Physical Chemistry

B.Sc. (II Semester)

Computer

Number System

Number systems are basically of two types:

1. Non-postional
2. Postional

In early days, human being counted on fingers. When ten fingers were not adequate then stones or sticks were used to indicate values. This method of counting uses an additive approach or the Nonpostional number systems.

In a Postional number system, there are only a few symbols, called digits and these symbols represent different values, depending on the position, they occupy in the number. The value of each digits in such a number is determined by three considerations:

1. The digit itself
2. The position of the digit in the number
3. The base of number system (where base is defined as the total numbers of digits available in the number system.)

Decimal Number System

The number system, which we use in our day-to-day life is called the Decimal Number System. In this system the base is equal to **10**. because there are altogether ten symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). In the decimal system, the successive positions to the left of the decimal point represent units, tens, hundreds, thousands, etc. However each position represents a specific power of the base (10). For example, the decimal number 2586 (2586_{10}) consists of the digit 6 in the units position, 8 in the tens position, 5 in the hundreds position and 2 in the thousands position and its value can be written as:

$$(2\times1000)+(5\times100)+(8\times10)+(6\times1) = 2000+500+80+6 = 2586$$

Hence, any number can be represented by using the available digits and arranging them in various positions.

Binary Number System :

The binary system is exactly like the decimal number system, except that base is 2, instead of 10. We have only two symbols or digits (0 and 1) which can be used in this number system.

Note that the largest single digit is 1 (one less than the base)

Each position in a binary number represents a power of the base (2). Hence in this system, the rightmost position is the units (2^0) position, the second position from the right is the 2's (2^1) position and proceeding in this way, we have 4's (2^2) position, 8's (2^3) position, 16's (2^4) position and so on. Therefore, the decimal equivalent of the binary number 10101 (10101_2) is:

$$(1\times2^4)+(0\times2^3)+(1\times2^2)+(0\times2^1)+(1\times2^0)=16+0+4+0+1=21$$

In order to be specific about which system we are referring to, it is common practice to indicate the base as a subscript. Hence

$$10101_2 = 21_{10}$$

Every computer stores numbers, letters and other special characters in binary form. When computer professionals need to know the raw data contained in a computer's memory. A commonly used way of doing this is to print out the memory contents on a printer. This printout is called Memory dump. Memory dumps which use binary numbers, would have many pages of 0s and 1s Hence, two number systems, Octal and Hexadecimal are often used as shortcut notations for binary.

3.

Octal Number System

In the Octal Number System, the base is 8. Here, there are only eight symbols or digits: 0,1,2,3,4,5,6,7 (8 and 9 do not exist in the system). The largest single digit is 7 (One less than the base). Each position in an octal number represents a power of the base (8). Therefore, the decimal equivalent of the octal number **2057** (2057_8) is:

$$(2\times 8^3) + (0\times 8^2) + (5\times 8^1) + (7\times 8^0) = 1024+0+40+7$$
$$= 1071$$

Hence $2057_8 = 1071_{10}$

Observe that there are only 8 digits in the Octal number System, 3 bits ($2^3=8$) are sufficient to represent any octal number in binary.

Hexadecimal Number System

In this number system one with a base of 16, having 16 single-character digits or symbols. The first 10 digits are the digits of the decimal number system – 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The remaining six digits are denoted by the symbols A, B, C, D, E and F, representing the decimal values 10, 11, 12, 13, 14 and 15 respectively. Hence the largest single digit is F or 15 (one less than the base). Each position in the hexadecimal system represents a power of the base (16). Therefore, the decimal equivalent of the hexadecimal number 1AF ($1AF_{16}$) is:

$$(1 \times 16^2) + (A \times 16^1) + (F \times 16^0) = (1 \times 256) + (10 \times 16) + (15 \times 16^0)$$
$$= 256 + 160 + 15 = 431$$

Hence $1AF_{16} = 431_{10}$

Observe that there are only 16 digits in the hexadecimal number system, 4 bits ($2^4 = 16$) are sufficient to represent any hexadecimal number in binary.

5.

Converting from One Number System to Another

(A) Converting to Decimal from Another Base

The following steps are used to convert a number to a base 10 value from any other number system:

Step 1: Determine the column (positional) value of each digit (this depends on the position of the digit and the base of the number system)

Step 2: Multiply the obtained column values (in step 1) by the digits in the corresponding columns.

Step 3: Sum the products calculated in step 2. The total is the equivalent value in decimals.

Examples. 1. $4052_6 = ?_{10}$

Solution: $4052_6 = 4 \times 6^3 + 0 \times 6^2 + 5 \times 6^1 + 2 \times 6^0$

$= 4 \times 216 + 0 \times 36 + 5 \times 6 + 2 \times 1$

$= 864 + 0 + 30 + 2 = 896_{10}$

Answer:- $4052_6 = 896_{10}$

2. $1AC_{13} = ?_{10}$

Solution: $1AC_{13} = 1 \times 13^2 + A \times 13^1 + C \times 13^0$

$= 1 \times 169 + 10 \times 13 + 12 \times 1$

$= 169 + 130 + 12$

$= 311_{10}$

3. $11001_2 = ?_{10}$

Solution: Step 1: Determine column values

Column Number (from right)	Column value
1	$2^0 = 1$
2	$2^1 = 2$
3	$2^2 = 4$
4	$2^3 = 8$
5	$2^4 = 16$

Step 2:

Multiply column values by Corresponding Column digits

16	8	4	2	1
×1	×1	×0	×0	×1
16	8	0	0	1

Step 3: Sum the product $16+8+0+0+1 = 25$

Hence $11001_2 = 25_{10}$

4. $4706_8 = ?_{10}$

Solution: <u>Step 1</u>: Determine column values

Column Number (from right)	Column value
1	$8^0 = 1$
2	$8^1 = 8$
3	$8^2 = 64$
4	$8^3 = 512$

<u>Step 2</u>: Multiply column values by corresponding column digits

512	64	8	1
X4	X7	X0	X6
2048	448	0	6

<u>Step 3</u>: Sum the products: $2048 + 448 + 0 + 6$

$= 2502$

Hence $4706_8 = 2502_{10}$

5. $1AC_{16} = ?_{10}$

Solution: $1AC_{16} = 1 \times 16^2 + A \times 16^1 + C \times 16^0$

$= 1 \times 256 + 10 \times 16 + 12 \times 1 = 256 + 160 + 12$

$= 428_{10}$

6. $11001_4 = ?_{10}$

Solution: $11001_4 = 1 \times 4^4 + 1 \times 4^3 + 0 \times 4^2 + 0 \times 4^1 + 1 \times 4^0$

$= 1 \times 256 + 1 \times 64 + 0 \times 16 + 0 \times 4 + 1 \times 1$

$= 256 + 64 + 0 + 0 + 1$

$= 321_{10}$

(B) Converting from Decimal to Another Base
(Division - Remainder Technique)

The following steps are used to convert a number from decimal to another base:

Step 1: Divide the decimal number to be converted by the value of the new base

Step 2: Record the Remainder from <u>Step 1</u> as the rightmost digit (least significant digit) of the new base number.

Step 3: Divide the quotient of the previous divide by the new base

Step 4: Record the remainder from step 3 as the next digit (to the left) of the new base number.

Repeat <u>Steps 3 and 4</u>, recording remainder from right to left until the quotient becomes zero in <u>Step 3</u>

<u>Note that</u> the <u>last remainder</u>, thus <u>obtained, will be the most significant digit of the new base number</u>.

Example

1. $952_{10} = ?_8$

Solution:

		Remainders in base 8
8	952	
	119	0
	14	7
	1	6
	0	1

Hence

$$952_{10} = 1670_8$$

2. $25_8 = ?_2$

Solution: Step 1 and 2 : $25/2 = 12$ and remainder 1
Step 3 and 4 : $12/2 = 6$ and remainder 0
Step 3 and 4 ; $6/2 = 3$ and remainder 0
Step 3 and 4 ; $3/2 = 1$ and remainder 1
Step 3 and 4 ; $1/2 = 0$ and remainder 1

As mentioned in steps 2 and 4, the remainders have to be arranged in the reverse order, making the first remainder the least significant digit (LSD) and the last remainder the most significant digit (MSD)

Hence $25_{10} = 11001_2$

3. $1715_{10} = ?_{12}$

Solution:

12	1715
	142
	11
	0

Remainders in base 12

$11 = B$

$10 = A$

$11 = B$

Hence $1715_{10} = BAB_{12}$

4. $428_{10} = ?_{16}$

Solution:

16	428
	26
	1
	0

Remainders in hexadecimal

$12 = C$

$10 = A$

$1 = 1$

Hence $428_{10} = 1AC_{16}$

5. $42_{10} = ?_{2}$

Solution:

		Remainders
2	42	0
	21	1
	10	0
	5	1
	2	0
	1	1
	0	

Hence $42_{10} = 101010_2$

Convert from a Base other than 10 to a Base other than 10

The following steps are used to convert a number from a base other than 10, to a base other than 10

Step 1: Convert the original number to a decimal number (base 10)

Step 2: Convert the decimal number obtained in Step 1 to the new base number.

Example

1. $11010011_2 = ?_{16}$

Solution: Step 1: Convert 11010011_2 to base 10

$$11010011_2 = 1\times2^7+1\times2^6+0\times2^5+1\times2^4+0\times2^3+0\times2^2+1\times2^1+1\times2^0$$

$$= 1\times128+1\times64+0\times32+1\times16+0\times8+0\times4+1\times2+1\times1$$

$$= 128+64+0+16+0+0+2+1 = 211_{10}$$

Step 2: Convert 211_{10} to base 16

		Remainder
16	211	
	13	3 = 3
	0	13 = D

Therefore, $11010011_2 = 211_{10} = D3_{16}$

2. $545_6 = ?_4$

Solution:

Step 1: Convert from base 6 to base 10

$545 = 5 \times 6^2 + 4 \times 6^1 + 5 \times 6^0$

$= 5 \times 36 + 4 \times 6 + 5 \times 1 = 180 + 24 + 5 = 209_{10}$

Step 2: Convert 209_{10} to base 4

4	209	Remainder
	52	1
	13	0
	3	1
	0	3

$209_{10} = 3101_4$

Therefore, $545_6 = 209_{10} = 3101_4$

Hence $545_6 = 3101_4$

13.

Shortcut Method for Binary to Octal Conversion

The following steps are used in this method.

Step 1: Divide the binary digits into groups for three (starting from the right)

Step 2: Convert each group of three binary digits to one octal digit. Since there are only 8 digits (0 to 7) in the octal number system, 3 bits ($2^3 = 8$) are sufficient to represent any octal number in binary. Moreover, decimal digits 0 to 7 are equal to octal digits 0 to 7, binary to decimal conversion can be used in this step.

Example

1. $1101010_2 = ?_8$

Solution.

$1101010_2 = \underline{001}\ \underline{101}\ \underline{010}$ (Group of 3 digit from right)

$= 152_8$ (Convert each group to an octal digit)

Hence $1101010_2 = 152_8$

2. $101110_2 = ?_8$

Solution: Step 1: Divide the binary digits into groups of 3, starting from right (LSD)

$\underline{101}\ \underline{110}$

Step 2. Convert each group into one digit of octal (use binary-to-decimal conversion)

$101_2 = 1\times2^2 + 0\times2^1 + 1\times2^0$
$= 4+0+1 = 5_8$

$110_2 = 1\times2^2 + 1\times2^1 + 0\times2^0$
$= 4+2+0 = 6_8$

Hence

$101110_2 = 56_8$

Shortcut Method for Octal to Binary Conversion

The following steps are used in this method.

Step 1. Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for the conversion)

Step 2. Combine all the resulting binary groups (of 3 digits each) into a single binary number.

15.

Example

1. $6751_8 = ?_2$

Solution: $6751_8 = \underset{6}{110}\ \underset{7}{111}\ \underset{5}{101}\ \underset{1}{001}$

$= 110111101001_2$

2. $562_8 = ?_2$

Solution. Step 1: Convert each octal digit to 3 binary digits

$5_8 = 101_2 \quad 6_8 = 110_2 \quad 2_8 = 010_2$

Step 2: Combine the binary group

$562_8 = \underset{5}{101}\ \underset{6}{110}\ \underset{2}{010}$

$562_8 = 101110010_2$

Shortcut Method for Binary to Hexadecimal Conversion

The following steps are used in this method:

Step 1: Divide the binary digits into groups of four (Starting from the right)

Step 2.: Convert each group of four binary digits to one hexadecimal digit. Remember that hexadecimal digits 0 to 9 are equal to decimal digit 0 to 9, and hexadecimal digits A to F are equal to decimal values 10 to 15.

16.

Example

1. $1011010110 0_2 = ?_{16}$

Solution $10110101100_2 = 0101\ 1010\ 1100$ (Group 4 digits from right)

$= 5AC$ (Convert each group to a hexadecimal digit.)

Hence

$10110101100_2 = 5AC_{16}$

2. $11010011_2 = ?_{16}$

Solution. Step 1 : Divide the binary digits into groups of 4, starting from the right (LSD)

$\underline{1101}\ \underline{0011}$

Step 2. : Convert each group of 4 binary digits to 1 hexadecimal digit.

$1101_2 = 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

$= 8+4+0+1 = 13_{10} = D_{16}$

$0011 = 0\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

$= 0+0+2+1 = 3_{16}$

Hence $11010011_2 = D3_{16}$

Shortcut Method for Hexadecimal to Binary Conversion

The following steps are used ~~to~~ this method.

Step 1: Convert the decimal equivalent of each hexadecimal digit to 4 binary digits.

Step 2 ; Combine all the resulting binary groups (of 4 digits each) into a single binary number.

Example:

1. $2ABC_{16} = ?_2$

Solution: Step 1: Convert the decimal equivalent of each hexadecimal digit to 4 binary digits.

$2_{16} = 2_{10} = 0010_2$ $A_{16} = 10_{10} = 1010_2$

$B_{16} = 11_{10} = 1011_2$ $C_{16} = 12_{10} = 1100_2$

Step 2: Combine the binary groups

$$2ABC_{16} = \underset{2}{\underline{0010}}\ \underset{A}{\underline{1010}}\ \underset{B}{\underline{1011}}\ \underset{C}{\underline{1100}}$$

Hence $2ABC_{16} = 0010101010111100_2$

Q: $ABC_{16} = ?_2$

Solution: $A_{16} = 1010$

$B_{16} = 1011$

$C_{16} = 1100$

i.e. $\underset{A}{1010}\ \underset{B}{1011}\ \underset{C}{1100}$

Hence $ABC_{16} = 101010111100_2$

Fractional Numbers

In binary number system, fractional numbers are formed in the same general way as in the decimal number system. For example in the decimal number system:

$0.325 = (3\times10^{-1})+(2\times10^{-2})+(5\times10^{-3})$

$68.325 = (6\times10^{1})+(8\times10^{0})+(3\times10^{-1})+2\times10^{-2}$

Similarly, in the binary system

$0.101 = (1\times2^{-1})+(0\times2^{-2})+(1\times2^{-3})$

$10.01 = (1\times2^{1})+(0\times2^{0})+(0\times2^{-1})+(1\times2^{-2})$

The binary point serves the same purpose as the decimal point. Some of the positional values in the binary number system are given:

Binary Point

Position	4	3	2	1	0	-1	-2	-3	-4
Position Value	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Quantity Represented	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

In general, a number in a number system with base 'b' would be written as

$$a_n a_{n-1} \cdots a_0 \, a_{-1} a_{-2} \cdots a_{-m}$$

and would be interpreted to mean

$$a_n \times b^n + a_{n-1} \times b^{n-1} + \cdots + a_0 \times b^0 + a_{-1} \times b^{-1} + a_{-2} \times b^{-2} + \cdots + a_{-m} \times b^{-m}$$

$46.32_8 = 4\times8^1 + 6\times8^0 + 3\times8^{-1} + 2\times8^{-2}$

$5A.3C_{16} = (5\times16^1)+(A\times16^0)+(3\times16^{-1})+(C\times16^{-2})$

Example :-

1. Find the decimal equivalent of the Octal Number 127.54

Solution: $127.54_8 = (1\times8^2)+(2\times8^1)+(7\times8^0)+(5\times8^{-1})+(4\times8^{-2})$

$= 64+16+7+5/8+5/64$

$= 87+0.625+0.0625 = 87.6675_{10}$

2. Find the decimal equivalent of the hexadecimal number 2B.C4

Solution: $2B.C4 = 2\times16^1 + B\times16^0 + C\times16^{-1} + 4\times16^{-2}$

$= 32 + B + C/16 + 4/256 = 32+11+12/16+4/256$

$= 43+0.75+0.015625 = 43.765625_{10}$

20.