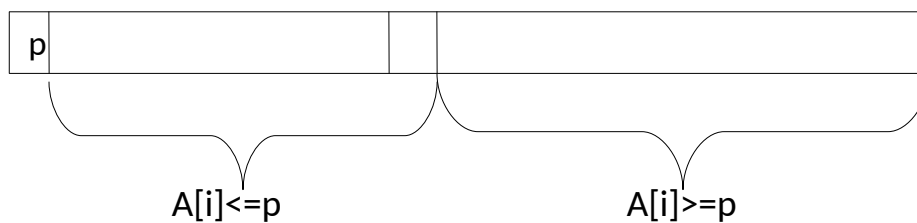


## Quick sort

Quick sort is application of divide and conquer. It is inplace algorithm. Quick sort is not stable. Following are the step for quick sort:

- Select a *pivot* (partitioning element) – here, the first element.
- Rearrange the list so that all the elements in the first  $s$  positions are smaller than or equal to the pivot and all the  $i$  elements in the remaining  $n-s$  positions are larger than or equal to the pivot.



- Exchange the pivot with the last element in the first (i.e.,  $\_$ ) sub-array — the pivot is now in its final position
- Sort the two sub-arrays recursively

### Partitioning Algorithm

```
Partition(arr , p ,q)
{
    x=arr[p], i=p;
    for(j=p+1; j<=q; j++)
    {
        if(arr[j]<=x)
        { i=i+1;
          Interchange(arr[i], arr[j]);
        }
    }
    Interchange(arr[i], arr[p]);
    return (i);
}
```

### Quick sort algorithm

```
Quick_sort(arr , p, q)
{
    if(p==q)
    {
        return (arr[p]);
    }
    else
    {
        m=Partition(arr, p, q);
        Quick_sort(arr, p, m-1);
        Quick_sort(arr, , m+1, q);
        return(arr)
    }
}
```

Example

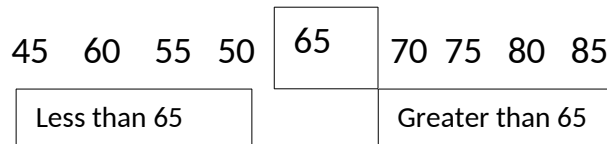
**Input:** 65 70 75 80 85 60 55 50 45

P: 65

**Pass 1:** 65 70 75 80 85 60 55 50 45

- i. i j if(arr[j]<=P) then swap (A[++i], A[j])  
65 70 75 80 85 60 55 50 45
- ii. i j if(arr[j]<=P) then swap (A[++i], A[j])  
65 70 75 80 85 60 55 50 45
- iii. i j if(arr[j]<=P) then swap (A[++i], A[j])  
65 70 75 80 85 60 55 50 45
- iv. i j if(arr[j]<=P) then swap (A[++i], A[j])  
65 70 75 80 85 60 55 50 45
- v. i j if(arr[j]<=P) then swap (A[++i], A[j])  
65 60 75 80 85 70 55 50 45
- vi. i j if(arr[j]<=P) then swap (A[++i], A[j])  
65 60 55 80 85 70 75 50 45
- vii. i j if(arr[j]<=P) then swap (A[++i], A[j])  
65 60 55 50 85 70 75 80 45
- viii. i j if(arr[j]<=P) then swap (A[++i], A[j])  
65 60 55 50 45 70 75 80 85

i j > 9 then swap (A[i],P)



**Apply same process in both the subsets**

45 60 55 50 & 70 75 80 85

Let  $T(n)$  be the amount of time taken by quick sort to sort an array of element of size  $n$ .

**Best case:** Occurs when every time partition algorithm divide elements into equal size of groups

$$\left. \begin{array}{l} O(1) \quad \text{if } n=1 \end{array} \right\}$$

$$T(n) = \begin{array}{ccc} O(n) & + & T(n/2) + T(n/2) \quad \text{if } n > 1 \\ | & & \swarrow \quad \searrow \\ \text{Partition /Divide cost} & & \text{2 sub-array} \end{array}$$

$T(n) = 2T(n/2) + n$  after solving this recurrence equation:

**Best case time complexity:  $\Theta(n \log_2 n)$**

**Worst case:** In the worst case the partition algorithm create 0 elements one side and n-1 elements on other side.

$$T(n) = \begin{cases} O(1) & \text{if } n=1 \\ O(n) & + \quad T(0) + T(n-1) \quad \text{if } n > 1 \end{cases}$$

|
\swarrow \quad \searrow  
 Partition /Divide cost                      2 sub-array

$T(n) = n + T(0) + T(n-1)$

$= n + T(n-1)$  after solving this recurrence equation:

**Worst case time complexity:  $O(n^2)$**

**Note:** when array is not at all sorted then quick sort performance best.

**Average case:** In this case some time we get lucky (equal partition) and some time we get unlucky in next recursion partition (unequal partition).

$T(n) = n + T(0) + T(n-1)$ ----- unlucky case

$T(n) = 2T(n/2) + n$ -----lucky case

**Average case time complexity=  $\Theta(n \log_2 n)$**

**Note:** Quick sort give worst case when array is sorted (whether ascending or descending)

**Stable sorting techniques:** The relative order of repeated elements is not change after sorting then the sorting technique is called stable sorting technique.

e.g:

input: 10<sub>a</sub> 11 1 10<sub>b</sub> 12

output 1 : 1 10<sub>a</sub> 10<sub>b</sub> 11 12 ----- stable sorting

output 2 : 1 10<sub>b</sub> 10<sub>a</sub> 11 12 ----- unstable sorting

**Randomized quick sort:** In this randomized quick sort difference is only pivot element is chosen randomly. But in normal quick sort pivot is first element.