Particle Spin

A particle moving through space around a point possesses angular momentum vector defined by

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \qquad (1)$$

Here **r** and **p** are the position vector and momentum respectively of the particle. Classically, there is no restriction on the magnitude or direction of orbital angular momentum. An electron carries a charge, hence from a classical perspective; its orbital motion will result in a tiny current loop that will produce a magnetic field. The magnetic moment μ of this magnetic field is defined as following

$$\boldsymbol{\mu}_{\mathbf{L}} = (q/2m)\, \mathbf{L} \qquad (2)$$

The spin angular momentum is associated with a rotating object such as the spinning ball or the spinning earth. The angular momentum or spin

$$\mathbf{S} = \mathbf{I}\boldsymbol{\omega} \qquad (3)$$

Here **I** is the moment of inertia and ω is the angular velocity. If the sphere possesses an electric charge, the circulation of the charge around the axis of rotation will constitute a current and hence will give rise to a magnetic field. The magnetic moment of this field for charge q is

$$\boldsymbol{\mu}_{\mathbf{S}} = (q/2m)\mathbf{S} \qquad (4)$$

exactly the same as in the orbital case.

Wave mechanics in general and the wave function in particular describe the properties of a particle moving through space, giving information on its position, momentum, energy, etc. It also provides a quantum description of the orbital angular momentum of a particle, such as that associated with an electron moving in an orbit around an atomic nucleus.

The magnitude of the angular momentum is limited to the values

$$\mathbf{L} = [\ell(\ell+1)]^{1/2} \qquad \ell = 0, 1, 2, \ldots \qquad (5)$$

If special relativity and quantum mechanics are combined, it is observed that even if a particle has zero momentum (i.e. the orbital angular momentum is zero) its total angular momentum is not zero. This angular momentum is due to the intrinsic spin of the particle. The possible values for the magnitude **S** of the spin angular momentum turn out to be

$$\mathbf{S} = [s(s+1)]^{1/2} \qquad s = 0, 1/2, 1, 3/2, 2 \ldots (6)$$

and any one vector component of S, say S_z, is restricted to the values $S_z = m_s$; $m_s = -s, -s+1, -s+2, \ldots s-2, s-1, s$ i.e. similar to orbital angular momentum, but with the significant difference of the appearance of half integer values for the spin quantum number s in addition to the integer values.

There is one classical property of angular momentum that does carry over to quantum mechanics. If the particle is charged, and if it possesses either orbital or spin angular momentum, then there arises a dipole magnetic field. In the case of the electron, the dipole moment is found to be given by

$$\mu_S = -(e/2m_e)g\mathbf{S} \qquad (7)$$

here m_e and $-e$ are the mass and charge of the electron, **S** is the spin angular momentum of the electron, and g is the gyromagnetic ratio, classically equal to one, but is known equal to two for an electron.

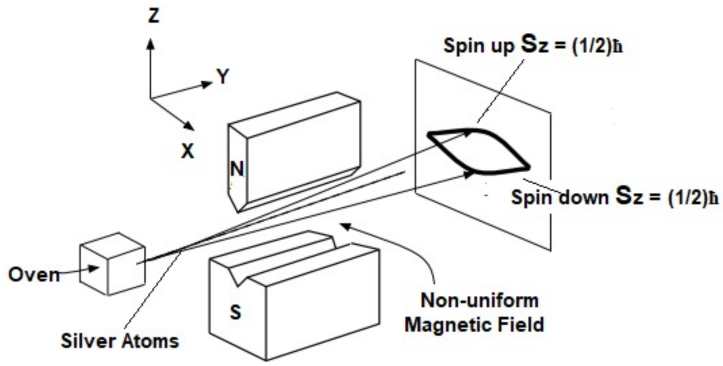
Electrons possess magnetic moment that makes it possible to perform experiments involving the spin of electrons that reveals the intrinsic quantum properties of spin.

The Stern-Gerlach Experiment

This experiment was first performed in **1922**. It is considered as the quintessential experiment that established that the electron possesses intrinsic angular momentum, i.e. the spin.

The original experiment had nothing to do with the discovery that the electron possessed spin. In fact, the first proposal by Uhlenbach and Goudsmit about the existence of the spin of the electron came in **1925**. This was based on the analysis of the atomic spectra. The goal of the original experiment was only to test the space-quantization of the orbital angular momentum of electrons. The prediction made by quantum theory developed out of Bohr's work, was that the spatial components of angular momentum could only take discrete values, so that the angular momentum vector had only discrete orientations.

An orbiting electron would give rise to a magnetic moment proportional to the orbital angular momentum of the electron. Thus, by measuring the magnetic moment of an atom, it would be possible to say if space quantization existed. The results of the experiment were in agreement with the expectations. The existence of electron spin was not at all under consideration. It was only later that it was realized that the interpretation of the results of the experiment needed revisit, and that the result of the experiment was direct evidence that electrons possessed spin. Thereafter, the results of the Stern-Gerlach experiment has been used to illustrate that electrons have spin.



The original experimental arrangement had a collimated beam of silver atoms moving in the y-direction passing through a non uniform magnetic field directed along z-direction. We assume that the silver atom has non-zero magnetic moment μ. The magnetic field has the following two effects:

i. The magnetic field will exert a torque on the magnetic dipole, so that the magnetic moment vector will *precess* about the magnetic field vector. This will not affect z-component of μ. However, the x and y components of μ will change with time.

ii. The atoms will experience a sideways force due to the non-uniformity of the field given by

$$F_z = -\partial \underline{U}/\partial z \qquad (8)$$

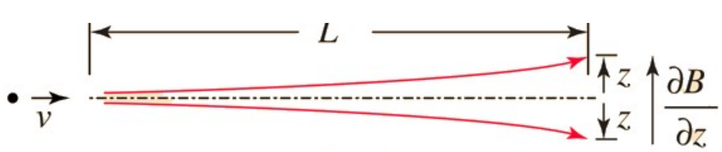
Here **U**=-**μ**.**B**=-μ_z **B** is the potential enegy of the silver atom. Thus

$$F_z = -\,\mu_z\, \partial \underline{B}/\partial z \qquad (9)$$

Classical physics predicts that due to thermal randomization in the oven, the magnetic dipole moment vectors of the atoms will be randomly oriented in space. This will lead to a continuous spread in the z-component of the magnetic moments of the silver atoms as they emerge from the oven, ranging from −|μz| to |μz|. Different values of μ_z will lead to different values of forces acting on the atoms depending on the magnitude of μ_z. As expected a line should then appear on the screen along the z-direction. However, the silver atoms arrived on the screen at two points that corresponding to the magnetic moments of

$$\mu_z = \pm\mu_B; \qquad \mu_B = e\hbar/2m_e \qquad (10)$$

Here μ_B is known as the Bohr magneton. The deflection can be shown to be proportional to the spin and to the magnitude of the magnetic field gradient. It is inversely proportional to the particle kinetic energy.



$$z = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m}\left[\frac{L}{v}\right]^2 = \pm\frac{\mu_B L^2}{4KE}\frac{\partial B}{\partial z}$$

Space quantization was confirmed by this experiment in no uncertain terms. However, the full significance of the results was not realized until sometime later, after the proposal by Uhlenbach and Goudsmit that the electron possessed intrinsic spin. The full explanation based on what is now known about the structure of the silver atom is as follows:

Silver atom has 47 electrons surrounding the nucleus. 46 of them form a closed inner core of total angular momentum zero, and the electrons with opposite spins pair off so the total angular momentum is zero. Thus, there is no contribution to magnetic moment due to the core of the silver atom. The 47^{th} electron also has zero orbital angular momentum. Hence, so the only source of any magnetic moment is that due to the intrinsic spin of the 47^{th} electron. In this way, the experiment represents a direct measurement of one component of the spin of the electron, the z-direction in the present experiment.

There are two possible values for S_z, corresponding to the two spots on the observation screen, as required by the fact that s = 1/2 for electrons, i.e. they are spin-1/2 particles. The allowed values for the z-component of spin are $S_z = \pm 1/2\hbar$ which, with the gyromagnetic value of two, yields the two values given in Eq. (7) for μ_z.

This experiment confirmed the quantization of electron spin into two orientations. This made a major contribution to the development of the quantum theory of the atom.