Transportation Problem :-

Transportation problem is a special kind of linear programming problem in which goods are transported from a set of sources to a set of destinations. where the objective is to minimize the cost of distributing a product from a no. of sources (supply points) to a no. of destinations (demands points).

Aim :- The main aim of transportation model is to find out optimum transportation schedule keeping in mind cost of transportation to be minimized.

Application :

- Minimizing shipping costs.
- Determine low cost location
- Find minimum cost production schedule
- Military distribution system.

⇒ **Types of transportation problem :-**

1) **Balanced transportation problem :-** Where the total supply equals total demand.
2) **Unbalanced transportation problem :-** Where the total supply is not equal to the total demands. In this type of problem, either a dummy row or a

dummy column is added according to the requirement to make it a balenced problem.

⇒ Phases of solution :-

Phase I : Obtain initial basic feasible solution

Phase II : Obtain the optimal basic solution.

⇒ Initial basic feasible solution :-

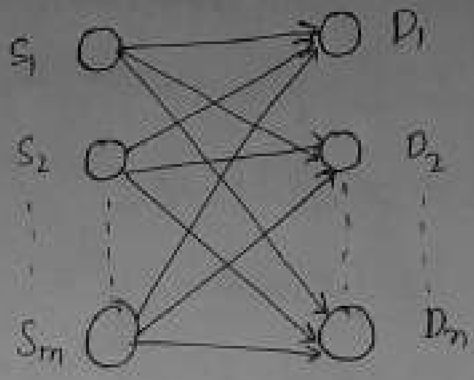
1) North-West corner rule.
2) Least cost method.
3) Vogele's approximation method.
4) Row minima method.
5) Column minima method.

⇒ Optimal basic solution :-

1) Modified distribution Method (MODI-Method).
2) Stepping stone method.

⇒ General Mathematical Model :-

Let $S_1, S_2 \cdots S_m$ are supply points and $D_1, D_2 \cdots D_n$ are the demand points.



Let x_{ij} be quantity transported from i to j.

where i - represent set of supply points and j represent set of demands point or destination points.

We have to minimize total cost of transpotation therefore,

Objective function is

$$\boxed{\text{Minimize } \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}}$$

where C_{ij} ÷ Transportation cost / unit

Now Constraints:

$$\sum_j x_{ij} \le a_i$$; a_i are the supply quantities available

$$\sum_i x_{ij} \geq b_j$$; b_j represent the demand quantities

$$x_{ij} \geq 0$$

The transportation problem is balenced when

$$\boxed{\sum_i a_i = \sum_j b_j}$$

For a feasible solution to exit, it is necessary that total capacity equal to the requirements.

Assumptions :-

1) Only a single type of commodity is being shipped from a supply point to a demand point.

2) Total supply is equal to total demand. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

3) The unit transportation cost of the item from all sources to destinations is constant and previously known.

⇒ Basic feasible solution of transportation problem:

Initial feasible solution of transportation problem is illustrated there with an example.

~~1) North~~

Problem① :- Suppose a manufacturing company owns three factories and distribute his products to five different retail agencies. The following table shows the capacity of three factories, the quantity of products required by the various retail agencies and the cost of shipping one unit of the product from each of three factories to each of five retail agencies.

	Retail Agencies					
Factories	1	2	3	4	5	Capacity
1	1	9	13	36	51	50
2	24	12	16	20	1	100
3	14	33	1	23	26	150
Requirement	100	60	50	50	40	300 / 300

Solution:

North-west corner method :-

the method starts at the North-West (upper-left) corner cell of the table.

step I: Allocate as much as posible to the selected cell and adjust the associated amounts of capacity (supply) and demand by subtracting the allocated amount.

Step 2: Cross out the row or column with zero supply or demand to indicate that no further assignments can be made in that row or column.

Step 3: If only one row or column is left uncrossed out, then stop. Otherwise, move to the cell to the right if a column has just been crossed or the one below if a row has been crossed out.

i)

	Retail Agencies					
Factories	1	2	3	4	5	supply
1	1 (50)	9	13	36	51	~~50~~
2	24	12	16	20	1	100
3	14	33	1	23	26	150
Demand	~~100~~ 50	60	50	50	40	250 / 250

ii)

	1	2	3	4	5	Supply
2	24 (50)	12	13	36	51	~~100~~ 50
3	14	33	1	23	26	150
Dem.	~~50~~	60	50	50	40	200 / 200

(iii)

	2	3	4	5	supply
2	12 (50)	13	36	51	~~50~~
3	33	1	23	26	150
Demand	~~60~~ 10	50	50	40	150 / 150

	2	3	4	5	Supply
3	33 ~~8~~ 10	1 50	23 50	26 40	~~150~~ ~~140~~ ~~90~~ ~~40~~
Demand	~~10~~	~~50~~	~~50~~	~~40~~	

Final Table :-

Retail Agencies

Factories	1	2	3	4	5	
1	1 50	9	13	36	51	
2	24 50	12 50	16	20	1	
3	14	33 10	1 50	23 50	26 40	

Transpotation cost $= 50\times1 + 50\times24 + 50\times12 + 10\times33$
$+ 50\times1 + 50\times23 + 40\times26$

$=$ Rs 4420/-

* There is no need to make a separate table for each step. Complete problem can be solved in a single table.

Least cost method :- (Matrix minimum method) :-

In the least cost method we look for the row or column corresponding to which unit transportation cost (C_{ij}) is minimum. This method finds a better initial basic feasible solution than that of North-West corner method by concentrating on cheapest route.

In this method we start by allocating as much as posible to the cell smallest unit cost. If there are two or more minimum costs then we should select the row and the column corresponding to the lower numbered row. If they appear in the same row we should select the lower numbered column. Then cross-out the satisfied row or column and adjust the amount of capacity and requirement accordingly. If both, a row and a column is satisfied simulteniously, only one is crossed out. Next, we look the uncrossed-out cell with the smallest unit cost and repeat the process untill we are left at the end with exactly one uncrossed out row or column.

Let us discuss the same problem with least cost method.

					supply
1 50	9	13	36	51	~~50~~
24	12	16	20	1	100
14	33	1	23	26	150
demand ~~100~~ 50	60	50	50	40	

24	12	16	20	1 40	~~100~~/60
14	33	1	23	16	150
50	60	50	50	~~40~~	

24	12	16	20	60
14	33	1 50	23	~~150~~/100
50	60	~~50~~	50	

24	12 60	20	~~60~~
14	33	23	100
50	~~60~~	50	

24	20	0
14 50	23	~~100~~/50
~~50~~	50	

→

20	0
23 50	~~50~~
~~50~~	

Final Table:-

1 (50)	9	13	36	51
24	12 (60)	16	20	1 (40)
14 (50)	33	1 (50)	23 (50)	26

Transportation cost $= 50\times1 + 60\times12 + 40\times1 + 50\times14 + 50\times1 + 50\times23$

$= \text{Rs } 2710/-$

* Note that the minimum transportation cost obtained by the least cost method is much lower than the corresponding cost of solution developed by using the north-west corner method.

Vogel's Approximation Method (VAM) or Method of Penalty :— VAM is an improved version of the least cost method that generally produces better solutions. The steps involved in this method are:

Step 1 :- For each row and column determine a penalty by subtracting the smallest unit cost element in row and column from the next smallest unit cost element in the same row and same column.

Step 2 :- Identify the row or column with the largest penalty among all the rows and columns. If the penalty ties corresponding to two or more rows or columns are equal we select the topmost row and extreme left column

step 3 :- We select x_{ij} as a basic variable if C_{ij} is the minimum cost in the row or column with largest penalty. We choose the numerical value of x_{ij} as high as possible subjected to the row and the column constraints.

Let us consider the same problem and solve by Vogel's approximation method :-

					Supply	Column penalty
1	9	13	36	51	50	8
24	12	16	20	1 40	~~100~~/60	11
14	33	1	23	26	150	13
Demand 100	60	50	50	~~40~~	300 / 300	
Row Penalty 13	3	12	3	25 ↑		

				Supply	Column penalty
1 50	9	13	36	~~50~~	8
24	12	16	20	60	4
14	33	1	23	150	13
Demand ~~100~~ 50	60	50	50	210 / 210	
Row Penalty 13 ↑	3	12	3		

				Supply	Column Penalty
24	12 60	16	20	~~60~~	4
14	33	1	23	150	13
Demand 50	~~60~~	50	50	150 / 150	
Row Penalty 10	21 ↑	15	3		

				Supply	Column Penalty
	24	16	20	0	4
	14	1 (50)	23	~~150~~/100	13
Demand	50	~~50~~	50	100/100	
Row Penalty	10	15	3		

↑

			Supply	Column Penalty
	24	20	0	4
	14 (50)	23	~~100~~/50	9
Demand	~~50~~	50	50/50	
Row Penalty	10	3		

↑

20	0
23 (50)	~~50~~/
~~50~~	

Final Table

1 (50)	9	13	36	51
24	12 (60)	16	20	1 (40)
14 (50)	33	1 (50)	23 (50)	26

Transpotation cost $= 50\times1 + 60\times12 + 40\times1 + 50\times14 + 50\times1 + 50\times23$

$= \text{Rs } 2710/-$

Problem ②

A company has three plants located throughout a state with production capacity 50, 75, 25 units. Each day the firm must furnish its four retail shops R_1, R_2, R_3 and R_4 with at least 20, 20, 50 and 60 units respectively. The transportation cost per unit (in Rs) are given below.

Company	Retail R1	R2	R3	R4	Supply
P_1	3	5	7	6	50
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	20	20	50	60	150 / 150

Distribute the available product to the different retailshops in such a way so that the total transpotation cost is minimum.

Find initial ~~basic~~ feasible solution by

i) North-West corner method.

ii) Least cost method.

iii) Method of penalty.