# SAMPLING INSPECTION

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## 1. Introduction

We have seen how the control charts enable a production process to be kept in control. But the process control does not imply lot control, that is, all the lots produced are good. This means that inspection of lots is required. We resort to sampling inspection, which is a procedure to determine whether a lot of manufactured items should be accepted or rejected on the base of the information supplied by random samples drawn from the lot under consideration. It is also called 'acceptance sampling'.

We usually deal with sampling inspection for attributes, i.e. the items are judged good or bad by inspection and the lot quality is judged by the sample fraction defective.

A sampling plan specifies the procedure for deciding when the lot under inspection is to be rejected or accepted. Usually corrective action is taken when the lot is rejected – it is inspected fully and all its defective items are replaced by good ones. This is known as rectifying inspection. Suppose the incoming lots have fraction defective $p_0$, then the outgoing lots, after rectifying inspection will have fraction defective $p_1 < p_0$

Before describing sampling plans in particular, we will introduce some terms relating to a sampling plans.

## 2. Acceptable Quality Level (AQL)

This represents the poorest level of quality of the items produced which the consumer would consider to be acceptable as a process average. It is denoted by p.

## 3. Lot Tolerance Proporation Defective (LTPD)

This represents the poorest level of quality that the consumer is willing to accept in an individual lot. The consumer will not accept lots having proporation defective more than LTPD. It is denoted by $p_t$ and $\bar{p} < p_t$

## 4. Producer's Risk

Suppose that the producer claims that he has standardized the quality of product at a level of fraction defective $\bar{p}$ (the producers process average). The probability of rejecting a lot under the sampling inspecting plan when the fraction defective is actually $\bar{p}$ is called the producer's risk, denoted by $\alpha$.

i.e.

Producer's risk = $\alpha$

= P (rejecting the lot of acceptable quality, $\bar{p}$)

## 5. Consumer's Risk

The consumer has to face the risk of accepting a lot of unsatisfactory quality, on the basis of sampling inspection the probability of accepting a lot with fraction defective equal to LTPD, under a sampling plan, is called the consumer's risk denoted by $\beta$.

i.e.

Consumer's Risk = $\beta$

= P (accepting the lot of rejectable quality $P_t$)

## 6. OC Function

This gives the probability of accepting the lot, as a function of the lot fraction defective.

i.e.,

$P_a(p) = L(p)$ = P (accepting the lot of quality p)

= P {lot is accepted /(lot contains fraction defectives p)}

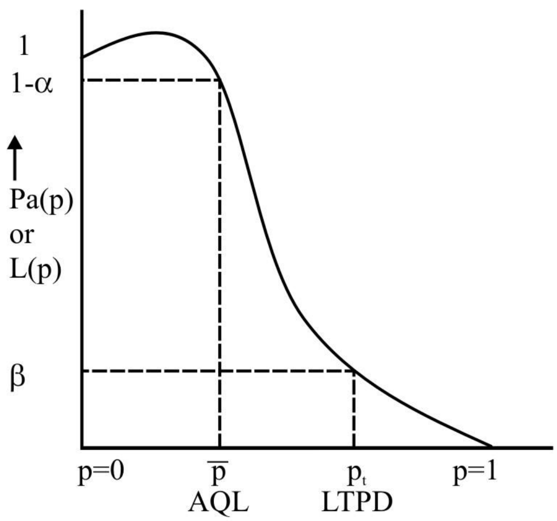
Also

$$\alpha = 1 - L(\bar{p})$$

or $[L(\bar{p}) = 1 - \alpha]$

and

$$[\beta = L(p_t)]$$



It is evident that the points on the oc curve corresponding to LTDP and AQL are $\beta$ and $1-\alpha$, respectively.

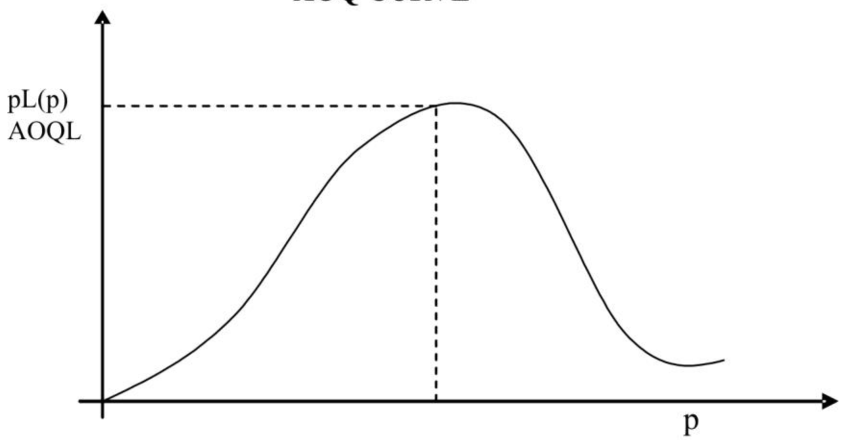
## 7. Average Outgoing Quality (AOQ)

The expected fraction defective remaining in the lot after the application of the sampling plan is called the average outgoing quality (AOQ). It is implied that rectifying inspection has been adopted. It is evident that AOQ will be the function of fraction defective in the lot.

i.e. $AOQ \sim p\, L(p) = p\, P_a(p)$

The maximum value of the AOQ function is known as the average outgoing quality limit AOQL. No matter how high the fraction defectives are in the incoming lots, the out going lots will never have a worse quality level, on the average than AOQL.

**AOQ CURVE**




## 8. Average Total Inspection (ATI)

IT represents the average total amount of inspection per lot, including the sampling inspection and sorting. This is made up of two parts

i) the ASN of the plan, and

ii) the cent percent inspection of the remainder of rejected lot

## 9. Average Sample Number (ASN) Function

This gives average (or expected) number of items inspected before a decision, regarding acceptance or rejecting, on the lot could be reached.

Now, we are in a position to discuss some specific sampling plans, i.e., single and double sampling plans.

Suppose a lot of size N, having ∞ defective items is submitted for inspection. Let lot fraction defective be $p = D/N$.

## 10. Single Sampling Plan

A single sampling plan is defined by two parameters, n – the sample size and c – the acceptance number. It is denoted by

$$\begin{pmatrix} N \\ n \\ c \end{pmatrix}$$

let d be the number of defective items in the sample. If

(i) $d \le c$, lot is accepted (the d defective items are replaced by good ones)

(ii) $d > c$, lot is rejected (the rejected lot is inspected fully and all its defective are replaced by good ones)

In general, d has hypergeometric distribution given by the probability function.

$$f_p(x) = P(d = x) = \frac{{}^{Np}C_x \, {}^{N(1-p)}C_{n-x}}{{}^{N}C_n}$$

If N is very large, it is approximated by the Binomial distribution.

$$f_p(x) = {}^{n}C_x p^x (1-p)^{n-x}$$

Moreover, if n is very large and p is very small such that $np = \lambda$ is finite, then it is approximated by the Poisson distribution.

$$\left[ f_\lambda(x) = \frac{e^{-\lambda}\lambda^x}{x!} \right]$$

The OC function of this plan is given by

$$\begin{aligned} L(p) &= Pa\ (p) \\ &= \text{probability of accepting the lot of quality p.} \\ &= P\ (d \le c) \\ &= P\ (0) + P(1) + \ldots\ldots\ldots.. + P\ (C) \\ &= \sum_{x=0}^{c} (\text{probability of getting x defectives out of n}) \\ &= \sum_{x=0}^{c} f_p(x) \qquad \ldots\ldots\ldots\ldots\ldots.. (10.1) \end{aligned}$$

Suppose the process average is p and LTPD is $p_t$ so that the producer's risk and consumer's risk are given by

$$\begin{aligned} PR &= \alpha = 1 - L(\bar{p}) \\ &= 1 - P_a(\bar{p}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots..(10.2) \end{aligned}$$

$$\begin{aligned} CR &= \beta = L(p_t) \\ &= P_a(p_t) \qquad \ldots\ldots\ldots\ldots\ldots\ldots..(10.3) \end{aligned}$$

Usually, p is kept at AQL.

If p be the actual fraction defective in the lot of size N, the AOQ under the sampling plan is given by

$$\left[AOQ = \sum_{x=0}^{c}\left(\frac{N_p - x}{N}\right).f_p(x)\right] \qquad \text{................(10.4)}$$

since the fraction defective in the lot after inspection is $\left(\frac{N_p - x}{N}\right)$, where x is the number of defectives found in the accepted sample (i.e $x \le c$) and it is zero for rejected samples (i.e x > c)

$$AOQ = \sum_{x=0}^{c} f_p(x)\left(\frac{N_p - x}{N}\right).f_p(x) \qquad \{\text{since } x = np\}$$

$$= p\left(\frac{N-n}{n}\right)\sum_{x=0}^{c} f_p(x)$$

$$= p\sum_{x=0}^{c} f_p(x) \qquad \left[\frac{N-n}{N} \simeq 1\right]$$

$$= pL(p)$$

$$= pP_a(p) \qquad \text{............... (10.5)}$$

The maximum AOQ with respect to p gives AOQL

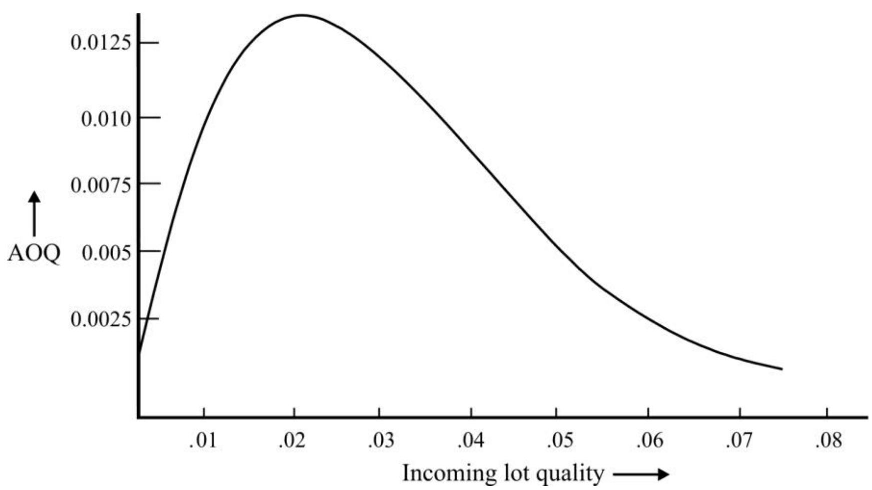
In this plan, the number of items inspected is always n, and therefore

$$ASN = n$$

If the lot is of quality p, the average total inspection ATI is given by

$$ATI = np_a + N(1 - P_a)$$

$$= n + (1 - P_a)(N - n) \qquad \text{............... (10.6)}$$




There are two approaches for this plan

a) **Lot Quality Protection**

The lots size N will be specified in any case, while the consumer's requirement will fix the value of $p_t$ and (c.r) $\beta$. Hence expression (4.3) gives an equation in the two unknown n and $\alpha$. This equation is satisfied for various combination of value of n and c to safeguard the producer's interest too one would select that value of n and c for which A.T.I. given by (4.6) is minimum for the specified value of p. The solution however is theoretically difficult to obtain Extensive tables have been prepared by Dodge and Romig who obtained the solution by numerical methods.

b) **Average Quality Protection**

The consumer's interests rests are taken care of by specifying the AOQL. Gives the value of N and AOQL, expression (4) gives an equation in n and c. In order to safeguard the producer interest, that pair of n and c satisfying (4) is selected for which A.T.I given by (6) is minimum, for specified value of p. Extensive tables for the sampling plan under this approach are also provided by Dodge and Romig.

## 11. Double Sampling Plan

A double sampling plan is a procedure in which, under certain circumstances, a second sample is required before a final decision on the lot may be made. This is defined by four parameters

$n_1$ = sample size of first sample

$c_1$ = acceptance number of first sample

$n_2$ = sample size of second sample

$c_2$ = acceptance number for both samples

It is denoted by

$$\begin{pmatrix} N \\ n_1 \\ c_1 \\ n_2 \\ c_2 \end{pmatrix}$$

Let $d_1$ be the number of defectives on the first sample. If

$d_1 \le c_1$, accept the lot (replace the $d_1$ defective items found by good ones)

$d_1 > c_2$, reject the lot (inspect 100% and replace all the defectives by non-defectives)

If $c_1 < d_1 \le c_2$, take the second sample of size $n_2$ from $(N - n_1)$ remaining items

Suppose the second sample has $d_2$ defectives. If

$d_1 + d_2 \le c_2$, accept the lot (replace the $d_1 + d_2$ defective found by good ones)

$d_1 + d_2 > c_2$, reject the lot (inspect 100% and replace all the defectives by non-defectives)

**Advantages & Disadvantages**

The principle advantage of a double sampling plan with respect to single sampling is that it may reduce the total amount of required inspection. Suppose, the first sample taken under a double sampling plan is smaller than the sample that would be required using a single sampling plan which offers the consumer the same protection. In all cases, then, in which a lot is accepted or rejected on the first sample, the cost of inspection will be lower for double sampling than it would be for single sampling plan. It is also possible to reject a lot without complete inspection of the second sample (curtailed sampling) i.e. to stop whenever the total number of defectives in the two samples exceed $c_2$. Consequently, the use of double sampling can often result in cutting total inspection cost.

Double sampling has two potential disadvantages. 1. Unless curtailment is used on the second samples, double sampling may require more total inspection than would be required in a single sampling plan that offers the same protection. Thus, unless double sampling is used carefully, its potential economic advantage may be lost. 2. The second disadvantage of double sampling is that it is more administratively complex which may increase the opportunity for the occurrence of inspection errors. These may be problems of storing and handing of items of the first sample which are awaiting a second sample to final decision

Let us denote

$$f(x, n; Np, N) = {}^{Np}C_x \frac{{}^{N(1-p)}C_{n-x}}{{}^{N}C_n}$$

$$\simeq {}^{n}C_x p^x q^{n-x}$$

$$\simeq \frac{e^{-\lambda}\lambda^x}{x!}; \qquad \lambda = np$$

according to as hyper geometric or binomial model is adopted

Then the OC function of this plan is given by

$$L(p) = P_a(p) \quad = \text{Probability of accepting the lot of quality p}$$

$$= P(d \le c_1) + P\left(d_1 + d_2 \le \frac{c_2}{c_1} < d_1 \le c_2\right)$$

$$= \sum_{x=0}^{c_1} \left\{ \frac{{}^{Np}C_x \, {}^{N(1-p)}C_{n_1-x}}{{}^{N}C_{n_1}} \right\} + \sum_{y=0}^{C_2-x} \left\{ \sum_{x=C_1+1}^{C_2} \frac{{}^{Np}C_x \, {}^{N(1-p)}C_{n_1-x}}{{}^{N}C_{n_1}} \right\} \times$$

$$\left\{ \frac{{}^{(N-n_1)}p_{cy} \, {}^{(N-n_1)(1-p)}C_{N_2-y}}{{}^{N-n_1}Cn_2} \right\}$$

or $$L(p) = P_a(p) = \sum_{x=0}^{C_1} f(x, n_1, Np, N) +$$

$$\sum_{y=0}^{C_2-x}\left\{\sum_{x=C_1+1}^{C_2} f(x, n_1; Np, N)\right\} \times f\{y, n_2; (N-n_1)p, N-n_1\}$$

or $$P_a(p) - P_a^{(1)}(p) + P_a^{(2)} \quad \text{say}$$

where,

$$P_a^{(1)}(p) = \sum_{x=0}^{C_1} f(x, n_1; N_p, N)$$

and $$P_a^{(2)}(p) = \sum_{y=0}^{C_2-x}\left\{\sum_{x=C_1+1}^{C_2} f(x, n_1, N_p, N)\right\} f(y, n_2; (N-n_1)p, (N-n_1))$$

If the process average $\overline{p}(=\text{AQL})$ and LTPD is $p_t$, then the producer's risk and consumer's risk are given by

$$P.R = \alpha = 1 - P_a(\overline{p})$$

$$C.R = \beta = P_a(p_t)$$

If p is the actual fraction defective in the lot, AOQ is given by

$$AOQ = \sum_{x=0}^{C_1}\left(\frac{N_p - x}{N}\right) f(x, n_1; N_p, N) + \sum_{y=0}^{C_2-x}\sum_{x=C_1+1}^{C_2}\left[\frac{N_p - (x+y)}{N}\right]$$

$$f(x, n_1; N_p, N) f(y, n_2; (N-n_1)p, N-n_1)$$

or $$AOQ \cong \frac{\left[P_a^{(1)}(p)(N-n_1) + P_a^{(2)}(p) + P_a^{(2)} p(N-n_1-n_2)\right]p}{N}$$

whose maximum gives AOQL

The ASN function of this plan is given by

$$ASN = n_1 P_1 + (n_1 + n_2)(1 - P_1)$$

where,

$P_1$ = P {only one sample is necessary}

= P {lot is accepted on the first sample} +
p {lot is rejected on the first sample}

$1 - P_1$ = P{the second sample is necessary}

The ATI function of this plan is given by

$$ATI = n_1 P_a^{(1)}(p) + (n_1 + n_2) P_a^{(2)}(p) + N(1 - P_a(p))$$

As in the case of single sampling plan, there are two approaches for double sampling plan viz, (i) lot quality protection and (ii) average quality protection.

Expensive tables for both the approaches have been provided by Dodge and Romig.

## 12. Sampling Inspection by Variables Sample

In sampling inspection by variable, for each item of the sample, measurements are taken on each quality characteristic along a continuous scale.

The primary advantage of variable sampling plan is that the same operating characteristic curve can be obtained with a smaller sample size than would be required by an attribute sampling plan. Thus, a variable acceptance-sampling plan that gives the same protection as an attribute acceptance- sampling plan would required less sampling. Through the measurement data for a variable sampling plan would probably cost more per observation than the collection of attributes data but the reduction in sample size may more than offset this increased cost. When destructive testing is employed variable sampling is particularly useful in reducing the cost of inspection.

A second advantage is that measurements data usually provide more information about the manufacturing process or the lot than does attributes data. Generally numerical measurements of quality characteristic are more useful than simple classification of the item as defective or non-defective.

A final point to be emphasized is that when acceptable quality levels are very small, the sample sizes required by attributes sampling plans are very large. Under these circumstances, there may be significant advantages in switching the variable measurement.

Variable sampling plans have some disadvantages. The primary disadvantage is that the distribution of the quality characteristic must be known-usually taken to be normal. If the distribution is not normal, serious departures from the advertised risks of accepting and rejecting lots of given quality may be experienced the second disadvantage of variable sampling is that a separate sampling plan must be employed for each quality characteristic.

Let x be the quality characteristic under question. It is assumed that x is normally distributed with mean $\mu$ and standard deviation $\sigma$, in the lot. There are two general types of variable sampling plans- plan that control the lot or process fraction defective, and plan that control a lot or process parameter. We shall discuss the former briefly.

Associated with x, there will be specification limits, only u, only z or both u and z. If only u is given, the proportion defective, p,u, is given by

$$
\begin{aligned}
p'U &= p\{x \geq U\} \\
&= 1 - \phi\left(\frac{U-\mu}{\sigma}\right)
\end{aligned}
$$

where, $\phi(x)$ is the distribution function of standard normal variable.

If only L is given, the proportion defective, $p'L$, is given by

$$p'L = P\{x \le L\}$$

$$= \phi\left(\frac{L-\mu}{\sigma}\right) = \phi\left(-\frac{\mu-L}{\sigma}\right)$$

If U and L, both are specified the proportion defective is

$$p'L + p'U$$

However $p'L$ and $p'U$ are unknown quantities, because $\mu$ and $\sigma$ are unknown sampling inspection provides us with estimates of $p'U$ and $p'L$ in other words of $\mu$ and $\sigma$, accepted or rejected.

**Case 1:** Known standard deviation when $\sigma$ is known, there exist MVUE of $p'L$ and $p'U$ viz.

$$pL = \phi\left[-\sqrt{\frac{n}{n-1}}\left(\frac{\bar{x}-L}{\sigma}\right)\right]$$

and $$pU = 1-\phi\left[\sqrt{\frac{n}{n-1}}\left(\frac{U-\bar{x}}{\sigma}\right)\right]$$

A sampling plan should naturally lead to the acceptance of lot if, and is small. Thus for a given USL, U, the lot is to be accepted if

$$pU \le M \text{ (say}$$

or equivalently, if

$$\frac{U-\bar{x}}{\sigma} \ge k$$

or $$\bar{x} + k\sigma \le U$$

where M is a quantity determined in accordance with the specified probability of errors and k is reacted to M by

$$k = \sqrt{\frac{n-1}{n}} Z_M \qquad \text{.................. (12.1)}$$

such that $Z_M$ is the upper 100 M% point of the standard normal variable

$$pU \le M$$

or $$1-\phi\left[\sqrt{\frac{n}{n-1}}\left(\frac{U-\bar{x}}{\sigma}\right)\right] \le M$$

$$\Rightarrow \phi\left[\sqrt{\frac{n}{n-1}}\left(\frac{U-\bar{x}}{\sigma}\right)\right] \geq 1-M$$

$$\Rightarrow \frac{U-\bar{x}}{\sigma} \geq \sqrt{\frac{n-1}{n}} Z_M$$

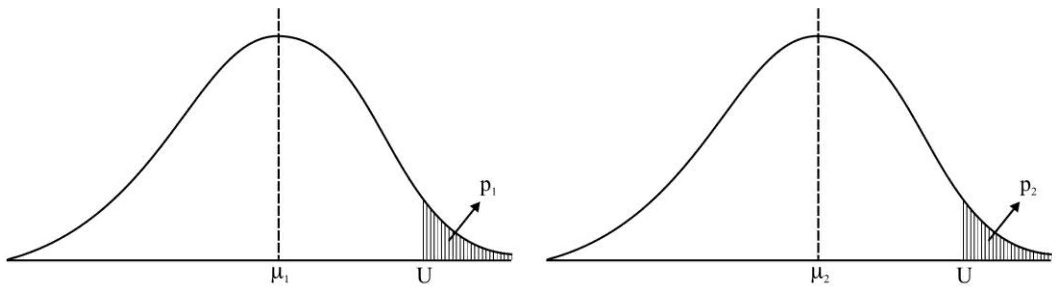
$$\Rightarrow \bar{x} \leq U - \sigma\sqrt{\frac{n-1}{n}} Z_M$$

**Derivation of n, k and M**

Suppose that we are given the acceptance quality level $p_1$ (like AQL), rejection quality level $p_2$ (like LTPD), producer's risk $\alpha$ and consumer's risk p, such that

P {acceptance of lot / $p_1$} = 1 – $\alpha$ ……………..(12.2)

P {acceptance of lot / $p_2$} = $\beta$ ……………..(12.3)



Corresponding to $p_1$ Corresponding to $p_2$

Evidently,

$$\mu_1 = U - Z_{p_1}\sigma \quad \text{or} \quad U = \mu_1 + Z_{p_1}\sigma$$
$$\mu_2 = U - Z_{p_2}\sigma \quad \text{or} \quad U = \mu_2 + Z_{p_2}\sigma$$

from (12.2) and (12.3) we get

$$p\left\{\bar{x}, k\sigma \leq \frac{U}{\mu_1}\right\} = 1-\alpha \qquad \text{……………(12.4)}$$

and $$p\left\{\bar{x}, k\sigma \leq \frac{U}{\mu_2}\right\} = \beta \qquad \text{……………(12.5)}$$

Equation (4.12.4) yields.

$$P\left[\frac{(\bar{x}+k\sigma)-(\mu_1+k\sigma)}{\sigma/\sqrt{n}} \leq \frac{U-(\mu_1+k\sigma)}{\sigma/\sqrt{n}}\right] = 1-\alpha$$

or $$P\left[\frac{(\bar{x}+k\sigma)-(\mu_1+k\sigma)}{\sigma/\sqrt{n}} \leq \frac{(Z_{p_1}+k\sigma)}{\sigma/\sqrt{n}}\right] = 1-\alpha$$

or $$\frac{\left(Z_{p_1}+k\right)\sigma}{\sigma/\sqrt{n}} = Z_\alpha$$

$$k = Z_{p_1} - \frac{Z_\alpha}{\sqrt{n}} \qquad \text{………… (12.6)}$$

similarly, equation (12.5) yields.

$$k = Z_{p_2} + \frac{Z_\beta}{\sqrt{n}} \qquad \text{………......(12.7)}$$

knowing $p_1$, $p_2$, α, β, we may get the value of n and k from (6) and (7)

$$n = \left[\frac{Z_\alpha + Z_\beta}{Z_{p_1} - Z_{p_2}}\right]^2 \qquad \text{………….. (12.8)}$$

and $$k = \frac{Z_\alpha Z_{p_2} + Z_\beta Z_{p_1}}{Z_\alpha + Z_\beta} \qquad \text{……………(12.9)}$$

from these values of n and k, we may easily obtain the value of M by use of (1).

If LsL, L, is given, lot is accepted if

$$pL \le M'$$

i.e. $$\frac{\bar{x} - L}{\sigma} \ge k$$

or if $$\bar{x} - k\sigma \ge L$$

And if both u and < are given, the lot is accepted if

$$pL + pU \le M$$

The value of k, according to the lot size, the sample size and specified AQ< are given by Bowker and Boode in "sampling inspection by variables".

**Case 2:** Unknown standard deviation

When σ is unknown, sampling inspection is based on sample mean $\bar{x}$ and sample standard deviation s given by $s = \sqrt{\Sigma \frac{\left(x_i - \bar{x}\right)^2}{\left(n-1\right)}}$ . The MVUE of $p'U$ and $p'L$ are given by pU and pL, functions of $\left(\frac{U - \bar{x}}{s}\right)$ and $\left(\frac{\bar{x} - L}{s}\right)$.

It can be shown that

(i) if USL, U, is given, the lot is accepted, if $\bar{x} + k's \le U$

(ii) if LSL, L, is given, the lot is accepted, if $\bar{x} - k's \ge L$

The value of $k'$, for different cases have been tabulated.

## 13. Summary

The main advantage of sampling inspection is that cost of inspection and time involved can be reduced dramatically. From economic considerations it is not practicable to inspect a ful lot. So one has to opt from some sampling inspection plan whether single sampling or double sampling, as the need may be.

The two main considerations on the basis of which the two plans my be compared are the operating characteristics and the average sample number. The average amount of inspection required per lot is more for single sampling plan it is for double sampling plan. Speaking generally, a double sampling plan often requires 25% to 33% less inspection on the average. For these two plans, we can say that:

1. It is easier for the sampling inspectors to understand the technique of single sampling plan.
2. The psychological satisfaction gained from giving the inspected lot more than one chance is absent in single sampling.

## 14. Solution/Answers

**Example:** Plot the operating characteristic curves for single sampling plan where N=5000, n=100, C=1,2,3. Assuming consumer's risk $\beta$=.01, determine the lot tolerance fraction defective. Also plot the average outgoing quality (AOQ) curve and determine AOQL.

**Solution:** Here we have $\frac{n}{N}=\frac{1}{50}$, which is very-very small. So the no. of defective items 'd' can be assumed to follow Poisson distribution. If, now, we use the notations already discussed, then

$$L(p) = P_a = \sum_{\lambda=0}^{C} \frac{e^{-np}(np)^{\lambda}}{\lambda!}$$

where 'P' is the lot quality.

'Pa' will be calculated using Biometrika table.

We further have-

$$A.O.Q. = \frac{P(N-n)}{N}.P_a$$

To determine lot tolerance fraction defective, we will draw lines parallel to 'p' axis at a distance $P_a$=0.10. Then draw perpendiculars to the p-axis from the points where this line meets the O.C. curves for C=1,2,3. The abcissa of these perpendiculars will give L.T.F.D.

Table for calculating Pa and A.O.Q.

| np | Pa | | | $100P\left(\frac{N-n}{N}\right)$ | A.O.Q. | | |
|---|---|---|---|---|---|---|---|
| | C=1 | C=2 | C=3 | | C=1 | C=2 | C=3 |
| .01 | 1.00 | 1.00 | 1.00 | .01 | .01 | .01 | .01 |
| .2 | .98 | .99 | 1.00 | .19 | .19 | .18 | .19 |
| .25 | .97 | .99 | 1.00 | .24 | .23 | .24 | .29 |
| .3 | .96 | .99 | 1.00 | .29 | .28 | .29 | .29 |
| .8 | .81 | .95 | .99 | .78 | .63 | .47 | .77 |
| 1.0 | .74 | .92 | .98 | .98 | .72 | .90 | .96 |
| 1.4 | .59 | .83 | .95 | 1.37 | .81 | 1.14 | 1.30 |
| 1.9 | .43 | .70 | .87 | 1.86 | .80 | 1.30 | 1.62 |
| 2.4 | .31 | .57 | .78 | 2.35 | .73 | 1.34 | 1.83 |
| 2.8 | .23 | .47 | .69 | 2.74 | .63 | 1.29 | 1.89 |
| 3.1 | .18 | .40 | .62 | 3.03 | .54 | 1.21 | 1.88 |
| 4.3 | .07 | .20 | .38 | 4.21 | .29 | .84 | 1.60 |
| 4.9 | .04 | .13 | .28 | 4.80 | .19 | .62 | 1.34 |
| 5.6 | .02 | .08 | .19 | 5.49 | .11 | .44 | 1.04 |
| 5.8 | .02 | .07 | .17 | 5.68 | .11 | .40 | .96 |
| 6.2 | .01 | .05 | .13 | 6.07 | .06 | .30 | .79 |
| 6.6 | .01 | .04 | .10 | 6.47 | .06 | .26 | .64 |
| 6.8 | .01 | .03 | .09 | 6.66 | .06 | .20 | .60 |
| 7.0 | .01 | .03 | .08 | 6.86 | .00 | .20 | .54 |
| 7.7 | .01 | .02 | .05 | 7.55 | .00 | .15 | .38 |

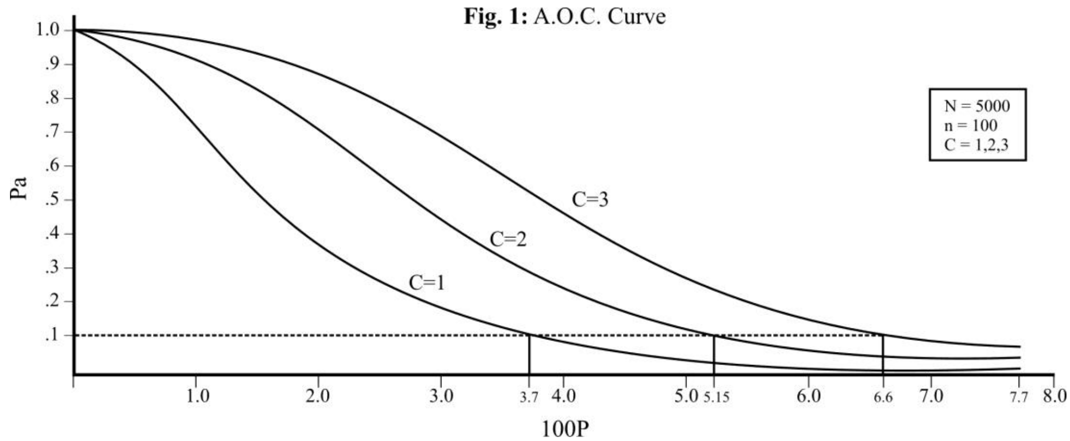



**Fig. 1:** A.O.C. Curve

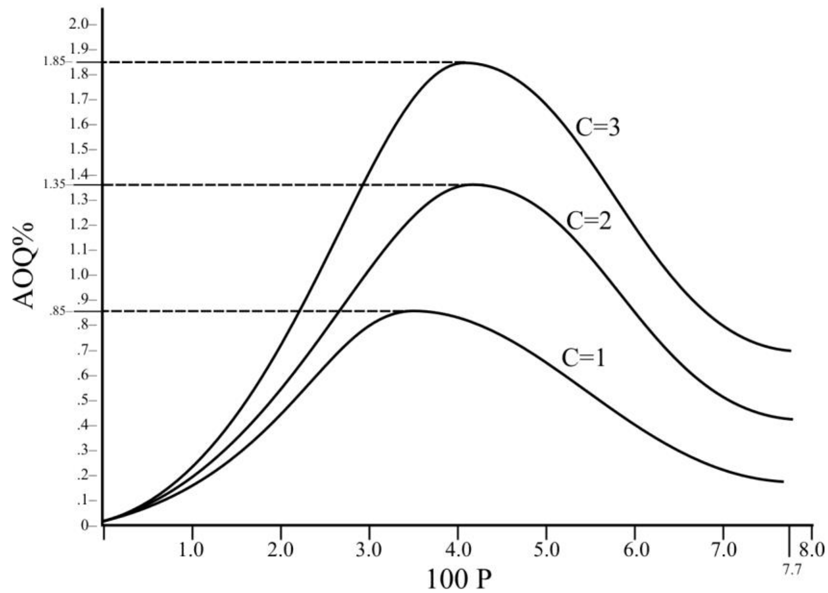
For $P_a$=.101 we get for Fig. (1)

for C=1 L.T.F.D. = 0.037

C=2 L.T.F.D. = 0.0515

C=3 L.T.F.D. = 0.066

**Fig. 2:** The A.O.Q. Curve for single sampling




From The AOQ curve, we get

for C=1 A.O.Q.L. = .85%

for C=2 A.O.Q.L. = 1.35%

for C=3 A.O.Q.L. = 1.85%

## 15. Exercises

1. Explain the terms producer's risk and consumer's risk.
2. Explain what is single sampling plan and what is double sampling plan.
3. For a single sampling plan, N=2000, n=100, C=2
   (i) Find Pa, when p = .005, .01, .05, .10
   (ii) Find AOQL for the same.

## 16. Further Readings


(1) Burr, I.W. Engineering Statistics and Quality Control. Mcgraw Hill.

(2) Cowden, D.J. Statistical Methods in Quality Control, Prentica Hall.

(3) Goon, Gupta, Dasgupta; Fundamentals of Statistics, Vol. Two, The World Press Pvt. Ltd.