Correlations: Partial and Multiple
For
M.Ed. And M.A. (Education) Classes

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Partial and multiple correlation

- It is an extension of simple or 2 variable linear correlation to problems which involve three or more variable
- Some time correlation between two variables is either zero or very little but correlation may be due to dependency of both variables on the third variables or several variables

Examples

- Height
- Weight
- Physical strength
- Vocabulary → Age from 5 to 20 years
- Reading skills
- General Intelligence
- Any two variables (Traits) showed correlation over age range(common maturity factor)
- If age eliminated, r may be zero

Controlling of Third variable

- There are **two methods**

A. **Experimentally**, by selecting all of whom are of the same age.

B. **Statistically**, by holding age variability constant through partial correlation.

Representation

- Let 1 =Height in inches
- 2= Vocabulary score

  And
- 3= age
- $r_{12.3}$ represent the partial correlation between 1 and 2 (Height and Vocabulary) when 3 (age) has been held constant or " Partialed out".
- 12.3 represent that variable is constant, leaving the net correlation between 1 and 2
- In the same way, $r_{12.34}$means that 2 variables namely, 3 and 4 are partialed out from the correlation between 1 and 2.
- The numbers to right the decimal point represent variables whose influence is ruled out

Use of partial correlation

- 1. In analysis in which the effect of some variable or variables are to be eliminated.
- 2. It unable us to setup a multiple regression equation of two or more variables by mean of which we can predict another variables.

Coefficient of multiple correlation

- The correlation between a set of obtained scores and same scores predicted from multiple regression equation is called a coefficient of multiple correlation.
- Represented by R (Multiple Correlation coefficient)
- Ex. $R_{1(234)}$ means score in variable (1) predicted from a multiple regression equation containing variables 2,3and 4
- In $R_{1(234)}$ 1 is criterion variable
- Symbol ( ) are independent variables inn regression equation.
- Multiple R is always positive.

Example

- N =450
- 1= Academic success (Result)
- 2= Intelligence
- 3= Average No. of hours spent in study per week

Q How well can we predict academic success from a knowledge of intelligence and hours spent to study?

- $M_1= 18.5$ $M_2= 100.6$ $M_3=24$
- $\sigma_1= 11.2$ $\sigma_2=15.8$ $\sigma_3=6$
- $r_{12}=.60$ $r_{13}=.32$ $r_{23}= -.35$

Step 1 : Writing of regression equations

Equations for multiple regression

$\bar{x}_1 = b_{12.3}\ x_2 + b_{13.2}\ x_3$ (Deviation form)

$\bar{X}_1 = b_{12.3}\ x_2 + b_{13.2}\ x_3 + K$ (Scores form)

Computation of partial r's

$$r_{12.3} = \frac{r_{12} - r_{13} . r_{23}}{\sqrt{1 - r_{13}^2} \cdot \sqrt{1 - r_{23}^2}}$$

$$r_{13.2} = \frac{r_{13} - r_{12} . r_{23}}{\sqrt{1 - r_{12}^2} \cdot \sqrt{1 - r_{23}^2}}$$

$$r_{23.1} = \frac{r_{23} - r_{12} . r_{13}}{\sqrt{1 - r_{12}^2} \cdot \sqrt{1 - r_{13}^2}}$$

$$r_{12.3} = \frac{r_{12} - r_{13} . r_{23}}{\sqrt{1 - r_{13}^2} \cdot \sqrt{1 - r_{23}^2}}$$

$$r_{12.3} = \frac{.60 + .112}{\sqrt{1 - .1024} \cdot \sqrt{1 - .1225}}$$

$$r_{12.3} = \frac{.712}{\sqrt{.8976}\ \sqrt{.877}}$$

$$r_{12.3} = \frac{.712}{.8871} = .80$$

$$r_{13.2} = \frac{r_{13} - r_{12} . r_{23}}{\sqrt{1 - r_{12}^2} \cdot \sqrt{1 - r_{23}^2}}$$

$$r_{13.2} = \frac{.32 - .60\ (-.35)}{.800 \text{ X } .937} = .71$$

$$r_{23.1} = \frac{r_{23} - r_{12} \cdot r_{13}}{\sqrt{1 - r_{12}^2} \cdot \sqrt{1 - r_{13}^2}}$$

$$r_{13.2} = \frac{-.35 - .60 \times .32}{.800 \times .937} = .72$$

Calculation of variability's
Variability's of all variables(Partial σ 's)

Computation of Partial σ 's

$$\sigma_{1.23} = \sigma_1 \sqrt{1 - r_{12}^2}\ \sqrt{1 - r_{13.2}^2} = .63$$
$$\sigma_{2.13} = \sigma_{2.31} = \sigma_2 \sqrt{1 - r_{23}^2}\ \sqrt{1 - r_{12.3}^2} = 8.9$$
$$\sigma_{3.12} = \sigma_{3.21} = \sigma_3 \sqrt{1 - r_{23}^2}\ \sqrt{1 - r_{13.2}^2} = 4.0$$

- $\sigma_{1.23} = \sigma_1 \sqrt{1 - r_{12}^2}\ \sqrt{1 - r_{13.2}^2}$

Calculation

$$\sigma_{1.23} = 11.2\ \sqrt{1 - (.60)^2}\ \sqrt{1 - (.71)^2}$$
$$= 11.2\ \sqrt{1 - .36}\ \sqrt{1 - .504}$$
$$= 11.2\ \sqrt{.64}\ \sqrt{.496}$$
$$= 11.2 \times .8 \times .704$$
$$= 6.31$$

$$\sigma_{2.13} = \sigma_{2.31} = \sigma_2 \sqrt{1 - r_{23}^2}\ \sqrt{1 - r_{12.3}^2}$$
$$\sigma_{2.13} = \sigma_2 \sqrt{1 - r_{23}^2}\ \sqrt{1 - r_{12.3}^2}$$
$$= 15.8\ \sqrt{1 - (-.35)^2}\ \sqrt{1 - (.80)^2}$$
$$= 15.8\ \sqrt{1 - .122}\ \sqrt{1 - .64}$$
$$= 15.8\ \sqrt{.878}\ \sqrt{.36}$$
$$= 15.8 \times .937 \times .6$$
$$= 8.88$$
$$= 8.9$$

$$\sigma_{3.12} = \sigma_{3.21} = \sigma_3 \sqrt{1 - r_{23}^2}\ \sqrt{1 - r_{13.2}^2}$$

$$r_{3.12} = \sigma_3\ \sqrt{1 - r_{23}^2} \sqrt{1 - r_{13.2}^2}$$
$$r_{3.12} = 6 \sqrt{1 - (.35)^2}\ \sqrt{1 - (.71)^2}$$
$$r_{3.12} = 6 \times .937 \times .704$$
$$r_{3.12} = 3.95$$
$$= 4.0$$

Computation of partial regression cofficent

$$b_{12.3} = r_{12.3} \times \frac{\sigma_{1.23}}{\sigma_{2.13}} = .80 \times \frac{6.3}{8.9} = .57$$

$$b_{13.2} = r_{13.2} \times \frac{\sigma_{1.23}}{\sigma_{3.12}} = .71 \times \frac{6.3}{4.0} = 1.12$$

Computation of partial regression equation

Regression equation

$\bar{x}_1 = b_{12.3}\ x_2 + b_{13.2}\ x_3$ (Deviation form)
$= .57\ x_2 + 1.12\ x_3$

$\bar{x}_1 = b_{12.3}\ x_2 + b_{13.2}\ x_3 + K$
$= .57\ x_2 + 1.12\ x_3 - 66$ (Scores form)

Prediction from multiple regression equation

- Ex. A particular student named Sandeep has an intelligence score 110 and studies on an average 18 hours per week.
- Q How many point in academic success should he receive after exam?

$\bar{X}_1 = b_{12.3}\ x_2 + b_{13.2}\ x_3 + K$
$= (.57 \times 110) + (1.12 \times 18) - 55$
$= 62.27 + 20.16 - 55$
$= 82.43 - .5577 = 27.47$

Most likely he will receive success point

Standard error of estimate

For a multiple regression $\sigma_{(est\ X_1)}$ is equal to $\sigma_{1.23}$ without any computation

$\sigma_{(est\ X_1)} = \sigma_{1.23} = 6.3$

Multiple Correlation Co-efficient

$$R_{1(23)} = 1 - \frac{\sigma_{(est\ X_1)}^{\ 2}}{\sigma_1^{\ 2}} = 1 - \frac{(6.3)^2}{11.2)^2}$$

$$R_{1(23)} = 1 - \frac{39.69}{125.44}$$

$= \sqrt{1 - .3164} = \sqrt{.6836}$
$= .83$

R= Multiple correlation coefficient
R^2(coefficient of multiple determination) shows common variance = .6889
K (coefficient of multiple non determination) shows remaining variance)
$K^2 = 1 - R^2$
$= 1 - .6889$
$K^2 = .311$
$K = .5577$

Thanks