is in the acceptance region since $R: |z| > 1.96$ and thus, H_0 is accepted and we may conclude that the sample mean is consistent with population mean i.e., the population mean 320 is supported by sample results.

Illustration: 3

Raju Restaurant near the railway station at Falna has been having average sales of 500 tea cups per day. Because of the development of bus stand nearby, it expects to increase its sales. During the first 12 days after the start of the bus stand, the daily sales were as under:

550, 570, 490, 615, 505, 580, 570, 460, 600, 580, 530, 526

On the basis of this sample information, can one conclude that Raju Restaurant's sales have increased? Use 5 per cent level of significance.

Solution: Taking the null hypothesis that sales average 500 tea cups per day and they have not increased unless proved, we can write:

$H_0 : \mu = 500$ cups per day

$H_a : \mu > 500$ (as we want to conclude that sales have increased).

As the sample size is small and the population standard deviation is not known, we shall use *t*-test assuming normal population and shall work out the test statistic *t* as:

$$t = \frac{\bar{X} - \mu}{\sigma_s / \sqrt{n}}$$

(To find $\bar{X}$ and σ_s we make the following computations:)

S. No.	X_i	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$
1	550	2	4
2	570	22	484
3	490	–58	3364
4	615	67	4489
5	505	–43	1849
6	580	32	1024
7	570	22	484
8	460	–88	7744
9	600	52	2704
10	580	32	1024
11	530	–18	324
12	526	–22	484
$n = 10$	$\Sigma X_i = 6576$		$\Sigma (X_i - \bar{X})^2 = 23978$

$\therefore$
$$\bar{X} = \frac{\Sigma X_i}{n} = \frac{6576}{12} = 548$$

and
$$\sigma_s = \sqrt{\frac{\Sigma (X_i - \bar{X})^2}{n - 1}} = \sqrt{\frac{23978}{12 - 1}} = 46.68$$

Hence,
$$t = \frac{548 - 500}{46.68/\sqrt{12}} = \frac{48}{13.49} = 3.558$$

Degree of freedom = $n - 1 = 12 - 1 = 11$

As H_a is one-sided, we shall determine the rejection region applying one-tailed test (in the right tail because H_a is of more than type) at 5 per cent level of significance and it comes to as under, using table of t-distribution for 11 degrees of freedom:

$$R : t > 1.796$$

The observed value of t is 3.558 which is in the rejection region and thus H_0 is rejected at 5 per cent level of significance and we can conclude that the sample data indicate that Raju restaurant's sales have increased.

Illustration 4

Sample of sales in similar shops in two towns are taken for a new product with the following results:

Town	*Mean sales*	*Variance*	*Size of sample*
A	57	5.3	5
B	61	4.8	7

Is there any evidence of difference in sales in the two towns? Use 5 per cent level of significance for testing this difference between the means of two samples.

Solution: Taking the null hypothesis that the means of two populations do not differ we can write:

$$H_0 : \mu_1 = \mu_2$$
$$H_a : \mu_1 \neq \mu_2$$

and the given information as follows:

Sample from town A as sample one	$\bar{X}_1 = 57$	$\sigma^2_{s_1} = 5.3$	$n_1 = 5$
Sample from town B As sample two	$\bar{X}_2 = 61$	$\sigma^2_{s_2} = 4.8$	$n_2 = 7$

Since in the given question variances of the population are not known and the size of samples is small, we shall use t-test for difference in means, assuming the populations to be normal and can work out the test statistic t as under:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(n_1 - 1)\sigma^2_{s_1} + (n_2 - 1)\sigma^2_{s_2}}{n_1 + n_2 - 2}} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

with d.f. = $(n_1 + n_2 - 2)$

$$= \frac{57 - 61}{\sqrt{\frac{4(5.3) + 6(4.8)}{5 + 7 - 2}} \times \sqrt{\frac{1}{5} + \frac{1}{7}}} = -3.053$$

Degrees of freedom = $(n_1 + n_2 - 2) = 5 + 7 - 2 = 10$

As H_a is two-sided, we shall apply a two-tailed test for determining the rejection regions at 5 per cent level which come to as under, using table of t-distribution for 10 degrees of freedom:

$$R : | t | > 2.228$$

The observed value of t is -3.053 which falls in the rejection region and thus, we reject H_0 and conclude that the difference in sales in the two towns is significant at 5 per cent level.

Limitations of the Test of Hypotheses

- Test do not explain the reasons as to why does the difference exist, say between the means of the two samples. They simply indicate whether the difference is due to fluctuations of sampling or because of other reasons but the tests do not tell us as to which is/are the other reason(s) causing the difference.
- Results of significance tests are based on probabilities and as such cannot be expressed with full certainty.
- Statistical inferences based on the significance tests cannot be said to be entirely correct evidences concerning the truth of the hypotheses.

Conclusion:

A hypothesis is an educated guess about something in the world around us. Hypotheses are theoretical guesses based on limited knowledge; they need to be tested. Thus, hypothesis testing is a decision-making process for evaluating claims about a population. We use various statistical analysis to test hypotheses and answer research questions. In formal hypothesis testing, we test the null hypothesis and usually want to reject the null because rejection of the null indirectly supports the alternative hypothesis to the null, the one we deduce from theory as a tentative explanation. Thus, a hypothesis test mutually exclusive statements about a population to determine which statement is best supported by the sample data.