Knapsack problem

Problem:

- input:
  - ✓ n objects.
  - ✓ each object i has a weight w_i and a profit p_i
  - ✓ Knapsack : M
- output:
  - ✓ Fill up the Knapsack s.t. the total profit is maximized.
  - ✓ Feasible solution: $(x_1, \dots \dots \dots ., x_n)$.

Formally:

- ✓ Let xi be the fraction of object i placed in the Knapsack, $O \le x_i \le 1$.

  For $1 \le i \le n$.

  Then :

$$P = \sum_{1 \le i \le n} p_i x_i \quad \text{And} \quad \sum_{1 \le i \le n} w_i x_i \le M$$

Assumptions:

- $\sum_{i=1}^{n} wi > M$; not all xi= 1.
- $\sum_{1 \le i \le n} wi\ xi = M$

Example:

- 3 objects (n=3).
- (w1,w2,w3)=(18,15,10)
- (p1,p2,p3)=(25,24,15)
- M=20(knapsack capacity)

Largest-profit strategy: (Greedy method)

- Pick always the object with largest profit.
- If the weight of the object exceeds the remaining Knapsack capacity, take a fraction of the object to fill up the Knapsack.

Example:

P=0(profit) , C=M=20 /* remaining capacity */

- Put object 1 in the Knapsack.
  P=25 Since $w_1 < M$ then $x_1=1$
  C=M-18=20-18=2
- Pick object 2
  Since $C< w_2$ then $x_2= C/w_2=2/15$.
  P=25+2/15*24 =25+3.2=28.2
- Since the Knapsack is full then $x_3=0$.

The feasible solution is (1, 2/15,0)   P=28.2

Smallest-weight strategy:

- be greedy in capacity: do not want to fill the knapsack quickly.
- Pick the object with the smallest weight.
- If the weight of the object exceeds the remaining knapsack capacity, take a fraction of the object.

Example:

cu=M=20

- Pick object 3
  Since $w_3 <$ cu then $x_3=1$
  P= 15   cu =20-10 = 10 , x_3 =1

- Pick object 2
  Since $w_2 >$ cu then x_2 = 10/15 = 2/3
  P = 15+ 2/3.24
  = 15+ 16 = 31   cu= 0.

- Since cu=0 then $x_1=0$

- Feasible solution : (0,2/3,1)   P (profit)=31.

Largest profit-weight ratio strategy:

- Order profit-weight ratios of all objects.
- Pi/wi >=(pi+1)/(wi+1) for $1 \leq i \leq n-1$
- Pick the object with the largest p/w

- If the weight of the object exceeds the remaining knapsack capacity, take a fraction of the object.

Example:

$P_1/w_1=25/18=1.389$
$P_2/w_2=24/15=1.6$
$P_3/w_3=15/10=1.5$

$P_2/w_2 > P_3/w_3 > P_1/w_1$

Cu=20; p=0

- Pick object 2
  Since cu >= w_2 then $x_2=1$
  cu=20-15=5 and p=24

- Pick object 3
  Since cu<w_3 then x_3=cu/w_3=5/10=1/2
  cu= 0 and P= 24+1/2.15=24+7.5=31.5

- Feasible solution (0,1,1/2) P=31.5

Largest-profit strategy: 28.2

Smallest-weight strategy: 31.

Largest profit-weight ratio strategy: 31.5

Largest profit-weight ratio strategy gives maximum profit

Analysis

Analysis of knapsack problem with **Largest profit-weight ratio strategy:**

Step 1- Calculating P/w for all n object required O(n).

Step 2- Arranging P/w for all n object required sorting technique so time required for sorting P/w for all object is O(nlogn).

Step 3- If the weight of the object exceeds the remaining knapsack capacity, take a fraction of the object required O(n)

So **time complexity of knapsack problem** is O(nlogn) because here bigger loop exists.