B.Sc (II Semester)

Physical chemistry

Binary Arithmetic

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In this Section, the four basic arithmetic operation are performed inside a computer by using binary numbers. Binary number system deals with only two digits – 0 and 1. Therefore, all the binary numbers are made up of only 0 and 1 and when arithmetic operations are performed on these numbers, the results are also in 0 and 1 only and the arithmetic is called Binary arithmetic.

Addition:- Binary addition is performed in the Same manner as decimal addition. However binary number system has only two digits, the addition table for binary arithmetic is very Simple, consisting of only four entries. For binary addition is as follows:

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 0$ plus a carry of 1 to Next higher Column.

For instance, 10 plus 10 binary requires the addition of two 1s in the Second position. Since $1+1 = 0$ plus a carry-over of 1, the sum of $10+10$ is 100 in binary.

By repeated use of the above rules, any two binary numbers can be added together by adding two bits at a time.

Examples:

$$\begin{array}{r} 0 \\ +\,0 \\ \hline 0 \end{array}, \quad \begin{array}{r} 1 \\ +\,0 \\ \hline 1 \end{array}, \quad \begin{array}{r} 1 \rightarrow \text{Carry} \\ 1 \\ +\,1 \\ \hline 1\,0 \\ 2^1\,2^0 \end{array} \quad \begin{array}{l} \rightarrow 1\times 2^0 = 1 \\ \rightarrow 1\times 2^0 = 1 \\ \hline 1\times 2^1 + 0\times 2^0 = 2 \end{array}$$

Carry → 1

$$\begin{array}{r} 1 \\ 1 \\ +\,1 \\ \hline 1\,1 \\ 2^1\,2^0 \end{array} \quad \begin{array}{l} \rightarrow 1\times 2^0 = 1 \\ \rightarrow 1\times 2^0 = 1 \\ \rightarrow 1\times 2^0 = 1 \\ \hline \rightarrow 1\times 2^1 + 1\times 2^0 = 3 \end{array}$$

Carry → 1

$$\begin{array}{r} 1010 \\ 1001 \\ \hline 10011 \\ 2^4\,2^3\,2^2\,2^1\,2^0 \end{array} \quad \begin{array}{l} \rightarrow 8+2 = 10 \\ \rightarrow 8+1 = 9 \\ \hline \rightarrow 16+2+1 = 19 \end{array}$$

Carry → 1   1

$$
\begin{array}{r l}
 & 100110 \rightarrow 38 \\
+ & 110101 \rightarrow 53 \\
\hline
 & 1011011 \rightarrow 91 \\
 & 2^6\,2^5\,2^4\,2^3\,2^2\,2^1\,2^0
\end{array}
$$

Carry → 1111 1

$$
\begin{array}{r l}
 & 101101 \rightarrow 32+08+04+1=45 \\
+ & 111101 \rightarrow 32+16+8+4+1=61 \\
\hline
 & 1101010 \rightarrow 64+32+0+8+0+2+0=106 \\
 & 2^6\,2^5\,2^4\,2^3\,2^2\,2^1\,2^0
\end{array}
$$

Carry → 11

$$
\begin{array}{r l}
 & 10011 \rightarrow 19 \leftrightarrow 16+0+0+2+1 \\
+ & 1001 \rightarrow 9 \leftrightarrow 8+0+0+1 \\
\hline
 & 11100 \rightarrow 28 \leftrightarrow 16+8+4+0+0 \\
 & 2^4\,2^3\,2^2\,2^1\,2^0
\end{array}
$$

$$
\begin{array}{r l}
 & 101 \rightarrow 5 \\
+ & 10 \rightarrow 2 \\
\hline
 & 111 \rightarrow 7 \\
 & 2^2\,2^1\,2^0
\end{array}
$$

Subtraction :

The principle of decimal Subtraction can as well be applied to subtraction of numbers in other number systems. It consists of two steps, which are repeated for each column of the numbers. The first step is to determine if it is necessary to borrow. If the lower digit is large than the upper digit, it is necessary to borrow from the column to the left. It is important to note that the value borrowed depends upon the base of the number system and is always decimals equivalent of the base. Hence, in decimal, 10 is borrowed, in binary, 2 is borrowed, in Octal, 8 is borrowed, in hexadecimal, 16 is borrowed. The Second Step is simply to subtract the lower value from the upper value. The Complete table for binary Subtraction is as follows;

$0-0=0$

$1-0=1$

$1-1=0$

$0-1=1$ with a borrow from the next column.

Observe that the only case in which it is necessary to borrow is when 1 is subtracted from 0.

<u>Examples:</u>

$$\begin{array}{rll} & 1 & \rightarrow 1\times 2^0 = 1 \\ - & 0 & \rightarrow 0\times 2^0 = 0 \\ \hline & 1 & \rightarrow 1\times 2^0 = \underline{\overline{1}} \end{array}$$

$$\begin{array}{rll} & 1 & \rightarrow 1\times 2^0 = 1 \\ - & 1 & \rightarrow 1\times 2^0 = 1 \\ \hline & 0 & \rightarrow 0\times 2^0 = \underline{\overline{0}} \end{array}$$

$$\begin{array}{rll} \text{borrow} \rightarrow & 0\;2 & \\ & \cancel{1}\cancel{0} & \rightarrow 1\times 2^1 + 0\times 2^0 \rightarrow 2 \\ - & 1 & \rightarrow 1\times 2^0 \rightarrow 1 \\ \hline & 0\,1 & \rightarrow 0\times 2^1 + 1\times 2^0 \rightarrow \underline{\overline{1}} \end{array}$$

$$\begin{array}{rll} \text{Borrow} \rightarrow & 0\;2\;\; & \\ & \cancel{1}\,0\,0 & \rightarrow 4 \\ - & 1\,0 & \rightarrow 2 \\ \hline & 0\,1\,0 & \rightarrow 2 \\ & 2^2\,2^1\,2^0 & \end{array}$$

$$
\begin{array}{lrcl}
\text{Borrow} & [\;0\;\overset{1}{\cancel{2}}\;2 & & \\
 & \cancel{1}\cancel{0}\cancel{0} & \longrightarrow & 4 \\
(-) & 1 & \longrightarrow & 1 \\
\hline
 & 011 & \longrightarrow & \underline{3} \\
 & 2^2\,2^1\,2^0 & &
\end{array}
$$

$$
\begin{array}{lrcl}
\text{Borrow} & [\;0\;\overset{1}{\cancel{2}}\;2 & & \\
 & \cancel{1}\cancel{0}\cancel{0} & \longrightarrow & 4 \\
(-) & 11 & \longrightarrow & 3 \\
\hline
 & 001 & \longrightarrow & \underline{1} \\
 & 2^2\,2^1\,2^0 & &
\end{array}
$$

$$
\begin{array}{lrclcl}
 & 0\;\overset{1}{\cancel{2}}\;\overset{1}{\cancel{2}}\;2 & & & & \\
 & \cancel{1}\cancel{0}\cancel{0}\cancel{0} & \longrightarrow & 8 & \leftrightarrow & 8+0+0+0 \\
(-) & 111 & \longrightarrow & 7 & \leftrightarrow & 4+2+1 \\
\hline
 & 0001 & \longrightarrow & \underline{1} & & \\
 & 2^3\,2^2\,2^1\,2^0 & & & &
\end{array}
$$

$$
\begin{array}{lrcl}
\text{Borrow} & [\;0\;\overset{1}{\cancel{2}}\;\overset{1}{\cancel{2}}\;2 & & \\
 & \cancel{1}\cancel{0}\cancel{0}\cancel{0} & \longrightarrow & 8 \\
(-) & 0001 & \longrightarrow & 1 \\
\hline
 & 0111 & \longrightarrow & \underline{7} \\
 & 2^3\,2^2\,2^1\,2^0 & &
\end{array}
$$

Borrow → 0 2

$\cancel{1}\cancel{0}1101 \rightarrow 32+0+8+4+0+1 = 45$

$(-)\ 10100 \rightarrow 16+0+4+0+0 = 20$

$011001 \rightarrow 0+16+8+0+0+1 = \underline{25}$

$2^5\, 2^4\, 2^3\, 2^2\, 2^1\, 2^0$

Borrow $\left[\begin{matrix} & 2 & 1 \\ 0 & \cancel{0} & \cancel{2} \end{matrix}\right.$ 2 0 2

$\cancel{1}\cancel{1}\cancel{0}\cancel{0}\cancel{1}\cancel{0}11 \rightarrow 203$

$(-)\ 1010110 \rightarrow 86$

$01110101 \rightarrow \underline{117}$

$2^7\, 2^6\, 2^5\, 2^4\, 2^3\, 2^2\, 2^1\, 2^0$

Borrow → 0 2

$\cancel{1}\cancel{0}11100 \rightarrow 64+16+8+4 = 92$

$(-)\ 0111000 \rightarrow 32+16+8 = 56$

$0100100 \rightarrow 32+4 = \underline{36}$

$2^6\, 2^5\, 2^4\, 2^3\, 2^2\, 2^1\, 2^0$

$1011 \rightarrow 11 \leftrightarrow 8+0+2+1$

$(-)\ 10 \rightarrow 2 \leftrightarrow 2+0$

$1001 \rightarrow 9 \leftrightarrow 8+0+0+1$

$2^3\, 2^2\, 2^1\, 2^0$

Multiplication:

Multiplication in the binary number system also follows the same general rules as multiplication in decimal number system. However, learning the binary multiplication is a trivial task, because the table for binary multiplication is very short, with only four enteries, instead of the 100 necessary for decimal multiplication. The complete table for binary multiplication is as follows:

$$0 \times 0 = 0$$
$$0 \times 1 = 0$$
$$1 \times 0 = 0$$
$$1 \times 1 = 1$$

The method of binary multiplication is illustrated with the example give below. It is only necessary to copy the multiplicand, if the digit in the multiplier is 1 and to copy all 0, if the digit in the multiplier is 0.

Examples:

$$
\begin{array}{lr l}
\text{Multiplicand} & 101 & \longrightarrow 4+0+1 = 5 \\
\text{Multiplier} & 11 & \longrightarrow 2+1 = 3 \\
\hline
 & 101 & \\
 & 1010 & \\
\hline
 & 1111 & \longrightarrow 8+4+2+1 = 15 \\
\hline
 & 2^3\,2^2\,2^1\,2^0 &
\end{array}
$$

$$
\begin{array}{lr l}
\text{Multiplicand} & 110 & \longrightarrow 6 \\
\text{Multiplier} & 101 & \longrightarrow 5 \\
\hline
 & 110 & \\
 & 0000 & \\
 & 11000 & \\
\hline
 & 11110 & \longrightarrow 30 \\
 & 2^4\,2^3\,2^2\,2^1\,2^0 &
\end{array}
$$

$$
\begin{array}{lr l}
\text{Multiplicand} & 1010 & \longrightarrow 10 \\
\text{Multiplier} & 100 & \longrightarrow 4 \\
\hline
 & 0000 & \\
 & 00000 & \\
 & 101000 & \\
\hline
 & 101000 & \longrightarrow 40 \\
\hline
 & 2^5\,2^4\,2^3\,2^2\,2^1\,2^0 &
\end{array}
$$

Multiplicand 1001 $\longrightarrow$ 9

Multiplier 1010 $\longrightarrow$ 10

$$
\begin{array}{r}
0000 \\
10010 \\
000000 \\
1001000 \\
\hline
1011010 \\
\hline
2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0
\end{array}
\longrightarrow 90
$$

Multiplicand 1010 $\longrightarrow$ 10

Multiplier 1001 $\longrightarrow$ 9

$$
\begin{array}{r}
1010 \\
00000 \\
000000 \\
1010000 \\
\hline
1011010 \\
\hline
2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0
\end{array}
\longrightarrow 90
$$

Division

Once again, division in binary number system is very simple. As in the decimal number system (or in any other number system), division by zero is meaningless. Hence, the complete table for binary division is as follows:

$$0 \div 1 = 0$$
$$1 \div 1 = 1$$

The division process is performed in a similar manner to decimal division. The rules for binary division are:

1. Start from the left of the dividend.
2. Perform a series of subtraction in which the divisor is subtracted from the dividend.
3. If subtraction is possible, put a 1 in the quotient and subtract the divisor from the corresponding digits of dividend.

Examples:-

$$100111 \div 11 = ?$$

Dividend ÷ Divisor = Quotient + Remainder

Solution:

$$\begin{array}{rl}
 & 001101 \\
11 & \overline{)\,100111} \\
 & -11 \\ \hline
 & 0011 \\
 & (-)\ 11 \\ \hline
 & 0011 \\
 & (-)\ 11 \\ \hline
 & 00
\end{array}$$

$11 > 01$

$11 > 10$

$11 < 100$

$11 > 001$

Verify the Quotient using binary number system.

$\underset{2^5 2^4 2^3 2^2 2^1 2^0}{100111} \longrightarrow 32+0+0+4+2+1 = 39$

$\underset{2^1 2^0}{11} \longrightarrow 2+1 = 3$

$39 \div 3 = 13$

$\underset{2^5 2^4 2^3 2^2 2^1 2^0}{001101} = 0+0+1\times 8+1\times 4+0+1\times 1 = 13$

$10100 \div 100 = ?$

$$
\begin{array}{r|l}
 & 00101 \\
\hline
100 & 10100 \\
 & -100 \\
\hline
 & 00100 \\
 & -100 \\
\hline
 & 000
\end{array}
$$

$100 > 1$

$100 > 10$

$100 < 101$

$100 > 10$

Ans:- 00101

Verify the Quotient by using Binary number.

$\underset{2^4 2^3 2^2 2^1 2^0}{10100} = 20$

$\underset{2^2 2^1 2^0}{100} = 4$

$20 \div 4 = 5$

$\underset{2^4 2^3 2^2 2^1 2^0}{00101} \rightarrow$ Quotient $\rightarrow 5$

$$
\begin{array}{r}
001001 \\
110 \overline{)110110} \\
-110 \\
\hline
0110 \\
-110 \\
\hline
000
\end{array}
$$

$110 > 1$
$110 > 10$
$110 = 110$
$110 > 01$
$110 > 011$

Quotient :- 001001

Verify the Quotient !

$\underset{2^5 2^4 2^3 2^2 2^1 2^0}{110110} \rightarrow 32+16+0+4+2+0 \times 1 = 54$

$\underset{2^2 2^1 2^0}{110} \rightarrow 4+2+0 = 6$

$54 \div 6 = 9$

$\underset{2^5 2^4 2^3 2^2 2^1 2^0}{001001} \rightarrow 8+1 = 9$

Divide 100001 by 110

$$
\begin{array}{r l}
 & 000101 \rightarrow \text{Quotient} \\
110 \,\big)\, & 100001 \\
 & -110\downarrow\downarrow \\
 & \overline{\quad 01001} \\
 & -\quad 110 \\
 & \overline{\quad 011} \rightarrow \text{Remainder}
\end{array}
$$

110 > 1

110 > 10

110 > 100

110 < 1001

Verify the Quotient and Remainder

$\underset{2^5 2^4 2^3 2^2 2^1 2^0}{100001} \longrightarrow 33$

$\underset{2^2 2^1 2^0}{110} \longrightarrow 6$

$33 \div 6 = 5$ and $3 \rightarrow$ Remainder

$\underset{2^5 2^4 2^3 2^2 2^1 2^0}{000101} \longrightarrow 5 \longrightarrow$ Quotient

$\underset{2^1 2^0}{11} \longrightarrow 3 \longrightarrow$ Remainder