cartesian

$$\nabla\cdot\bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$ [

40

$$\nabla\cdot\bar{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$ [cylindrical]

$$= \frac{1}{r^2\sin\theta}\frac{\partial}{\partial r}(r^2\sin\theta A_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(r\sin\theta A_\theta)$$

$$+ \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\phi}(r A_\phi)$$

$$\nabla\cdot\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

[spherical]

4. curl

$$\nabla\times\bar{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1 a_u & h_2 a_v & h_3 a_w \\ \partial/\partial u & \partial/\partial v & \partial/\partial w \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$$

Static Electric fields:

- Fundamental of colombs law behind is the Gauss law.

(41)

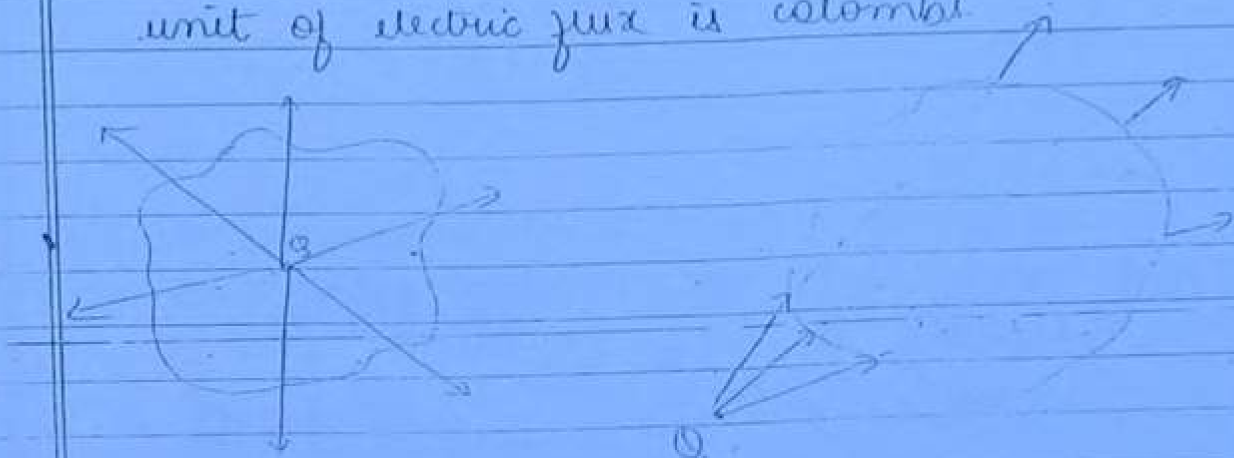
Gauss Law:

Statement: The net (Total) electric flux leaving (Effects) any closed surface is always equal to the charge (cause) enclosed in that volume

The complete effects from any cause are analysed by considering an encapsulating surface i.e closed surface. Hence

$$\boxed{\psi_{e(total)} = Q}$$

unit of electric flux is colombs



If the charge is inside the surface there are net flux line crossing the surface outwardly.

If the same charge is outside flux entering the volume or surface should be equal to flux leaving.

Summary: charge - source / sink for flux lines

Note: Gauss law never define for an open surface i.e for an open surface flux only exits through it we cannot define entering / leaving flux

$$\boxed{\oint D \cdot ds = Q}$$ Integral form of Gauss law.

$D =$ Flux density displacement

Strength of flux $= \frac{\text{Flux}}{\text{area}} = \frac{C}{m^2}$ = Instantaneous flux at every point

$$\boxed{\int D \cdot ds = \frac{\psi}{\epsilon_0}}$$ Not gauss law.

$$\boxed{\nabla \cdot D = \rho_v}$$ point form of Gauss law.

$$\nabla \cdot D = \frac{\text{outflow}}{\text{volume}} = \text{flux density} = \frac{\text{charge}}{\text{volume}} = \rho_v$$

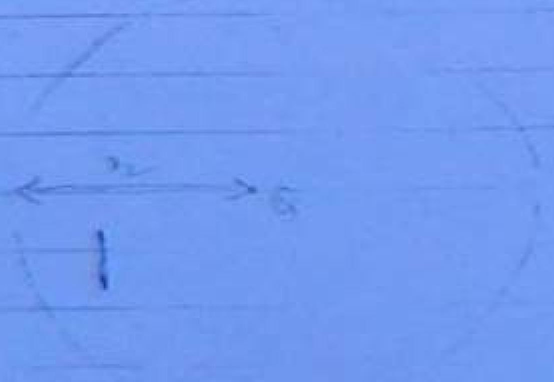
$$\frac{1}{m} \times \frac{C}{m^2} = \frac{C}{m^3}$$

Diverging ability always depends on charge density.

Gauss law: Application 1: Strength of the field due to a point charge Q

$$\oint D \cdot ds = Q$$

r = small, spherical surface
Equidis[illegible] from the [illegible]



Strength [illegible] everywhere around the charge at [illegible]
[illegible] law [illegible]

be used for any closed surface.
In this example we choose a symmetric spherical surface for applying Gauss law.

The choice of a sphere is because the surface is equidistant from the charge and hence the strength is is const. and hence the integration converges to multiplication

$$D(r) \times \text{Area of the sphere} = Q$$

$$D(r) = \frac{Q}{4\pi r^2} \cdot a_r$$

C/m^2

Chosen surface is an r = const sphere having a_r direction so by logic D also have same direction as $a_r \cdot a_r = 1$. Hence the field is radially outward and divergent from the cause or charge

Colomb have a different measure of field strength which was in terms of force b/w charges per unit charge

$$E = \frac{F}{q}$$

He called it as intensity or electric field intensity with unit $\frac{\text{Newton}}{\text{Colomb}}$.

He also proved that charge having a mass should have force and hence we the word E and related

$$\therefore E = \frac{D}{\epsilon}$$

Hence $E = \frac{Q}{4\pi\epsilon r^2} \cdot a_r$

N/C

44

Note 1: Electric field direction E is the direction of flux line, it is the direction of the repulsive force on a positive charge and hence it is always outward from a charge.

+q → repulsive test charge

−q ← test charge

Static Magnetic fields.

Biot Savart's Law. [Ampere's law for current element]

- Biot Savart's law derived from Ampere's law.
- applying that a small length I carrying wire can be treated as the Basic cause of magnetic fields.

Ampere's law = Ampere's circuital law

Biot Savart's law states -

Imp: $I\bar{dl}$ → small current element - vector quantity.

assumes - cause.

Imp: H → Magnetic field strength - field Intensity

- Effect.

Imp for exam

$$\boxed{\overline{H(r)} = \frac{I\,dl \times a_r}{4\pi r^2}}$$

The strength expression ... very similar to ... static ... field and ...

but the direction is not as if in electric field. 45

The direction of magnetic field is always current direction multiplied with radial direction to the point from the current.

current direction $\times$ radial dir^n to point from the current

Intensity $H(r) = \dfrac{\text{Amp} \cdot m}{m^2} \times a_r$

$= \dfrac{\text{Amp}}{m^2}$

Lorentz's basic force Eq^n defines the field strength in magnetic field as flux density $\left(\dfrac{\text{weber}}{m^2}\right)$ Hence as shown below

$$B = \frac{\bar{F}}{I\,dl} = \frac{\text{Force}}{\text{Basic cause}}$$

$F = q(\underline{v} \times B) \longrightarrow$ Lorenz's Force Eq^n

$$dF = dq\left(\frac{dl}{dt} \times B\right) = I\,dl \times B$$

q = charge

v = velocity of the moving charge

$\dfrac{dl}{dt} = v_d$ = drift in a conductor of length l

Given $B(r) = \mu H$

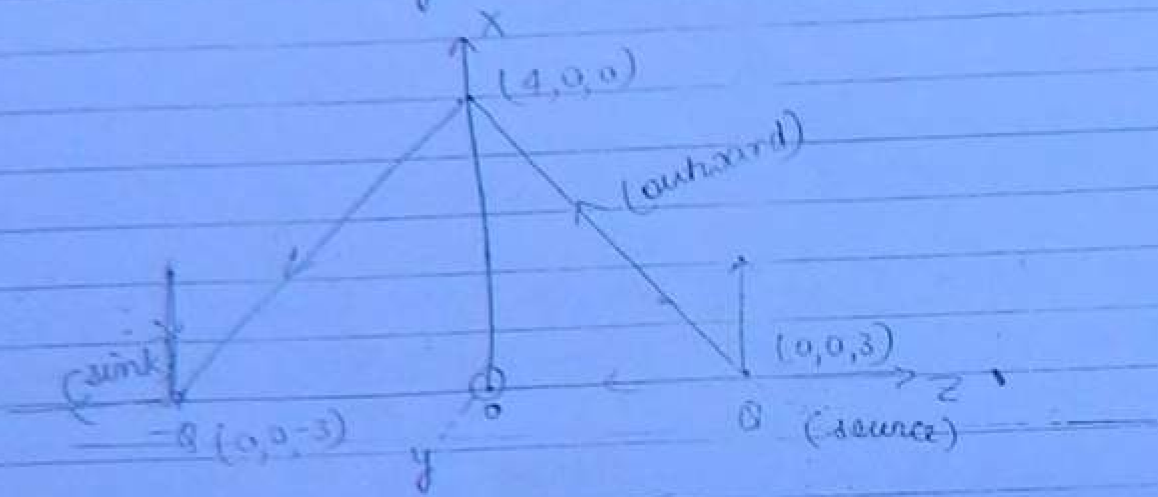
$B(r) = \dfrac{F}{I\,dl} = \dfrac{\text{Newton}}{\text{A} \cdot m}$

It relates current and force b/w currents (46)

Note: B's direction is physically the direction which a moving charges tracing when it enters a magnetic field.

Workbook:-

1. Static electromagnetic fields



$$r = \sqrt{4^2+0^2+(-3)^2} = \sqrt{16+9} = 5$$

$$E_1 = \frac{Q}{4\pi\varepsilon_0 r^2} \frac{[4a_x - 3a_z]}{\sqrt{4^2+3^2}}$$

$$\left\{ a_r = \hat{r} = \frac{\bar{r}}{|r|} \right\}$$

$$E_1 = \frac{Q}{4\pi\varepsilon_0 (5)^2} \frac{[4a_x - 3a_z]}{\sqrt{4^2+3^2}}$$

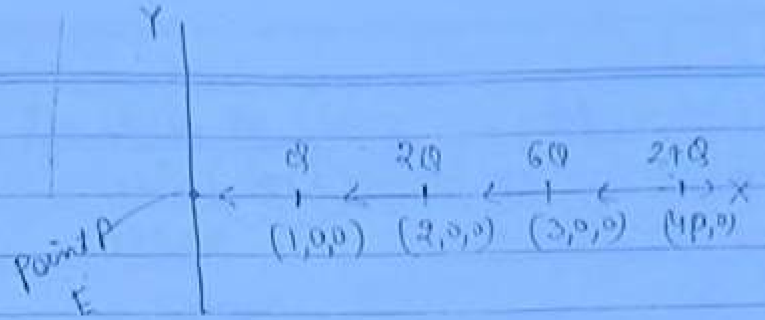
$E_1 = a_x, -a_z$

$E_2 = -\cancel{a_x} - a_z$

$$E_2 = \frac{Q}{4\pi\varepsilon_0 r^2} \frac{[4a_x + 3a_z]}{\sqrt{4^2+3^2}}$$

$$r = \sqrt{4^2 + \ldots (-3)^2} \quad 5$$

2.



$\Rightarrow n_1(Q)$ 47

direction $(-a_x)$

$$E_T = \left(\frac{Q}{4\pi\varepsilon_0 (1)^2} + \frac{2Q}{4\pi\varepsilon_0 (2)^2} + \frac{6Q}{4\pi\varepsilon_0 (3)^2} + \frac{24Q}{4\pi\varepsilon_0 (4)^2} \right)(-a_x)$$

$$E_T = \frac{Q}{4\pi\varepsilon_0}\left[1 + \frac{1}{2} + \frac{2}{3} + \frac{3}{2} \right](-a_x)$$

minimum maximum

as distance increases the charge is also increases. so 4th one has strongest charge and the 2^{nd} one has the least charge contributed

4. $-5 < x < 5$, $-5 < y < 5$, $-5 < z < 5$ } cube

$2C \rightarrow (7,2,3)$ Entering flux is dominating

(inside) $8C \rightarrow (2,-1,-3)$

(inside) $-12C \rightarrow (-4,0,1)$

$-4C$ Ans

5 centre - origin

$d_1 = \sqrt{4^2+8^2+3} = \sqrt{16+64+9} = \sqrt{25+64} > 6$

$d_2 = \sqrt{2^2+1^2+1^2} = \sqrt{4+1+1} = \sqrt{6} < 6$ — 8C (Enclosed

$d_3 = \sqrt{4^2+1^2} = \sqrt{16+1} = \sqrt{17} < 6$ — 12C (Enclosed

$-4C$ Ans

6 $d_1 = \sqrt{2^2+1^2+1^2} > 6$

$d_2 = \sqrt{2^2+2^2+5^2} \not< 6$ — 86

$d_3 = \sqrt{6^2+3^2+2^2} \not> 6$

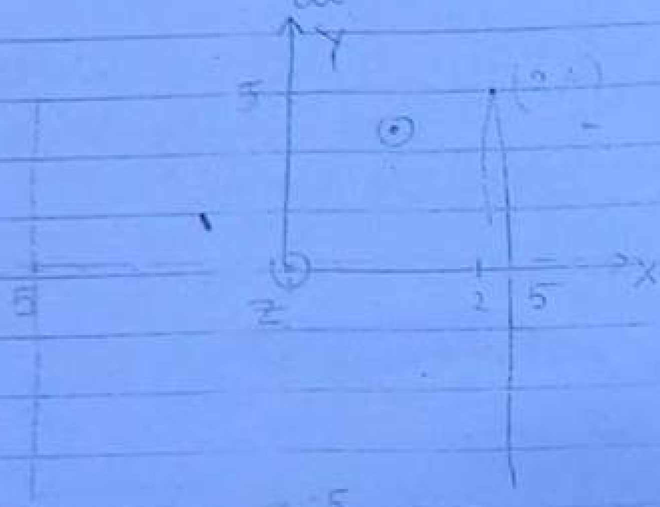
48

86 Ans

Extension Prob: calculate the net flux leaving the surface $-5<x<5$, $-5<y<5$, $-5<z<5$ due to a line charge $\rho_l = 10nC/m$ located at $x=2$, $y=1$ for all z

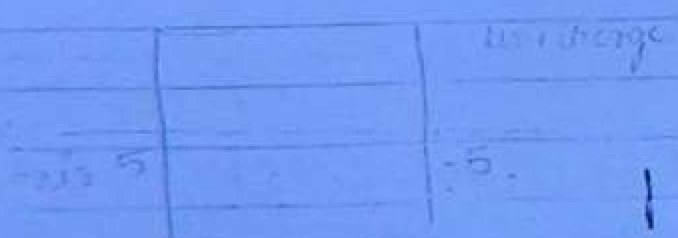
Sol^n $x=2$, $y=4$ $\rho_l = \frac{dq}{dl}$

$\rho_L = 10nC/m$



flux leaving = charge enclosed = part of a line charge
How much part of the line charge is in the cube.

2nd view



length of the line inside the cube = 10m
(-5 to 5 of z)

$Q = \rho_l \cdot l$

H.W Q If line is along $y = x$ line in the $z = 4$ plane $100\sqrt{2}$

P Hint = Repeat the question $z = a$, $y = x$ line

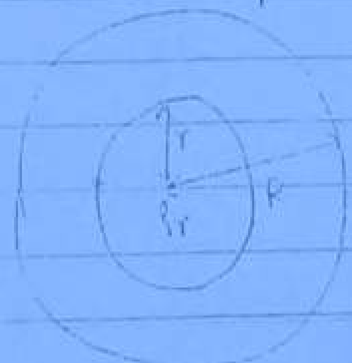
Hint $z = b$, $y = x$ line 49

Q. colombs law cannot apply because it is for point charge not for volume. we use $\oint D \cdot ds = Q$

E/D inside the charge.

The gaussian surface considered is concentric sphere of $r < R$ so that the strength on the surface same everywhere and hence loop $\oint D \cdot ds = Q$.

D. area = charge



$\rho_v \times \frac{4}{3} \pi r^3$

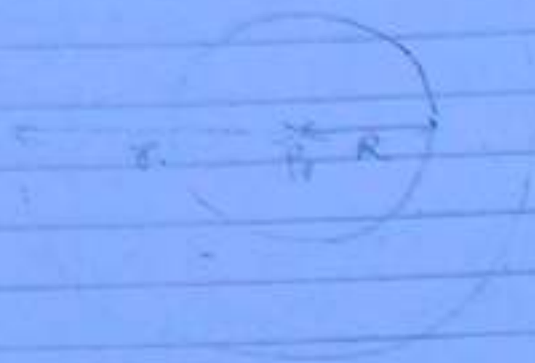
$C/m^3 \times m^3 = C$

$$D(r) = \frac{\text{charge inside}}{\text{Area}} = \frac{\rho_v \frac{4\pi r^3}{3}}{4\pi r^2} = \frac{\rho_v r}{3}$$

$$E(r) = \frac{\rho_v r}{3\epsilon}$$

$$\boxed{E(R/2) = \frac{\rho_v R}{6\epsilon}}$$

E at the centre is perfectly zero on the and maximum on the surface



(58)

$$\oint D \cdot ds = Q$$

$$D(r) = \frac{\text{charge inside}}{\text{area}} = \rho_v \frac{4}{3}\pi r^3$$

$$= \frac{\rho_v \frac{4}{3}\cancel{\pi} R^3}{4\pi r^2} = \frac{\rho_v R^3}{3r^2}$$

$$E(r) = \frac{\rho_v R^3}{3r^2 \epsilon}$$

$$E(2R) = \frac{\rho_v (2R)^3}{3r^2 \epsilon} = \rho_v 8R$$

$$E(2R) = \frac{\rho_v R}{12\epsilon}$$

$$D(r) = \frac{\rho_v R^3}{3r^2}$$

$$E(r) = \frac{\rho_v (R)^3}{3\epsilon r^2}$$

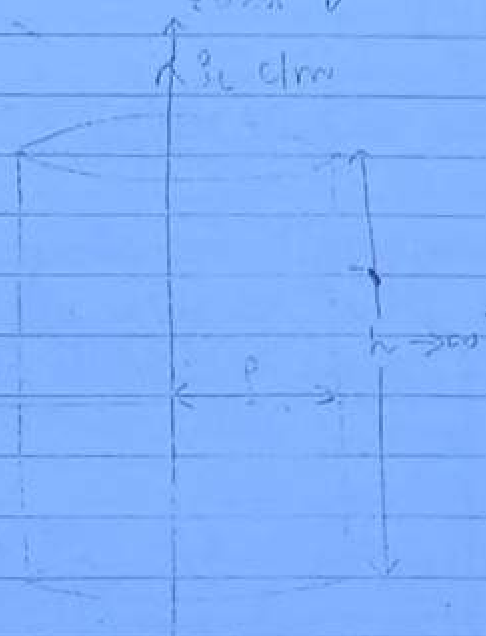
$$E(2R) = \frac{\rho_v R^3}{3\epsilon (2R)^2} = \frac{\rho_v R^3}{12\epsilon R^2} \Rightarrow \frac{\rho_v R}{12\epsilon}$$

57

Line charges & I carrying wires

Gauss Law. Application 2: Strength E due to an infinite length line charge.

The application of gauss law involves chosing an cylindrical surface If ρ = constt value. The surface have equi-distant nature from the charge and Hence D is constt everywhere.



$$\oint D \cdot ds = Q$$

$D(\rho) \times$ area = charge enclosed

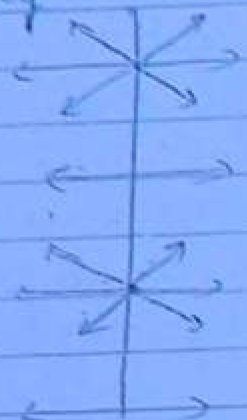
$$\boxed{D(\rho) = \frac{\rho_L h}{2\pi \rho h} = \frac{\rho_L}{2\pi\rho}}$$

The closed surface is a circular surface area hence the flux will only through the surface with the top and bottom surfaces are ignored.

As the ccw surface it ρ = const surface ds is a_ρ directed
Hence by logic D is a_ρ directed

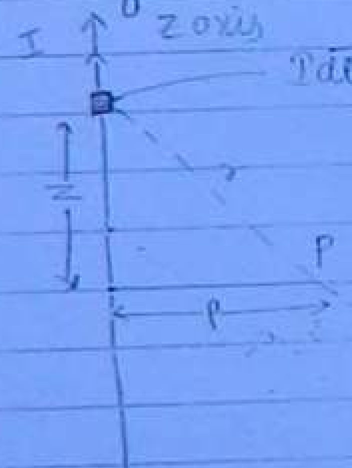
(52)

a_ρ = radially outward from the line



Strength H due to an infinite length I carrying wire

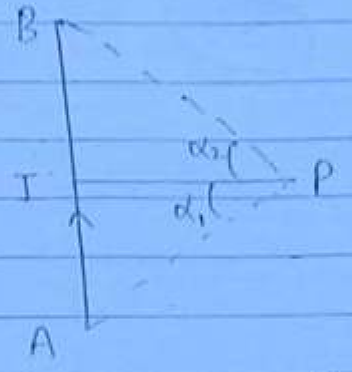
Consider a current carrying wire along the z axis
Lets us calculate the field strength at ρ distance from it



- For any infinite length or finite length.
- Identify a small incremental length element dl, such that $|dl| \to 0$
- Find the incremental strength dH due to this length.

$$dH = \frac{Idl}{4\pi r^2} \times a_r$$

3. When applied for a finite length current carrying wire



(53)

$$H = \frac{I}{4\pi\rho}(\sin\alpha_1 + \sin\alpha_2)\, a_\phi$$

* If E is to be calculated in line charge in Electrostatics

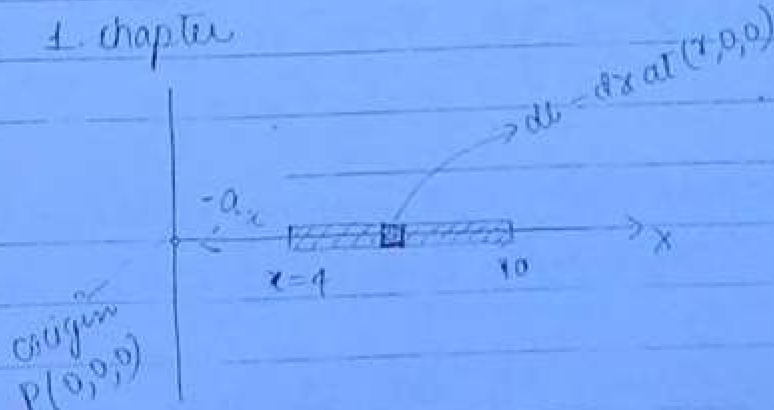
$$E(\rho) = \frac{\rho_L}{2\pi\epsilon\rho} a_\rho = \frac{D(\rho)}{\epsilon}$$

If B is to be calculated in I carrying wire in Magnetostatics.

$$B(\rho) = \mu H = \frac{\mu_0 I}{2\pi\rho} a_\phi$$

Work Book 1. chapter

3.



Let small length dl on this wire

find dH due to this length using

$$Q = \rho_L dx$$

$$R = x$$

$$a_r = -a_x$$

(54)

$$dE = \frac{\rho_L dx}{4\pi\varepsilon_0 x^2}(-a_x)$$

$$E = \int_{x=4}^{10} \frac{\rho_L dx}{4\pi\varepsilon_0 x^2}(-a_x)$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\int_4^{10} \frac{1}{x^2}\,dx \quad (-a_x)$$

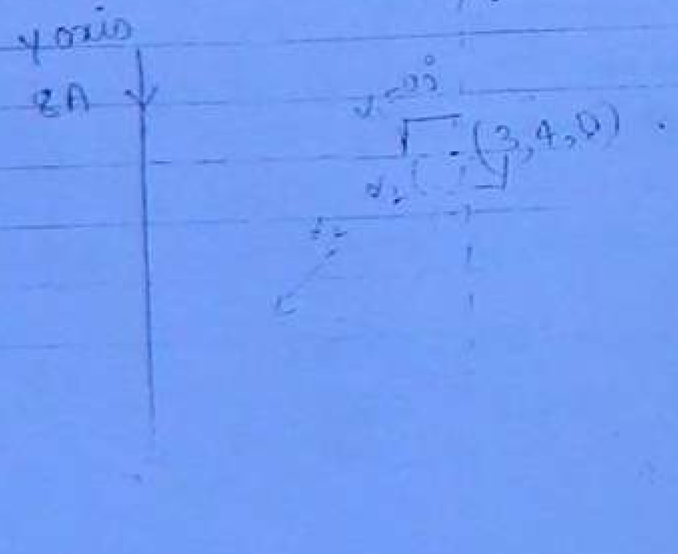
$$= \frac{\rho_L}{4\pi\varepsilon_0}\left[-x^{-2+1}\right]_4^{10} \Rightarrow \frac{\rho_L}{4\pi\varepsilon_0}\left[-\left[\frac{1}{10}-\frac{1}{4}\right]\right]$$

$$\Rightarrow \frac{\rho_L}{4\pi\varepsilon_0}\left[-\frac{1}{10}+\frac{1}{4}\right] \Rightarrow \frac{\rho_L}{4\pi\varepsilon_0}\left[\frac{-4+10}{10\times4}\right] \Rightarrow \frac{\rho_L}{4\pi\varepsilon_0}\left(\frac{6}{40}\right)(-a_x)$$

10 Ans 3, 4

(1) Volume is same then no effect on flux ϕ_E

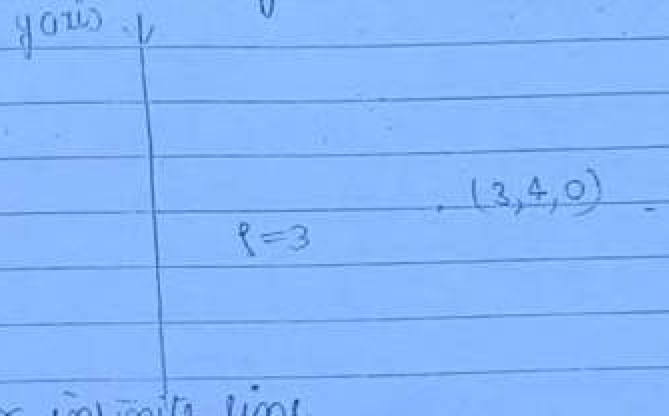
(2) gauss law. the flux leaving the surface is equal to the cause i.e charge.

11. 

Date ______
Page ______

Q Repeat the same quest if the current was flowing on the entire y-axis.

55

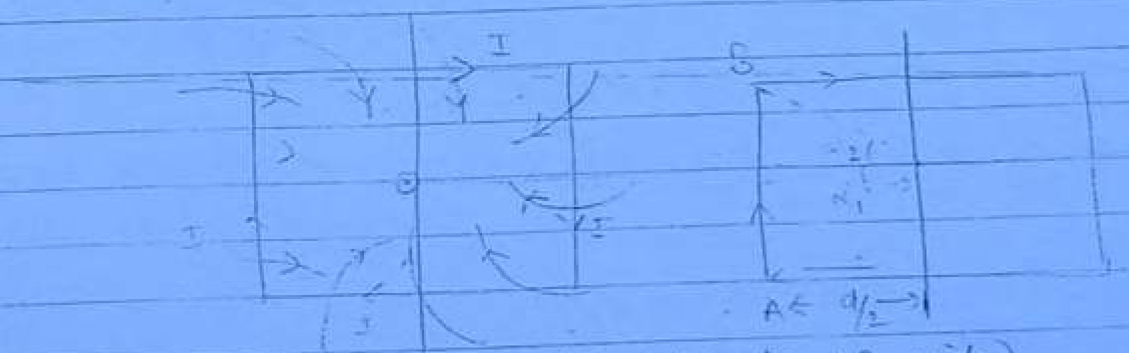


For infinite line

$$H = \frac{I}{2\pi\rho} = \frac{8}{2\pi(3)} a_z = \frac{1.33}{\pi} a_z$$

The field in L shape is stronger and in infinite line is weak as in L shape the same come close.)

W.B 12.



H direction at the centre (due to AB side)

$$a_y \times a_x = -a_z$$

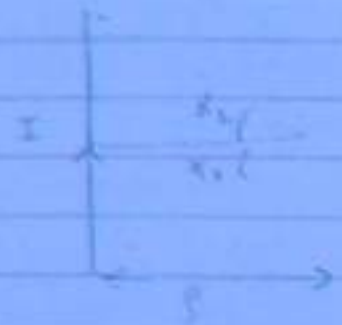
Note for a symmetric current distribution magnetic field at the geometric centre is always maximum.

H magnitude due to side AB

$$H = \frac{I}{4\pi\rho}(\sin\alpha_1 + \sin\alpha_2)$$

Break the L-shape wire into two part and then calculate the H for y-axis and H for x-axis seprately and then take the vector sum.

56



$$H = \frac{I}{4\pi\rho}\left(\sin\alpha_1 + \sin\alpha_2\right) a_\phi$$

$$\underset{(y\text{-axis})}{H} = \frac{8}{4\pi(3)}\left(\sin 90^\circ + \sin\alpha_2\right) a_\phi$$

$$\underset{(y\text{-axis})}{H} = \frac{8}{4\pi(3)}\left(1 + \frac{4}{5}\right) a_\phi$$

direction current direction $\times$ radial dir^n to the point from the current

$$(-a_y \times a_x) = a_z$$

$$x \rightarrow y \rightarrow z$$

$$\underset{(x\text{-axis})}{H} = \frac{8}{4\pi(4)}\left(\sin\alpha_1 + \sin\alpha_2\right)$$

$$= \frac{8}{16\pi}\left(1 + \frac{3}{5}\right)$$

$$= \frac{8}{16\pi}\left(\frac{8}{5}\right)$$

$$= \frac{64}{16\pi 5} \Rightarrow \frac{8}{2\times\pi\times5} = \frac{8}{10\pi} = \frac{4}{5\pi} a_z$$

$x \quad y \rightarrow z$

$$\frac{I}{4\pi(d/2)}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)(-a_z)$$

H at the centre, totally due to 4 sides

57

$$H = \frac{4\times I}{4\pi d/2}\left(\frac{2}{\sqrt{2}}\right)(-a_z)$$

$$= \frac{2I}{\pi d}\left(\frac{2}{\sqrt{2}}\right)(-a_z)$$

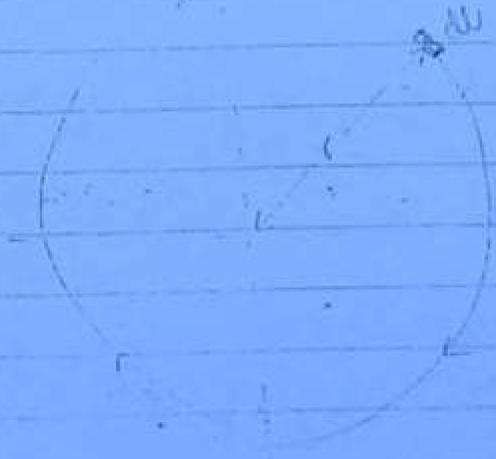
$$= \frac{4I}{\sqrt{2}\pi d}(-a_z)$$

$$= \frac{2\sqrt{2}\sqrt{2}\,I}{\cancel{\sqrt{2}}\pi d}(-a_z)$$

$$= \frac{2\sqrt{2}}{\pi d}I(-a_z)$$

$$H = \frac{2(\times 1.41)}{3.14}\frac{I}{d} = \frac{0.9\,I}{d}$$

Q Repeat the same question for circle. find the H at the centre. O



using Biot savart law.

$$dH = \frac{I\,dl \times a_r}{4\pi r^2}$$

$dl = r\,d\phi$

$$H = \frac{I}{4\pi}\int \frac{r\,d\phi}{r^2}$$

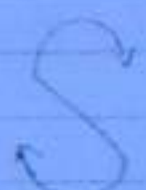
$$H = \frac{I}{4\pi}\int \frac{d\phi}{r}$$

$$H = \frac{I}{4\pi r}\int_0^{2\pi} d\phi$$

$$H = \frac{I}{4\pi r}(2\pi) \Rightarrow \frac{I}{2r} = \frac{I}{d}$$

1. clockwise current - H field direction into the paper.
2. All current carrying wires that are closed (square, circle) and have a finite area of encloser are refered as magnetic dipole

clockwise current I flow side — South pole



anticlockwise current I flow side North pole

Atom is a example of dipole.
dipole, closed current carrying wire

$\tan^{-1}\infty = \pi/2$
$\tan^{-1}(-\infty) = -\pi/2$
$\tan^{-1}(1) = 45$
$\tan^{-1}(0) = 0$

classmate

Date
Page

using Biot savart's law.

$$dH = \frac{I\,dz\,a_z}{4\pi(\rho^2+z^2)} \times (a_r)$$

59

$$r = \sqrt{\rho^2+z^2}$$

$$a_r = \hat{r} = \frac{\bar{r}}{|r|}$$

$$\hat{r} = \frac{\bar{r}}{|r|} = \frac{(\rho a_\rho - z a_z)}{\sqrt{\rho^2+z^2}}$$

$$dH = \frac{I\,dz\,a_z}{4\pi(\rho^2+z^2)} \times \frac{(\rho a_\rho - z a_z)}{\sqrt{\rho^2+z^2}}$$

The total strength $\bar{H}$ is

$$H = \int_{z=-\infty}^{\infty} dH = \int \frac{\rho I\,dz}{4\pi(\rho^2+z^2)^{3/2}}\, a_z \times (\rho a_\rho - z a_z)$$

$$H = \int \frac{\rho I\,dz}{4\pi(\rho^2+z^2)^{3/2}}\, a_\phi$$

$$= \frac{I\cdot\rho}{4\pi}\int_{-\infty}^{\infty} \frac{dz}{(\rho^2+z^2)^{3/2}}$$

put $z = \rho\tan\theta$

$dz \quad \rho\sec^2\theta\,d\theta$

$\rho^2 + \rho^2 \tan^2\theta = \rho^2 \sec^2\theta$

60

$H = \int dH$

$= \frac{I \cdot \rho}{4\pi} \int_{-\pi/2}^{\pi/2} \rho \frac{\sec^2\theta}{\rho^3 \sec^3\theta} d\theta$

$= \frac{I \cdot \rho}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{1}{\rho^2 \sec\theta} d\theta$

$= \frac{I \cdot \rho}{4\pi \rho^2} \int_{-\pi/2}^{\pi/2} \cos\theta$

$= \frac{I}{4\pi\rho} \left[\sin\theta\right]_{-\pi/2}^{\pi/2}$

$= \frac{I}{4\pi\rho} \left[\sin \pi/2 - \sin(-\pi/2)\right]$

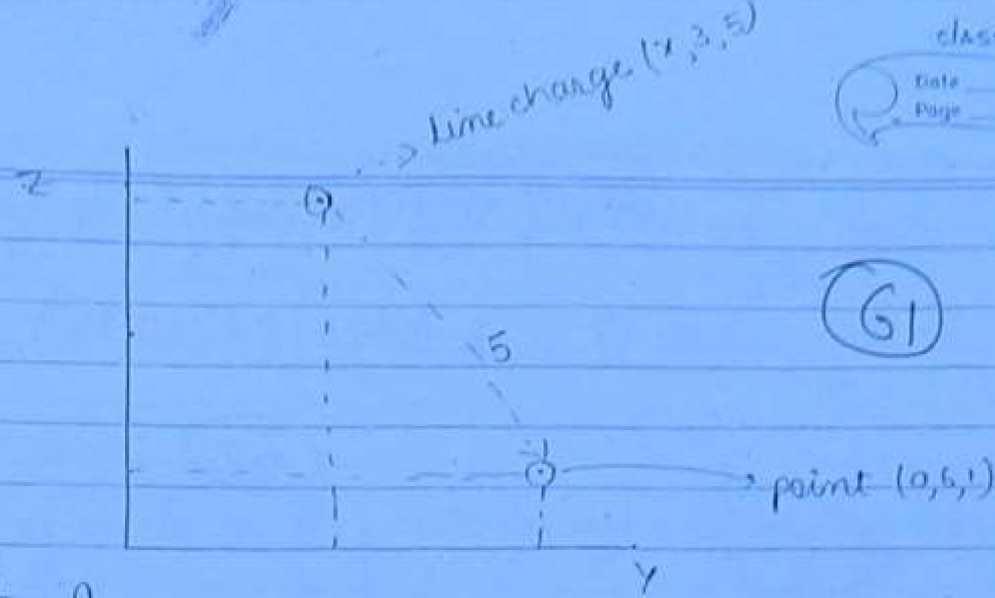
$= \frac{I}{4\pi\rho} \left[1 - (-1)\right] \Rightarrow \frac{I}{4\pi\rho} \times 2 \Rightarrow \frac{I}{2\pi\rho}.$

$$\boxed{H = \frac{I}{2\pi\rho} a_\phi}$$

a_ϕ - The H direction which is $(a_z \times a_\rho)$ is circulatory around the current

The expression is similar to $D = \frac{\rho_l}{2\pi\rho} a_\rho$

61

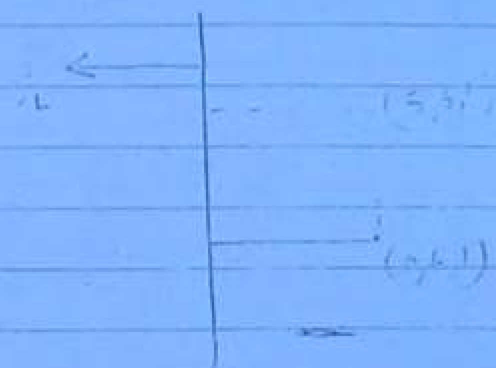


$$E = \frac{\rho_L}{2\pi\epsilon\rho}\, a_\rho$$

$$E \propto \frac{1}{\rho}$$

ρ between $(0,6,1)$ & $(7,3,5)$

$$\rho = \sqrt{3^2+4^2} = 5$$



$E = ?$ at $(5,6,1)$

$$E \propto \frac{1}{\rho}$$

$\rho = 5$ in both cases

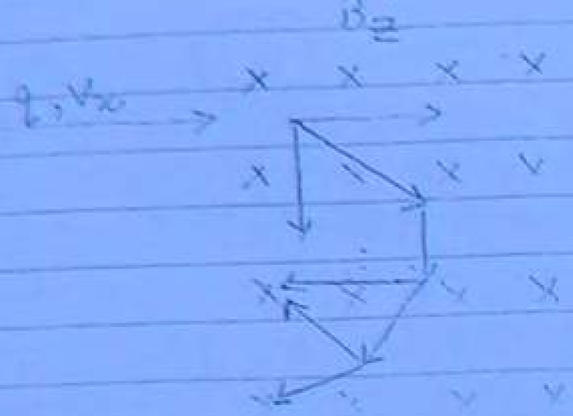
E is the same

Note: If z axis is defined as $(0,0,z)$
Any point (x,y,z) had a radial distance

$$\rho = \sqrt{x^2+y^2}$$

which is independent of z

Maxwell's III Eq^n – closed surface integral of $\vec{B}$.



$F_y = q(v_x \times B_z)$

62

1. The nature of magnetic field line is always to form closed loop around the current as seen in H due to a line current carrying wire (a_ϕ direction)

2. Any phenomena that is circulatory or closed never has a distinct starting / Ending point

There are no source / sink points for magnetic flux lines

$$\nabla \cdot B = 0$$

Because:

- Hence divergence of magnetic flux density is zero everywhere because divergence needs a distinct Start or an End

Maxwell's IIIrd Eq in point form.

$$\boxed{\nabla \cdot B = 0}$$

Mathematically this property is called as solenoidal nature of H fields

(2) $+Q$ $-Q$ (monopole + dipole may exist)

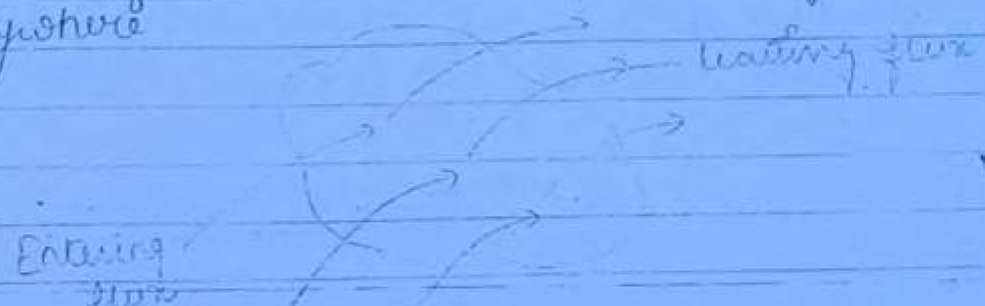
Applying divergence theorem for the point form. (63)

$$\oint B \cdot ds = \int (\nabla \cdot B) \cdot dv \quad \text{[Divergence theorem]}$$

$\therefore \nabla \cdot B = 0$

$$\boxed{\oint B \cdot ds = 0}$$ Maxwell's III Eq^n - Integral form.

Magnetic Monopoles donot exist because entering flux into any closed surface should always be equal to leaving flux as flux cannot start from anywhere or terminate anywhere



Hence we conclude that only dipole exist in magnetic fields i.e the basic cause of magnetic fields is a current I which flows only in closed circuits.
It flows only when Both the polarities (dipoles) exist.

Summary.- E fields Vs H fields

1. Q - Basic cause in E-fields - scalar quantity | Idl - Point current - in H-field. elements - vector quantity.

2. $\rho_l \rightarrow$ line charge density | $I \rightarrow$ flowing in line

3. [illegible]

4	$\bar{D}$ is called as flux density in E-fields	$\bar{B}$ is called as flux density in magnetic field.
	It is always a measure of strength in terms of charge	It is always a measure of strength - force
	It is always independent of ε	It is always μ dependent
	$\bar{E}$ - field Intensity in E fields.	$\bar{H} \rightarrow$ field intensity in magnetic fields
	It is always a measure of strength - force	It is always a measure of strength in terms of current (cause)
	It is always $\frac{1}{\varepsilon}$ dependent	It is always independent μ.
5.	$F \propto E \propto \frac{1}{\varepsilon} (10^{12})$	$F \propto B \propto \mu (10^{-7})$
	Strongest	Weakest
	E field is one of the strongest force	B field is one of the weakest force in nature

Potential, Gradient, closed line Integral of E

Potential

1 A scalar measure of field strength of E field in terms of the energy at a point or in terms of work done to reach the point.

... its ... work done to reach the point from a infinity charge.

$E = \frac{F}{Q}$ $V = \frac{W}{Q}$

classmate Date Page

$V = \frac{W}{Q} = \frac{\text{Joule}}{\text{coloumb}} = \text{volts}$

65

Note: If work done by the charge is the measure of potential and never work done on the charge.

work = force · displacement

$W = F \cdot l$

$dW = F \cdot dl$

$dW = -Q E \cdot dl$

Note: work is done by the charge only when it goes against the field (against the repulsive force) (thats y -ve sign)

$W = -\int Q \cdot E \cdot dl$

$W = -Q \int E \cdot dl$

$$\boxed{V = \frac{W}{Q} = -\int \bar{E} \cdot \bar{dl}}$$

V → Potential function of space but it is a scalar function

→ It is similar to intensity function which is an vector function

If the potential is evaluated b/w two distinct point with reference at B then V_{AB} is called potential difference with b/w A & B.

(Potential difference) $V_{AB} = -\int_{\text{ref } B}^{\text{at } A} E \cdot dl$ (Potential difference b/w A & B)

66

→ If ref B is assumed to be zero value; then
V_A = absolute potential w.r.t B.

eg. Ground is taken zero in most Electric ckts
Infinite distance is zero potential

14/7/11 Thursday

$$V = -\int \bar{E} \cdot \bar{dl}$$

$$= -\int \frac{Q}{4\pi\varepsilon_0 r^2} a_r \cdot dr\, a_r$$

When the field intensity is radially directed the potential calculation is simplified when dl is $dr\, a_r$

$$V = -\int \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

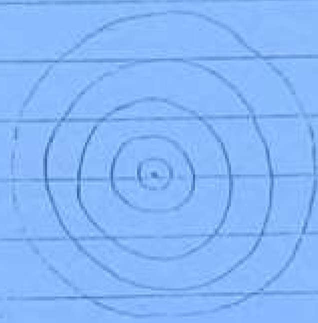
$$= -\frac{Q}{4\pi\varepsilon_0} \int \frac{dr}{r^2}$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 r}}$$

Note: If E decreases as $\frac{1}{r^2}$ V decreases as $\frac{1}{r}$.
$E\left(\frac{1}{r^2}\right)$ → vector
$V\left(\frac{1}{r}\right)$

If r = constt then V = constt the locus of all these points forms a sphere around the charge where the potential is constt this is called as equipotential surface. The family of equipotential surfaces graphically represent the changes in potential

67



If r = constt
→ concentric sphere

Potential of a line charge

$$V = -\int \bar{E} \cdot \bar{dl}$$

$$= -\int \frac{\rho_L}{2\pi\epsilon_0 \rho} a_\rho \cdot d\rho \, a_\rho$$

Always use natural log not a

$$= -\frac{\rho_L}{2\pi\epsilon_0}\int \frac{d\rho}{\rho}$$

$$\boxed{V = \frac{\rho_L}{2\pi\epsilon_0} \ln\left(\frac{1}{\rho}\right)}$$

$$\left\{ -\int \frac{d\rho}{\rho} = \ln\left(\frac{1}{\rho}\right) \right\}$$

1. $E\left(\frac{1}{\rho}\right) \rightarrow$ vector

$V\left\{ \ln\left(\frac{1}{\rho}\right) \right\} \rightarrow$ scalar

If ρ = constt then V = constt then we get concentric cylinders.

$$-\frac{1}{2}\left(\frac{x^{1/2+1}}{1/2+1}\right)\Bigg|_0^4 - 4\left[\frac{y^3}{3}\right]_0^1$$

(68)

$$\Rightarrow -\frac{1}{2}\left|\frac{x^{3/2}}{3/2}\right|_0^4 - 4\left[\frac{1}{3}\right]$$

$$\Rightarrow -\frac{1}{2}\left[\times\frac{2}{3}\right]\left[(4)^{3/2}\right] - \frac{4}{3}$$

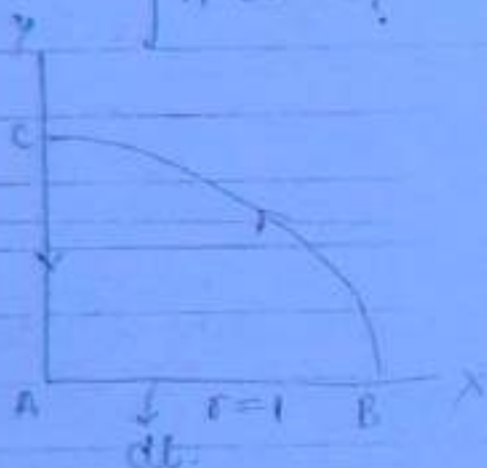
$$\Rightarrow \frac{-1}{3}\left[4^{3/2}\right] - \frac{4}{3}$$

$$\Rightarrow \frac{-8}{3} - \frac{4}{3} \Rightarrow -4$$

Q 21 W-B

$$\bar{A} = 2\rho\cos\phi\, a_\rho$$

$$\oint \bar{A}\cdot\bar{dl} = ?$$



$\oint A\cdot dl$ = 3 lines

$$\oint A\cdot dl = \oint_A^B \bar{A}\cdot\bar{dl} + \int_B^C \bar{A}\cdot\bar{dl} + \oint_C^A \bar{A}\cdot\bar{dl}$$

(1) $$\int_A^B \bar{A}\cdot\bar{dl} = \int_{\rho=0}^{1} 2\rho\, d\rho = 1$$

$dl = d\rho\, a_\rho$

$\phi = 0°$

$A = 2\rho\, d\rho$

2. B to C

$$dl = \rho\, d\phi\, a_\phi$$
$$\bar{A} = 2\rho \cos\phi\, a_\rho$$
$$\oint_B^C \bar{A}\cdot \bar{dl} = 0$$

(64)

3. C to A

$$dl = d\rho\, a_\rho$$
$$\phi = 90^\circ$$
$$\oint_B^C \bar{A}\cdot\bar{dl} = \int_B^C 2\rho\cos\phi$$
$$= 0.$$

Potential Gradient

Scalar Eq^n of a surface $\xrightarrow{\text{gradient}}$ Vector Direction of the surface.

In maths the gradient is used to find direction vector of any scalar surface Eq^n i.e gradient is used to find a normal vector everywhere given to the surface

eg. Linear surface

$$f = 3x - 4y - 8z = 100$$
$$\nabla\cdot f = 3a_x \;\; 4a_y - 8a_z$$

Non linear.

$$g = 4x^2y - 8xz = 100$$
$$\nabla\cdot g = (8xy - 8z)a_x + 4x^2a_y - 8x\,a_z$$

$$V = -\int \vec{E} \cdot \vec{dl}$$

(70)

$$dV = -\vec{E} \cdot \vec{dl}$$

$$dV = -|\vec{E}| \cdot |\vec{dl}| \cos\theta$$

$$\frac{dV}{dl} = -E\cos\theta$$

case I

If $\theta = 90^\circ$ i.e the change of potential per unit length is analysed orthogonal to or $\perp$ to the electric field direction. The potential has the same value. Hence the locus of all the points $\perp$ to E field constitutes equipotential surface

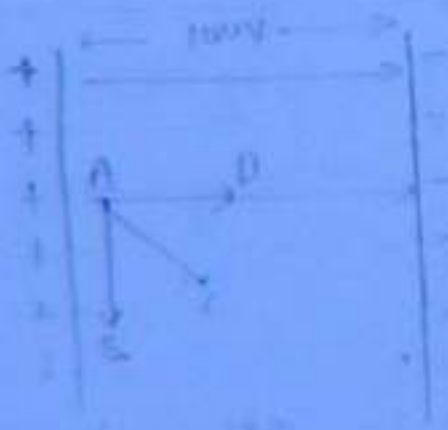
$$\theta = 90^\circ$$
$$V = \text{const}$$

case II

If $\theta = 0^\circ / 180^\circ$

$$\left.\frac{dV}{dl}\right|_{max} = |E|$$

→ The magnitude of the E field intensity is the maximum rate of change of potential per unit length



case(i) $\theta = 0^\circ$

$$\left.\frac{dV}{dl}\right|_{max} = -E$$

(71)

The direction of E field intensity is the direction in which potential decreases at a maximum rate

Hence every scalar can have a vector defined from a unique direction of change by maximum and the rate of change by maximum. This is called as gradient operation. If the vector is E the scalar is V. then

$$\boxed{E = -\nabla V}$$

$\frac{\text{volts}}{\text{mete}}$

Potential gradient means E field intensity (E)

Potential Eqn $\xrightarrow{\text{Gradient}}$ vector Intensity

unit of electric field intensity (E) is $\frac{\text{volts}}{\text{meter}}$

Formula for gradient operation
If V = scalar function of space

$V(u, v, w)$

$$\boxed{\nabla \cdot V = \frac{1}{h_1}\frac{\partial V}{\partial u} a_u + \frac{1}{h_2}\frac{\partial V}{\partial v} a_v + \frac{1}{h_3}\frac{\partial V}{\partial w} a_w}$$

Q given the potential fun V = [illegible] $(x^2 - y^2)$ for all z. find the Eqn of the [illegible] surface passing through the [illegible]

Sol^n (3,1,1)

22

Eq^n of equipotential surface is

$$V = 25(x^2-y^2) = K$$

(as voltage is const on equipotential surface)

$$25(x^2-y^2) = K$$

$$x^2-y^2 = k'$$

$25(x^2-y^2)=k$ at $(3,1,1)$

$25(9-1) = K = 200$

$$25(x^2-y^2) = 200 \Rightarrow x^2-y^2 = 8$$

The potential fun^n given in the question is in itself equipotential surface definition

Q given $V = \frac{4\cos\theta}{r^2}$. find $\bar{E}$ at $(2, \pi/2, \pi/2)$

$$V = \frac{4\cos\theta}{r^2}$$

$$\nabla V = \frac{1}{h_1}\frac{\partial}{\partial r}\left(\frac{4\cos\theta}{r^2}\right)a_r + \frac{1}{h_2}\frac{\partial}{\partial\theta}\left(\frac{4\cos\theta}{r^2}\right)a_\theta + \frac{1}{h_3}$$

$$\nabla V = \frac{4}{1}\cos\theta\left(r^{-2+1}\right)(-2) + \frac{4}{r^2}(-\sin\theta)\,a_\theta$$

$$= -8\cos\theta\left(\frac{1}{r^3}\right) + \frac{4}{r^3}(-\sin\theta)$$

$$= -8\cos\pi/2\left(\frac{1}{8}\right) + \frac{4}{8}(-\sin\pi/2)$$

$$\frac{1}{2}(-1) \quad a_\theta$$

closed line integral of E - Maxwell's II Eqn

$$\int E \cdot dl = \text{potential}$$

(73)

$$\oint E \cdot dl = 0$$

→ Potential at a point in space is always unique at a point of time

→ Potential cannot be a multivalued function

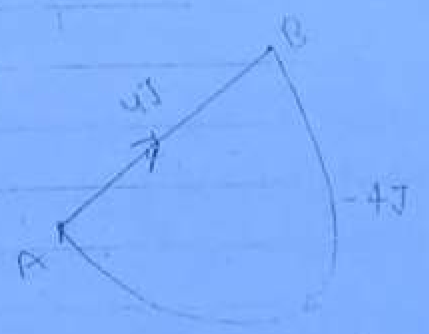
→ The work done in moving a charge in any closed loop is zero. i.e in a closed loop we sometime aquire energy sometimes loss energy. such that energy is conserved.

Hence E field is a conservative field.

→ E-field lines never forms closed loop, the lines are always outwardly divergent from a charge.

E field is an irrotational vector

$$\boxed{\nabla \times E = 0}$$



Work done in moving a charge b/w two points is independent of path of consideration

Note: $\oint E \cdot dl = 0$ - Maxwell's II Eqn in integral form but not $\int E \cdot dl = -V$

74

Similarly $\nabla \times E = 0$ — Maxwell's II eq^n in point form.

but not $E = -\nabla V$

Note: To identify whether a given vector/field is a valid E or H put $\nabla \times E = 0$ = Valid Electric (E) field

put $\nabla \cdot B = 0$ = Valid Magnetic field.

* -- In static E/H only.

Potential, Vector Potential, Maxwell IV Eq^n (Ampere's law)

Potential in Magnetic fields expressed as a scalar quantity is called as MMF (Magneto motive force)

$$V_m = \int H \cdot dl \quad \text{(amperes)}$$

$$H = \nabla V_m$$

Its unit is ampere but it is never similar to current It is equal to current when analysed for a closed path. As current flows only in closed circuits hence

$$\boxed{\oint H \cdot dl = I}$$ Ampere's law in integral form.

Maxwell IV Eq^n in integral form.

but not $V_m = \int H \cdot dl$

(75)

Statement of Ampere's Law:

The circulation of magnetic field intensity in any $\boxed{\text{closed}}$ $\boxed{\text{loop}}$ is always equal to the $\boxed{\text{current}}$ crossing the $\boxed{\text{surface}}$ enclosed.

→ circulation means the effects which are around the current.

→ current means the cause of the effects

→

$$\nabla \times H = \text{curl of } H = \frac{\text{circulation}}{\text{area}} = \frac{\text{current}}{\text{area}} = J \quad A/m^2$$

$$\boxed{\nabla \times H = J}$$

Ampere's law in point form

Maxwell' IV Eq^n in point form

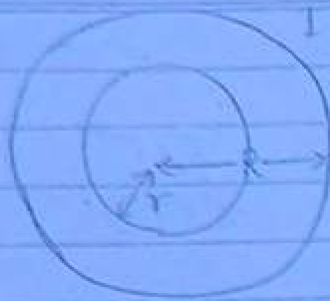
eg:

$$\oint H \cdot dl = I$$

$$H(\rho) = \frac{I}{2\pi\rho}$$

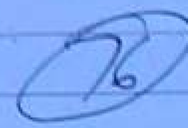
As the length of the circulation increases the strength of the effect is reduces. If the circulation is at a length of $2\pi\rho$ strength is $\frac{I}{2\pi\rho}$

26.W.B



given $r<R$

$H(r) = ?$



Case I $r<R$

To apply ampere's law consider a circular line concentric and symmetric with the current. so the strength (H) is const everywhere

$$\oint H \cdot dl = I$$

$H(r) \times$ length of circulation $=$ current in the area

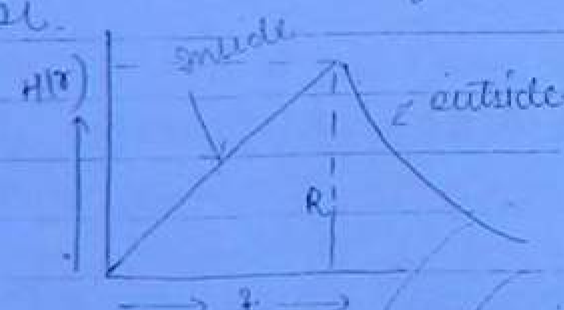
current density $\rightarrow \left(\frac{I}{\pi R^2}\right) \pi r^2 =$ current crossing the ~~area~~ formed by the closed line

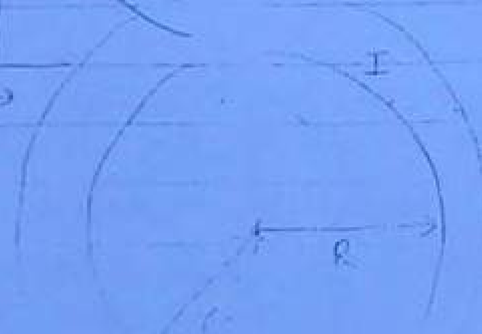
Total area

$$H(r) = \frac{\frac{I}{\pi R^2} \times \pi r^2}{2\pi r}$$

$$= \frac{I r^2}{2\pi R^2 r} \Rightarrow \frac{I r}{2\pi R^2}$$

Note 1. The field is zero at the centre of the conductor and max^m of the conductor.





Case II $r=R$

$$H(r) = \frac{I}{\pi r}$$

22 W.B

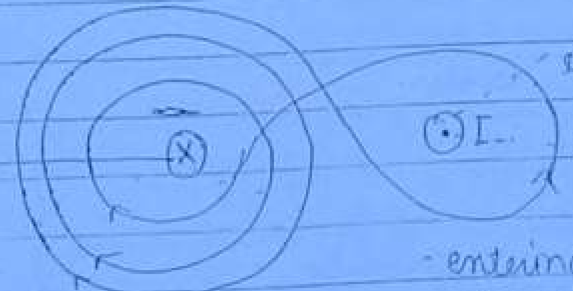
A sphere has the same geometry as that of point and hence a potential can be calculated on the same basis as.

$$V = \frac{Q}{4\pi \varepsilon_0 r}$$

77

$$V = \frac{2\times 10^{-8}}{4\pi\varepsilon_0 (10\ cm)} \Rightarrow \frac{2\times 10^{-8}}{4\pi \times}$$

25 W.B



$$I + I + I - (-I) = 4I$$

Vector Magnetic Potential $\bar{A}$

$E = -\nabla V$

$\nabla \times E = 0$

$\nabla \times (-\nabla V) = 0$

failure of MMF in certain regions

$$H = \nabla V_m$$

$$\nabla \times H = J$$

$$\nabla \times (\nabla V_m) = 0 = J$$

Hence $H = \nabla V_m$ definition is correct and exist only in those regions where there is no current density. i.e in free space or current free - regions only But not inside conductor.

If the curl of vector potential is B. as divergence of. B is 0 everywhere this definition satisfied for any point in magnetic fields

$$B = \nabla \times \bar{A}$$

$$\nabla \cdot B = 0$$

78

$$\nabla \cdot (\nabla \cdot A) = 0$$

$$\text{Div} \cdot (\text{curl of vector}) = 0$$

$\bar{A}$ has the unit of $-\frac{\text{weber}}{\text{meter}}$

From faraday's law.

$$\frac{\text{weber}}{\text{second}} = \text{volts} = \frac{\text{Joule}}{\text{colombs}}$$

$$\left\{ \therefore \text{volts} = \frac{W}{Q} = \frac{\text{Joule}}{\text{colombs}} \right\}$$

$$\text{weber} = \frac{\text{Joule}}{\text{colombs}} \times \text{sec} \Rightarrow \frac{\frac{\text{Joule}}{\text{colombs}}}{\text{sec}}$$

$$\text{weber} = \frac{\text{Joule}}{\text{Amp}}$$

$$\frac{\text{weber}}{\text{meter}} = \frac{\text{Joule}}{\text{Amp-meter}}$$

$$\bar{A} = \frac{W}{I \cdot \bar{dl}}$$

potential signifies work or energy measured per basic source [illegible] field [illegible] signifies the basic source of [illegible] current element $I\bar{dl}$ is a vector quantity.

$$\bar{A} = \frac{W}{I\,dl}$$

Note: If $E = \frac{F}{Q}$ $\quad \bar{B} = \frac{F}{I\,dl}$

79

If $\bar{V} = \frac{W}{Q}$ $\quad \bar{A} = \frac{W}{I\,dl}$

$E = \nabla V$ $\quad B = \nabla \times \bar{A}$

If $V = \frac{Q}{4\pi\epsilon_0 r}$ — point

Expression of A is obtained by duality of expression of V

$$\boxed{\bar{A} = \frac{\mu_0 I\,dl}{4\pi r}}$$ point current element.

The direction of $\bar{A}$ is always the current direction itself.

Q. The closed line integral of vector magnetic potential is

soln

$$\oint A \cdot dl = \frac{weber}{metre} \times metre - weber$$

Magnetic flux crossing the open surface formed by the closed line

$$\oint A \cdot dl = \int (\nabla \times A) \cdot ds$$

$\int B \cdot ds$

Magnetic flux ... crossing the surface

Note: $\oint B \cdot ds = 0$ = Entering flux
=
Leaving flux

Maxwell III Eq^n

But $\int B \cdot ds = \psi_m$ = cutting flux

This is not Maxwell III Eq^n

15/7/11 Friday

Laplace / Poisson's Equation -

The most common ~~or~~ form of charge is a volume charge moving inside materials. Hence $\boxed{\rho_v = ne}$ for most application

~~where~~ n = no of carriers per unit volume

e = charge of the carrier

Poisson Eq^n relates in potential develop around any volume charge.

$$V \longrightarrow \rho_v$$

$$\nabla \cdot D = \rho_v$$

$$D = \epsilon E$$

$$E = -\nabla V$$

$$\nabla \cdot (\epsilon E) = \rho_v$$

$$\nabla \cdot (\epsilon(-\nabla V)) = \rho_v$$

$$V = \frac{-6r^5}{\epsilon_0}$$

81

$$\nabla^2 V = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial r}\left(h_2 h_3 \frac{\partial}{\partial r}\left(\frac{-6r^5}{\epsilon_0}\right)\right)\right]$$

$$= \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial r}\left[r^2 \sin\theta \frac{\partial}{\partial r}\left(\frac{-6r^5}{\epsilon_0}\right)\right]$$

$$= \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial r}\left[\frac{r^2 \sin\theta}{\epsilon_0}(-6\times 5 r^4)\right]$$

$$= \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial r}\left[\frac{-30 r^6 \sin\theta}{\epsilon_0}\right]$$

$$= \frac{1}{r^2 \sin\theta}\left(\frac{-30\sin\theta}{\epsilon_0}\right)\frac{\partial}{\partial r}\left[r^6\right]$$

$$= \frac{1}{r^2 \sin\theta}\left(\frac{-30\sin\theta}{\epsilon_0}\right) 6r^5$$

$$= \frac{-180 r^3}{\epsilon_0}$$

as $\nabla^2 V = \frac{-\rho_v}{\epsilon}$

$$\nabla^2 V = \frac{-180 r^3}{\epsilon_0} = \frac{-\rho_v}{\epsilon}$$

$$\rho_v = 180 r^3$$

step 2 $Q = \int \rho_v \, dv$

$$= \int_{r=0}^{1}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} 180 r^3 r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$180\left[8\frac{x^9}{6}\right]_0^1\left[-\cos\theta\right]_0^{\pi}\left[\phi\right]_0^{2\pi}$$

$$180\times\frac{1}{6}(2\pi) = 120\pi$$ 82

Alternate Method:

$D \leftarrow E = -\nabla V$

$\downarrow$

$\oint D \cdot dl = Q$

31 W.B $\nabla^2 V = -\frac{\rho_V}{\epsilon}$

given E & V are zero } Intial condition

function zero

$\nabla^2 = E = \frac{dy}{d\rho}$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\cdot\frac{\partial V}{\partial\rho}\right) = +\frac{\left[10^{-8}(1+10\rho)\right]}{\frac{1}{36\pi\times10^9}}$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\cdot\frac{\partial V}{\partial\rho}\right) - 360\pi(1+10\rho)$$

$$\rho\frac{\partial V}{\partial\rho} = \int 360\pi(\rho+10\rho^2)\,d\rho$$

$$\rho\cdot\frac{\partial V}{\partial\rho} = 360\pi\left(\frac{\rho^2}{2}+10\frac{\rho^3}{3}\right)$$

$$V = \int^{\rho=5cm} 360\pi\left(\frac{\rho}{2}+\frac{10\rho^2}{3}\right)d\rho$$

$$V = 360\pi \int_{\rho=2cm}^{5cm} \left(\frac{\rho^2}{4} + \frac{10\rho^3}{9}\right)$$

83

$$V = 360\pi \left[\frac{\rho^2}{4} + \frac{10\rho^3}{9}\right]_{2\times10^{-2}}^{5\times10^{-2}}$$

$V =$

32. $\nabla^2 V = \frac{-\rho_v}{\epsilon}$ $\qquad V = [20x^3 + 10y^4]$

$$\nabla^2 V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left[60x^2 \cancel{+ 40y^3}\right] + \frac{\partial}{\partial y}\left[40y^3\right]$$

$$\Rightarrow (120x + 120y)_{(2,0)} = \frac{-\rho_v}{\epsilon_0}$$

$$\Rightarrow 240 = \frac{-\rho_v}{\epsilon_0} \Rightarrow \boxed{-240\epsilon_0 = \rho_v} \text{ Ans}$$

34. $\phi \rightarrow$ 1 dimensional fn - Laplace

II derivative of $\phi = 0$

$$\nabla^2 \phi = 0$$

$\phi \rightarrow$ linear fun

$\phi \rightarrow$ changes at constt rate (charge per unit length) $\frac{\phi_2 - \phi_1}{d} = \frac{\phi_3 - \phi_2}{2d}$

$$\frac{\phi_2 - \phi_1}{d} = \frac{\phi_3 - \phi_2}{2}.$$

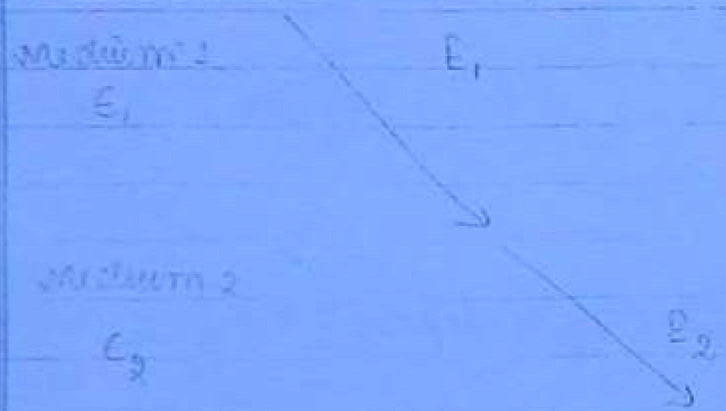
84

$2\phi_2 - 2\phi_1 = \phi_3 - \phi_2 \Rightarrow$ $\phi_2 = \frac{2\phi_1 + \phi_3}{3}$

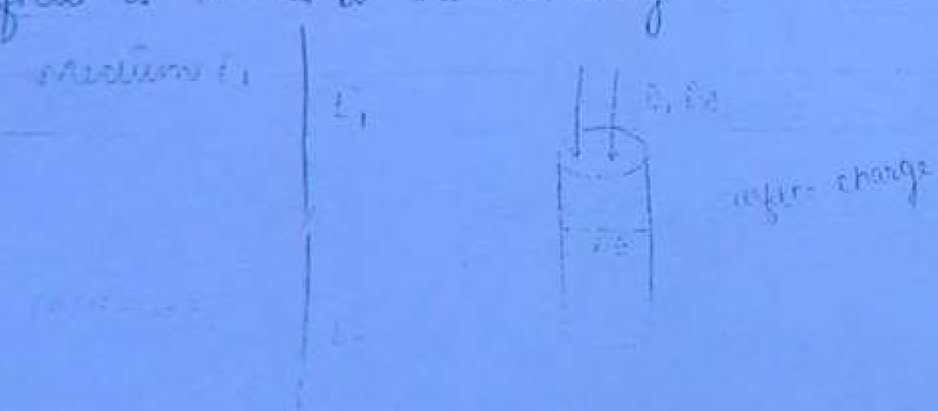
Imp conv

Boundary conditions (dielectric - dielectric)

- If a field is known in one medium and the field is to be calculated in the adjacent medium we use boundary conditions
- Boundary condition can be defined for only two types of fields for normal and tangential directions only.



case I E field is normal to the boundary



$\epsilon = \text{const}$

$$\boxed{\nabla^2 V = \frac{-\rho_v}{\epsilon}}$$ Poission's Eqn.

85

If $\rho_v = 0$ charge free regions

eg. free space

$$\boxed{\nabla^2 V = 0}$$ Laplace Eqn

∇^2 - scalar Laplacian operator

Note 1 Laplace and poission's are second order differential Eqn but donot have two solutions we always have a unique solution. This is called as Uniqueness Theorem.

Voltage at a point is unique it cannot have multiple value. This is the physical meaning of uniqueness theorem.

Laplace and Poission in magnetic fields

In magnetic field the same relationship exist b/w vector potential $\bar{A}$ and current density J

$$\nabla \cdot D = \rho_v \longrightarrow \nabla \times H = J$$

$$D = \epsilon E \longrightarrow B = \mu H$$

$$E = -\nabla V \qquad B = \nabla \times A$$

$$\nabla \times (\nabla \times A) = \mu J$$

$$\nabla(\nabla \cdot A) - \nabla^2 A = \mu J$$

{$(\nabla \cdot A) = 0$ as magnetic field doesnot have divergence it has only curly nature)

$$\boxed{\nabla^2 A = -\mu J}$$

86

$$\boxed{\nabla^2 A = 0}$$ If $J = 0$ current free region

Formula for laplace operation on scalar V

If $V = V(u, v, w)$ = scalar function of space

$$\nabla \cdot \underbrace{(\nabla V)}_{\text{vector}} = \underbrace{\nabla^2 V}_{\text{scalar}}$$

Divergence of vector gives scalar (∇^2)

$$\nabla^2 V = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2 h_3}{h_1}\frac{\partial V}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_3 h_1}{h_2}\frac{\partial V}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_1 h_2}{h_3}\frac{\partial V}{\partial w}\right)\right]$$

D.W.B

$G \quad \int \rho_v \, dv$ step 2.

$\nabla^2 V = -\frac{\rho_v}{\epsilon}$ step 1

V(x) only so it reduce the calculation

consider a cylinder symmetrically in both the medium and use the $\oint D \cdot ds = Q$ (87)

v/s there is no charge enclosed in the cylinder entering flux equals to the leaving flux because cylinder is consist of material and atoms has no charge inside it is electrically neutral.

so. $D_1 \Delta s = D_2 \Delta s$

$$D_1 = D_2$$

$$\epsilon E_1 = \epsilon E_2$$

If the Boundary has surface charge ρ_s C/m^2

$$D_2 \cdot \Delta s = D_1 \cdot \Delta s + \rho_s \cdot \Delta s$$

$$(D_2 - D_1) = \rho_s$$

Words statement of Boundary Condition

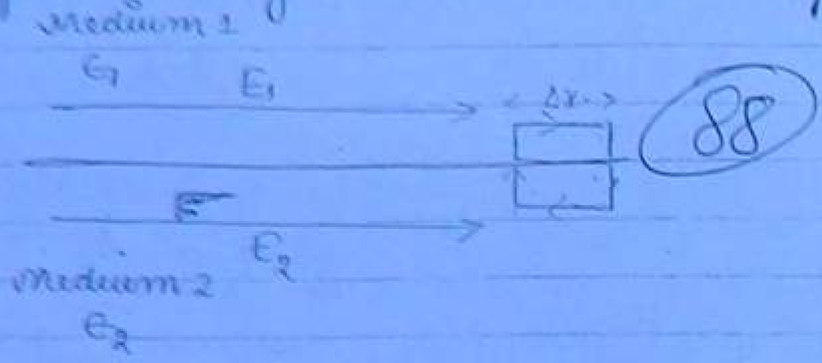
1. The normal component of flux density are continous on either sides if there is no surface charge else discontinous (not equal) (some gap) by amount equal to the surface charge density on the boundary

$$\boxed{D_{n1} = D_{n2}}$$

or. $\boxed{D_{n2} - D_{n1} = \rho_s}$

n indication that it is for normal

Case II — E field is tangential to the Boundary.



Using Maxwell Eq^n (II) $\oint E \cdot dl = 0$

Take a closed line symmetrical in both the media that has Δx length.

$$\oint E \cdot dl = 0$$

$$E_1 \Delta x - E_2 \Delta x = 0$$

$$\boxed{E_1 = E_2}$$

Statement.
The tangential components of electric field intensity are always continous.

36 WB

$x = 0$

$x < 0$	$x > 0$
$\epsilon_1 = 1.5\epsilon_0$	$\epsilon_2 = 2.5\epsilon_0$
$E_1 = 2a_x - 3a_y + a_z$	
$E_{n_1} = 2a_x$	
$E_{t_1} = -3a_y + a_z$	$E_{t_2} = -3a_y + a_z$

$x =$ const surface. (89) $\{ x-5=0$ (take gradient)

Normal is always a_x

direction " " a_x $1a_x$

$D_{n_1} = \epsilon E$ $\longrightarrow$ $D_{n_2} = 3\epsilon_0 a_x$

$= 1.5\epsilon_0 \, 2a_x$

$= 3\epsilon_0 a_x$

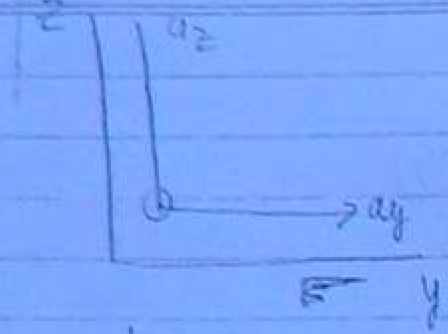
$E_{n_2} = \frac{D_{n2}}{\epsilon} = \frac{3\epsilon_0 a_x}{2.5\epsilon_0} = 1.2a_x$

$E_2 = 1.2a_x - 3a_y + a_z$ $\underline{\underline{Ans}}$

$D_1 = \epsilon_0(3a_x - 4.5a_y + 1.5a_z)$ $D_2 = \epsilon_0(3a_x - 7.5a_y + 2.5a_z)$

note :

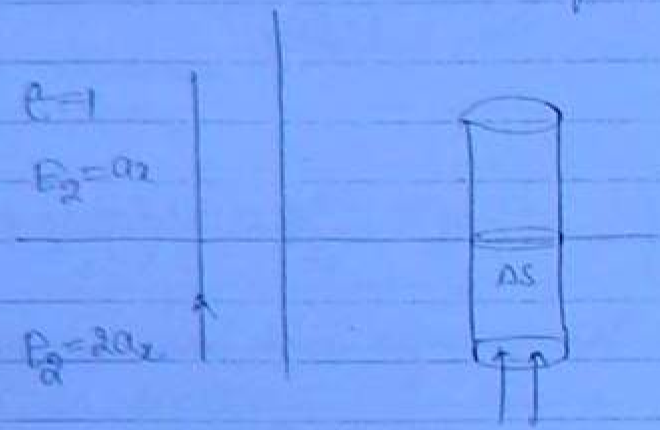
As seen in the diagram the field in the second medium shifting away from the normal which can be under stood the decrease normal compoments in E_2.
It can also be understood that the field is shifting towards the boundary increase tangential component as seen in D_2



yz plane
$x=0$ plane
$a_x \to$ normal
$a_y, a_z \to$ tangential

90

37 W.B



$2 \cdot 2\epsilon_0 \cdot \Delta S$ = Entering. sink at the boundary
$1 \cdot 1\epsilon_0 \cdot \Delta S$ = leaving.
$\rho_s \cdot \Delta S = -3\epsilon_0 \, \Delta S$
$\rho_s = -3\epsilon_0$

Extension : Magnetic Boundary conditions

1. $B_{n_1} = B_{n_2}$ Apply Maxwell III Eqn
Always $\oint B \cdot ds = 0$

2. $H_{t_1} = H_{t_2}$ Apply Maxwell IV Eqn
not always $\oint H \cdot dl = I$

$H_{t_1} \; H_{t_2} \; I$

$\bar{K}$ = surface current density Amp/m

91

Note: D and ρ_s have same units C/m^2. $\bar{H}$ and $\bar{K}$ have the same units. A/m

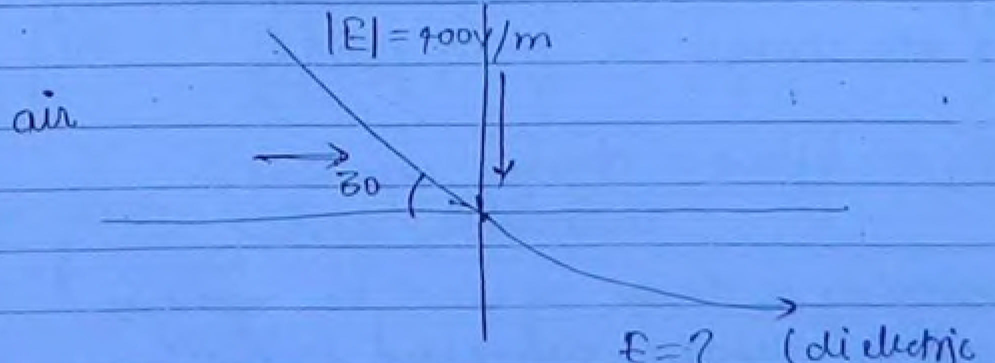
38 W.B

$Z=0$

$Z<0$	$Z>0$
$\mu_{R1} = 2$	$\mu_{R2} = 1$
$\bar{B}_1 = 1.2\,\bar{a}_x + 0.8a_y + 0.4a_z$	B_2
$B_{n1} = 0.4a_z$ $\{\because z=0\}$	$H_2 = ?$ $B_{n2} = 0.4a_z$
$B_{t1} = 1.2a_x + 0.8a_y$	
$H_{t1} = \dfrac{B_{t1}}{\mu_{R1}}$	$H_{t2} = \dfrac{1}{\mu_0}(0.6a_x + 0.4a_y)$
$= \dfrac{1}{2\mu_0}(1.2a_x + 0.8a_y)$	$H_{n2} = \dfrac{1}{\mu_0}(0.4a_z)$
$= \dfrac{1}{\mu_0}(0.6a_x + 0.4a_y)$	

$\left(H = \dfrac{B}{\mu}\right)$

39 W.B



cos is tangential as it is adjacent

sin is taken as normal.

$E_{t1} = 400 \cos 30^\circ = 400 \times \frac{\sqrt{3}}{2} = 200\sqrt{3} = E_{t2}$

$E_{n1} = 400 \sin 30^\circ = 200$

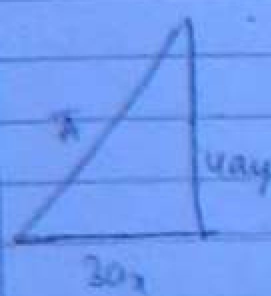
92

$D_{n1} = 200\epsilon_0 = D_{n2}$

$E_{n2} = \frac{200\epsilon_0}{\epsilon_r}$ $\{\epsilon_r = 20\epsilon_0\}$

$E_{n2} = \frac{200\epsilon_0}{20\epsilon_0} = 10$

(magnitude) $E_2 = \sqrt{(200\sqrt{3})^2 + (10)^2} \approx 200\sqrt{3}$



$|A| = \sqrt{3^2 + 4^2}$

Energy Density in a electric field.

Consider a system of n discrete point charges having a electric field E and the total energy W_E.
The total energy can be equal to the energy expanded in assembling the charges in their positions

Total energy in E field = Energy expanded in assembly the charges

It involves bringing a charge against the repulsive forces of the quantle already assembled charges.

$$W_E \begin{cases} W_1 = 0 \\ W_2 = -Q_2 V_{21} \\ W_3 = -Q_3 V_{31} - Q_3 V_{32} \\ W_4 = -Q_4 V_{41} - Q_4 V_{42} - Q_4 V_{43} \\ \vdots \\ W_n = -Q_n V_{n1} - Q_n V_{n2} \cdots - Q_n V_{n(n-1)} \end{cases}$$

V_{21} = potential of 2 due to charge 1

93

The total energy W_E is the sum of all the energy

The subscript can be interchange without changing the meaning Hence

$$Q_2 \cdot V_{21} = Q_2 \cdot \frac{Q_1}{4\pi\varepsilon_0 r_{21}}$$

$$Q_1 V_{12} = \frac{Q_1 \cdot Q_2}{4\pi\varepsilon_0 r_{12}}$$

$$W_E \begin{cases} W_1 = 0 \\ W_2 = -Q_1 V_{12} \\ W_3 = -Q_1 V_{13} - Q_2 V_{23} \\ W_4 = -Q_1 V_{14} - Q_2 V_{24} - Q_3 V_{34} \\ \vdots \\ W_n = -Q_1 V_{1n} \cdots - Q_{n-1} V_{n-1n} \end{cases}$$

$$2W_E = -Q_1 V_1 - Q_2 V_2 - Q_3 V_3 \cdots Q_n V_n$$

Total Energy $$\boxed{W_E = -\frac{1}{2} \sum_{i=1}^{n} Q_i V_i}$$

for continous charges & E fields

$$W_E = -\frac{1}{2} \int \rho_v V \, dv \qquad \{ Q = \rho_v dv \}$$

$$= -\frac{1}{2} \int (\nabla \cdot D) V \, dv$$

$$W_E = \frac{1}{2}\int D(\nabla \cdot V)\, dv$$

$$W_E = \int \frac{1}{2}(D\cdot E)\, dv$$ (94)

$$\frac{dW_E}{dv} = \frac{1}{2} D\cdot E$$

$$\boxed{\frac{dW_E}{dv} = \frac{1}{2}\epsilon E^2}$$

$\frac{dW_E}{dv}$ = strength of the energy at every point in the E field.

$$\frac{dW_E}{dv} = \frac{1}{2}\epsilon E^2$$

Note 1. $D\cdot E = \frac{\text{Joule}}{m^3} = \frac{\text{Newton} \times \text{meter}}{m^3} = \frac{\text{Newton}}{m^2}$

$\frac{N}{C}\cdot\frac{C}{m^2} \Rightarrow \frac{\text{Newton}}{m^2}$ = Pressure.

2. $W_E = \frac{1}{2}CV^2$ is similar to $\frac{1}{2}\epsilon E^2$

Extension:- $\frac{dW_H}{dv} = \frac{1}{2}\mu H^2$

$$= \frac{1}{2} B\cdot H$$

It is similar to $\frac{1}{2}LI^2$

1 Ohm's law and Continuity Eq^n.

$I = \frac{Q}{t} = \frac{Ne}{\frac{l}{v}} = \frac{NevA}{lA}$

95

$I = neAv$

$$I = \frac{dQ}{dt} = \frac{d}{dt}\int \rho_v \, dv = \int \frac{\partial \rho_v}{\partial t} dv$$

Q = continuous volume charge movement

the current can be considered as the current crossing the closed surface formed by the volume charge. Hence in this case

$$I = \oint J \cdot ds$$

Note: Generally $\oint J \cdot ds = 0$ (loop integral) means law of conservation of charge because entering charge is equal to the leaving charge.

Apply divergence theorem

$$\oint J \cdot ds = \int (\nabla \cdot J) \, dv$$

Hence by comparison

$$\boxed{\nabla \cdot J = \frac{\partial \rho_v}{\partial t}}$$ continuity Eq^n.

This is called as continuity Eq^n which is the definition of current in field theory.

- outflow of current depends on movement of volume charges density

$\frac{\partial J}{\partial x}$

Apply for field.

$\nabla \cdot J = \frac{\partial \rho_v}{\partial t}$

(96)

$\nabla \cdot J = \frac{\partial J}{\partial x} = \frac{\partial \rho_v}{\partial t}$

$\partial J = \partial \rho_v \cdot \frac{\partial x}{\partial t}$ $\left\{ \because \frac{\partial x}{\partial t} = \underbrace{v_d}_{\text{drift velocity}} \right\}$

$$\boxed{J = \rho_v v_d}$$

$v_d \propto E$

$v_d = \mu E$

μ → mobility (ability to move)

$J = \rho_v \mu E$ — In point form

$\boxed{J = \sigma E}$ Ohm's law & continuity Eq'n in

here $\boxed{\sigma = \rho_v \mu}$ conductivity

Hence conductivity is ability to allow current in to the medium. It is the product of availibility and ability to move.

case 1 $\sigma = \infty$ very good conductor

$\frac{J}{\sigma} = E$

$\frac{J}{\infty} = E$

$E = 0$

- E field cannot exist inside a good conductor
- only flow exist but not accumulation of charge - Hence only current exist but not E field.

$$V = \int D \cdot dl = \text{const}$$

(97)

Every conductor is an equipotential surface
Potential diff. b/w any 2 points is zero.

For conductor surface as $\sigma = \infty$ on the surface

1. E along the surface = $E_{tan} = 0 = E_t$
   (Electric field never be parallel) to the conductor surface

2. D_{normal} to the surface = $D_{normal} \neq 0$

$$\boxed{D_n = \rho_s}$$

case 2. $\sigma = 0$ very good dielectric

$J = \sigma E \Rightarrow J = 0 \times E$

$J = 0$

- E field can exist inside a good dielectric
- Not flow exist but accumulation exists hence only E field exist

$$\nabla \cdot J = \frac{\partial \rho_v}{\partial t}$$

$$J = \sigma E$$

$$\nabla \cdot (\sigma E) = \frac{\partial \rho_v}{\partial t}$$

$$\sigma\ \nabla\cdot\left(\frac{D}{\epsilon}\right) = \frac{\partial \rho_v}{\partial t}$$

$$\boxed{\frac{\partial \rho_v}{\partial t} = \frac{\sigma}{\epsilon}\rho_v}$$

98

The solution of the Eq^n is - an exponentially decaying function(derivative back the same function is exponential) since

$$\rho_v(t) = \rho_{v0}\cdot e^{-\frac{t\sigma}{\epsilon}}$$

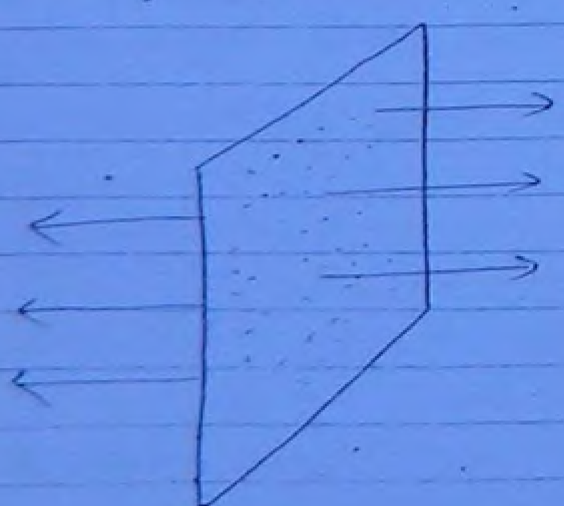
The Eq^n shows the any charge placed in any medium exponentially spread into ~~the~~ medium with the time constt depending on (ϵ/σ)

This ~~co~~ time constt is called as relaxation time or Average spreading time

$$\boxed{\frac{\epsilon}{\sigma} = \text{Relaxation time}}$$

If $\sigma = \infty$, good conductor $\frac{\epsilon}{\sigma} = 0$

Sheet charges of ρ_s C/m^2 & uniform fields



$D = \frac{\rho_s}{2}$

$E\left(\frac{1}{r^2}\right)$

$E\left(\frac{1}{\rho}\right)$

E (uniform)

$E(x)$

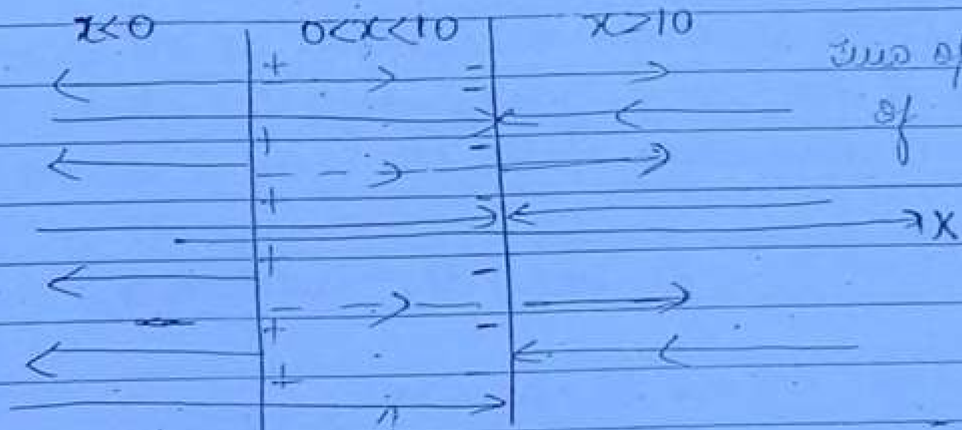
$I\,\overline{dl} = \overline{K}\,ds = \overline{J}\,dv$

$H = \frac{K}{2} \times a_N$

99

$D = \frac{\rho_s}{2}\, a_N$

7/11



Two opposite field of equal strength cancel each other

$+\rho_s$ uniform $-\rho_s$

$E_1 = \left|\frac{\rho_s}{2\epsilon_0}\right| (-a_x)$

$E_2 = \left|\frac{\rho_s}{2\epsilon_0}\right| (a_x)$

left side $E = 0$

Right side two fields because of two sheets.

$E_1 = \left|\frac{\rho_s}{2\epsilon_0}\right| a_x$

$E_2 = \left|\frac{\rho_s}{2\epsilon_0}\right| (-a_x)$

$E_0 = 0$ on right also.

field in b/w.

$E = \frac{\rho_s}{\epsilon_0}\, a_x$

- consider an infinite sheet of charge density ρ_s C/m^2 the field can be found Normal to the sheet and only to the right or left of the sheet. (TCD)

- The sheet is being infinitely large the lines are not divergent but are parallel to themselves.

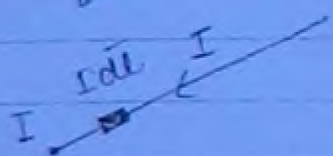
Laser lines are best example

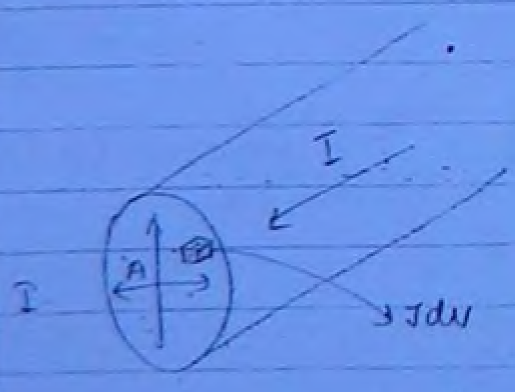
- Hence the strength is same everywhere as the spacing b/w the lines is the same everywhere the flux density is same everywhere Hence the field is a uniform field.

Hence the density: $D = \frac{\rho_s}{2}$ with the direction a_N

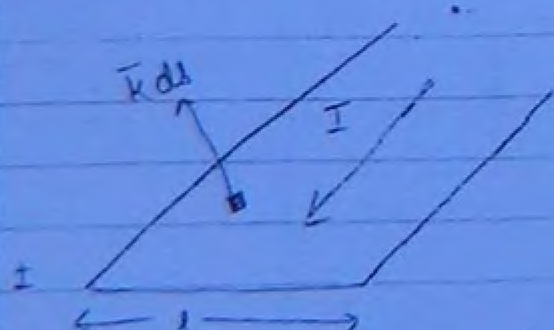
where a_N = unit normal to the sheet.

~~sheet of currents $\bar{K}$ A/m~~





$$J = \frac{I}{A} = \frac{\text{Ampere}}{m^2}$$



$$K = \frac{I}{l} = \frac{\text{Amp}}{\text{meter}}$$

Two infinite sheets of charge density equivalent opposite nature have field only b/w the sheets and cancel out everywhere else

(Q1)

W.B 3 infinite sheets — $18\,nC/m^2 \pm a_x$ at $x=4$

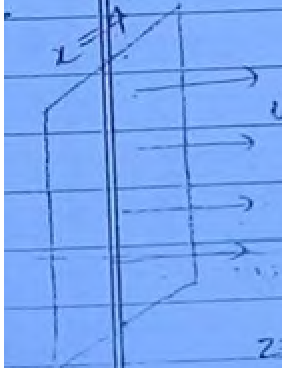
$9\,nC/m^2 \pm a_y$ at $y=3$;

$-24\,nC/m^2 \pm a_z$ at $z=0$

$$E = \frac{D}{\epsilon} = \frac{\rho_s}{2\epsilon}(a_N)$$

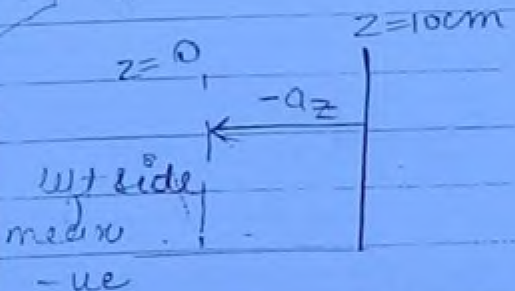
$$E = E_1 a_x + E_2 a_y + E_3 a_z$$

$$E_1 = \frac{18}{2\epsilon_0} a_x \qquad E_2 = \frac{+9}{2\epsilon_0} a_y \;, \qquad E_3 = \frac{-24}{2\epsilon_0} a_z$$



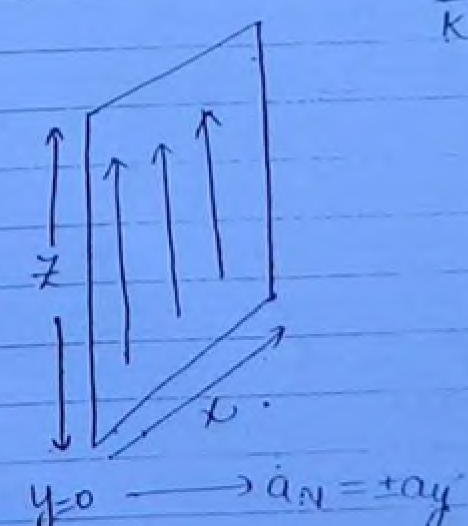
increasing x i.e +ve

18 W.B



$$E = \frac{\rho_s}{2\epsilon} = \frac{20\times10^{-9}}{2\times\frac{1}{36\pi\times10^{9}}} = -360\pi\, a_z \text{ V/m}$$

19 W.B



$y=0 \longrightarrow a_N = \pm a_y$

xz plane.

$$\bar{K} = 30\hat{k} = 30 a_z \text{ mA/m}$$

H at $(1, 20, -2) = ?$

$$H = \frac{\bar{K}}{2} \times a_N$$

$$= \frac{30 \times a_N}{2}$$

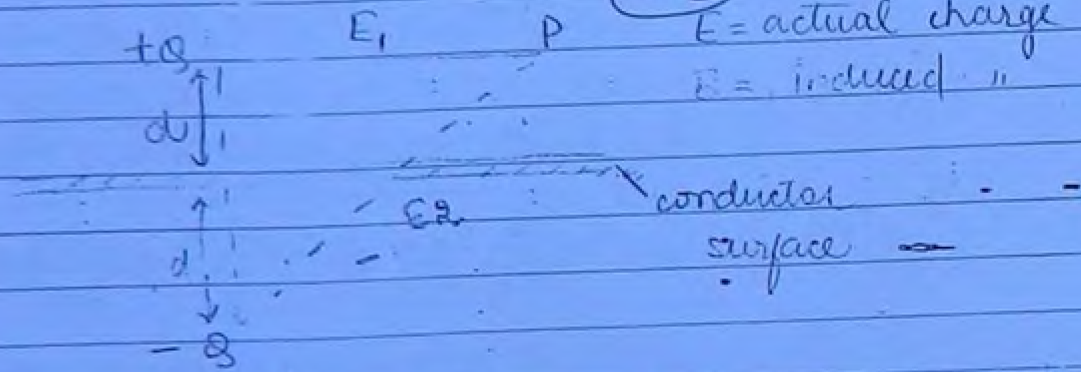
current direction

$$= 15(a_z \times a_y)$$

$$= \cancel{-15}\ -15\, a_x$$

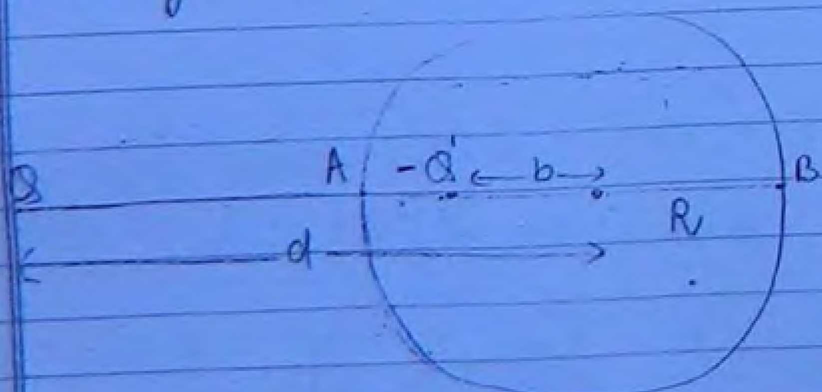
When charge is brought to the near a conductor surface. The charges inside a conductor are realigned due to the field of the charge that is the charges are displaced. Hence the final field anywhere is sum of the field of the actual charge and the field due to this induced charge.

102



This induced charge is represented with a image charge following the dymaics of a mirror. Hence for a flat conductor the charges are equal the image has opposite sign.

9 w.R Image should always be with coaxial.



Let induced charge = image charge = -Q'
Let the image be at 'b' distance

Date
Page

Every conductor is a equipotential surface

$$\text{eq}^n \quad V_A = \frac{Q}{4\pi\epsilon_0(d-R)} - \frac{Q'}{4\pi\epsilon_0(R-b)} = 0 \qquad (103)$$

Let find potential at B also so that eq^n become simple.

$$V_B = \frac{Q}{4\pi\epsilon_0(d+R)} - \frac{Q'}{4\pi\epsilon_0(R+b)} = 0$$

Potential at A and B are zero because the sphere is given a grounded sphere.

$$\frac{Q}{Q'} = \frac{d-R}{R-b} = \frac{d+R}{R+b}$$

29 W.B

$$W_E = \frac{-1}{2}\sum_{i=1}^{n} Q_i V_i$$

$$W_E = \frac{-1}{2}\sum Q_i \frac{Q}{4\pi\epsilon_0 r}$$

$$\boxed{W_E \propto \frac{1}{r}}$$

$$\frac{W_1}{W_2} = \frac{1}{2}$$

another method

1

Q $\quad \frac{1}{2} \quad$ Q $\quad \frac{1}{2} \quad$ Q

104

$W_1 = 0 \qquad W_2 = Q \dfrac{Q}{4\pi\varepsilon_0\left(\frac{1}{2}\right)} \qquad W_3 = \dfrac{Q \cdot Q}{4\pi\varepsilon_0(1)} + \dfrac{Q\, Q}{4\pi\varepsilon_0\left(\frac{1}{2}\right)}$

$W_{E_1} = \dfrac{5Q^2}{4\pi\varepsilon_0} \qquad W_{E_2} = \dfrac{5Q^2}{8\pi\varepsilon_0}$

22 W.B

$1\mu C$ at $(2,1,5)$ $\qquad$ $4\mu C$ at $(1,3,-1)$

$W_1 = 0 \qquad W_2 = \dfrac{Q_1 \cdot Q_2}{4\pi\varepsilon_0 r}$

$W_2 = \dfrac{1\times 4\times 10^{-12}}{4\pi\varepsilon_0 \sqrt{3^2+2^2+6^2}}$

$W_E = W_1 + W_2 = 5.14\,mJ$

We can also use $\quad W_E = \dfrac{1}{2}\sum_{i=1}^{2} Q_i V_i$

40 W.B

$D_n = \rho_s$ as the surface charge density is

$\rho_s = 2\sqrt{(1)^2+(\sqrt{3})^2}$

$2\sqrt{4}$

$\rho_s = 2\times 2$

$\rho_s = 4$ Ans

$\oint D \cdot ds = 0$ $\oint B \cdot ds = 0$

i) $Q = 0$

ii) $+Q, -Q$.

105

Magnetic dipole in magnetic field.

- Whenever a current flows in a closed line it is treated as Magnetic dipole.
- finite Area

Dipole moment $m = I \times \text{area} = I \cdot A$

Electric dipole in Electric fields

- Whenever two charges of equal and opposite are separated by a finite distance

Electric - dipole

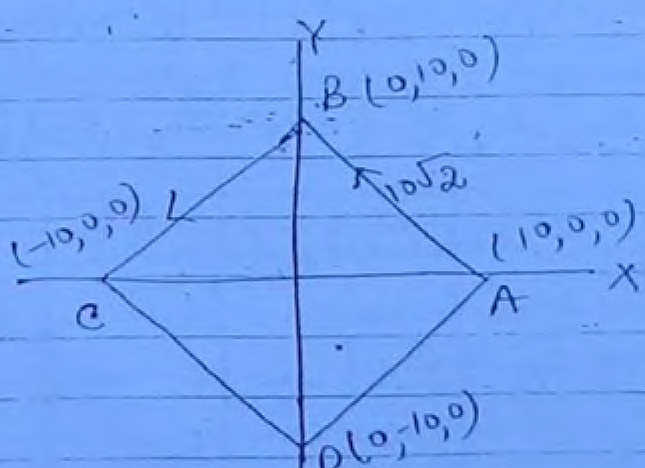
finite distance

Dipole moment $p = Qd$

The importance of dipole moment is it inside the torque or the moment of the dipole in an external field such that magnetic torque is equal $= T = M \times B$

Electric torque is equal $T = P \times E$

6 W.B



$M = I \times \text{Area}$

$= 0.01 \times (10\sqrt{2})^2$

$= 2$

- M's direction is the direction of the area surface of the loop

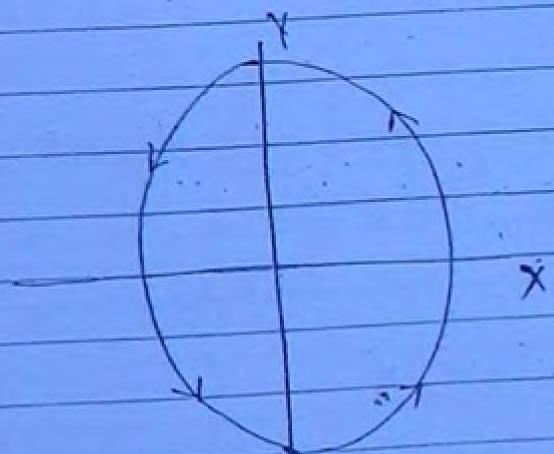
Surface $z = 0$, xy plane

Direction $= \pm a_z$ (1 of 6)

current is anticlockwise as per RHS thumb direction

M direction (+ve $+a_z$) Ans $+2a_z$

45 W.B.



Magnetic dipole moment (~~Torque~~) M $= I \times A$

$= I \times (\pi r^2)$

$= (0.1) \times \pi \times (10^{-3})^2 a_z$

Torque $= M \times B$

$= [(0.1) \times \pi \times (10^{-3})^2 a_z] \times [10^{-5}(2a_x - 2a_y + a_z)]$

$= [10^{-7}\pi\, a_z] \times [10^{-5}(2a_x - 2a_y + a_z)]$

$= [2\times10^{-12}\pi a_y + 2\times10^{-12}\pi a_x]$

$= 2\times10^{-12}\pi[a_y + a_x]$

capacitors & Inductors:

107

- capacitors -

Ability to hold E field confining it into a small region

$$C = \text{Farads} = \frac{\oint D \cdot ds}{\int E \cdot dl} = \frac{\epsilon \oint E \cdot ds}{\int E \cdot dl} = \frac{Q}{V}$$

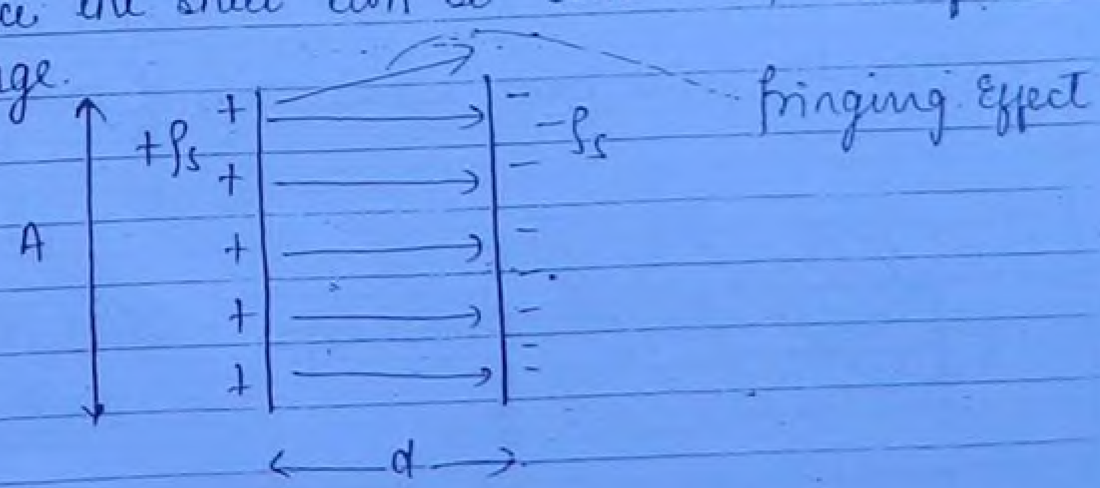
- It is always measure in terms of charge utilize and the potential develop by this charge. because potential is accumulation and hence the measure of holding ability.

- The best examples of capacitor are geometry involving
  eg. Parallel plates
  concentric cylinders
  concentric spheres.

Parallel Plate capacitors.

- Two sheet charge of Area A
- seperation d (A>>d)

Hence the sheet can be considered an infinite sheet of charge.



$$C = \frac{Q}{V} \qquad Q = \rho_s A$$

$$V = Ed = \frac{\rho_s}{\epsilon} d$$

108

so
$$\boxed{C = \frac{\epsilon A}{d}}$$

$$C = \frac{\epsilon \oint E \cdot ds}{\int E \cdot dl}$$ E = const. for uniform field.

$$C = \frac{\epsilon E A}{E \cdot d} = \frac{\epsilon A}{d}$$

Total energy held by the field

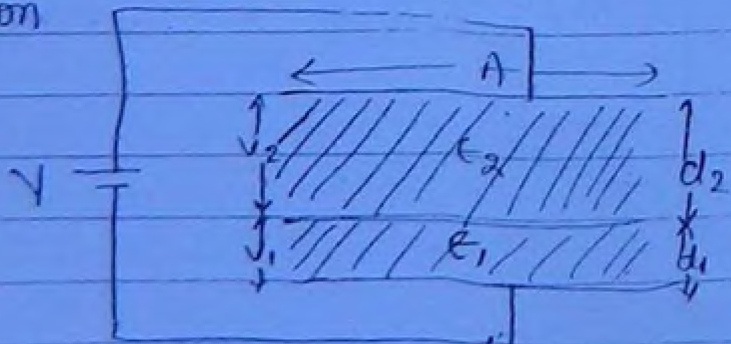
$$W_E = \left(\frac{1}{2}\epsilon E^2\right)(Ad)$$

$$= \frac{1}{2}\frac{\epsilon A}{d}(Ed)^2$$

$$W_E = \frac{1}{2} C V^2$$

Multiple dielectrics in parallel plate capacitor

(i) Dielectric & capacitor plates have equal area of cross section



C_1 and C_2 $\longrightarrow$ series

$$C_1 = \frac{\epsilon_1 A}{d_1} \qquad C_2 = \frac{\epsilon_2 A}{d_2}$$

109

voltage divides b/w the dielectrics.

$$\boxed{V_1 + V_2 = V}$$

$$C_{eq} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{(in series)}$$

$$= \frac{C_1 + C_2}{C_1 C_2}$$

$$C_{eq} \Rightarrow \frac{C_1 C_2}{C_1 + C_2}$$

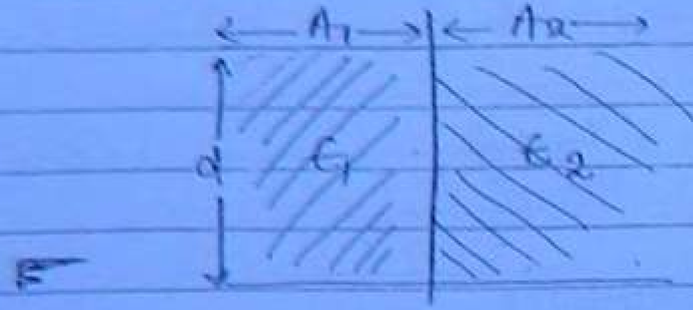
$$C_{eq} = \frac{\frac{\epsilon_1 A}{d_1} \cdot \frac{\epsilon_2 A}{d_2}}{\frac{\epsilon_1 A}{d_1} + \frac{\epsilon_2 A}{d_2}}$$

$C_1 V_1 = C_2 V_2$ (as charge is common and voltage divides in series)

$$\frac{V_1}{V_2} = \frac{C_2}{C_1}$$

$$\frac{V_1}{V_2} = \frac{\epsilon_2}{d_2} \cdot \frac{d_1}{\epsilon_1}$$

...e(ii) Dielectrics & capacitor plates have equal seperation / thickness



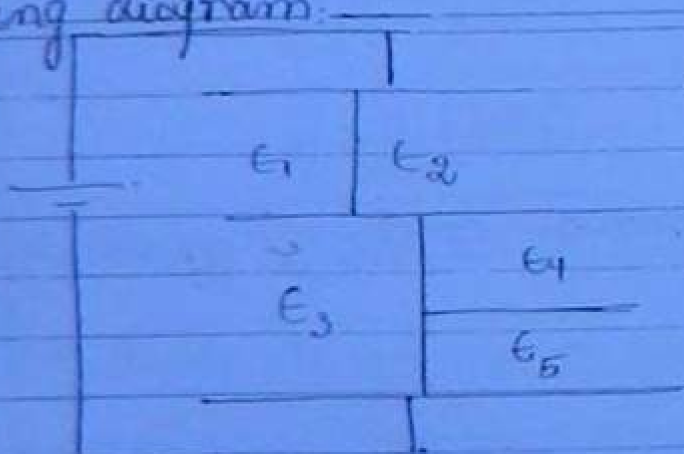
$C_1\ C_2 \longrightarrow$ are in shunt parallel.

voltage applied to plates = voltage b/w the dielectrice

$$C_{eq} = C_1 + C_2$$

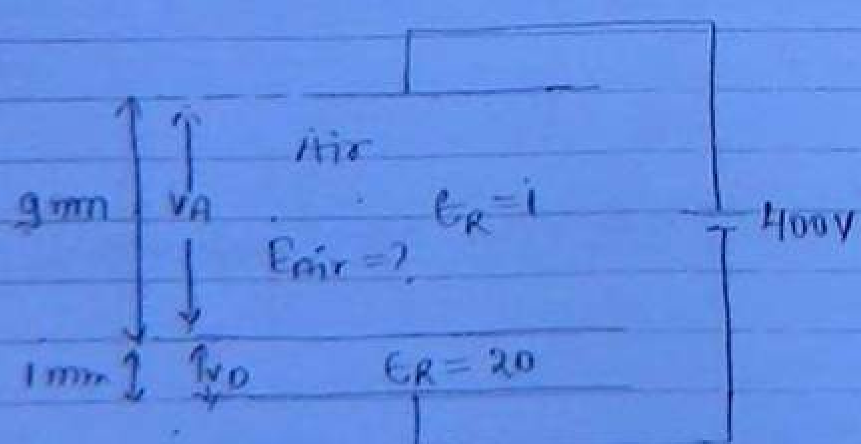
$$= \frac{\epsilon_1 A_1}{d} + \frac{\epsilon_2 A_2}{d}$$

Q Identify the equivalent capacitance in the following diagram.



$C_{eq} = (C_1 \text{ parallel } C_2)$ series $(C_3 \text{ parallel } (C_4 \text{ series } C_5))$

12 W.8



$V_A + V_D = 400$

$\frac{V_A}{V_D} = \frac{20}{1} \times \frac{9}{1}$

$\frac{V_A}{V_D} = 180\, V_D$

$180 V_D + V_D = 400$

$181 V_D = 400$

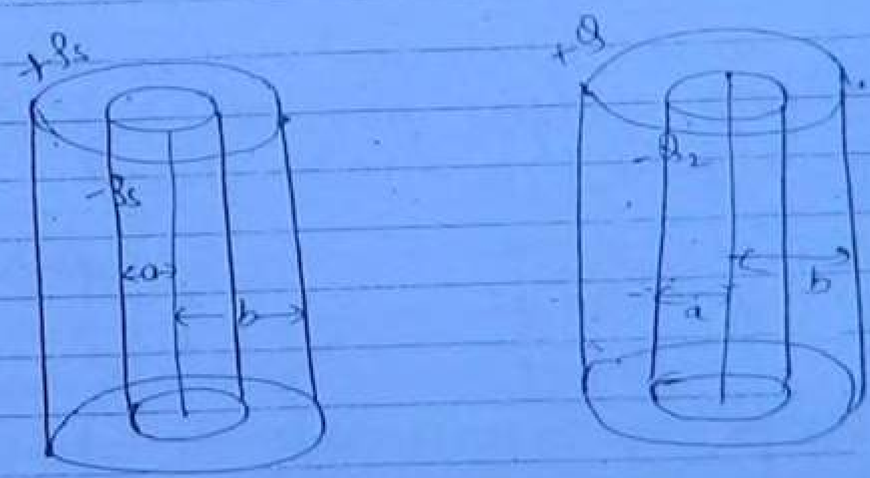
$V_D = 2.21$

$V_A + 2.21 = 400$

$V_A = 397.8V$

$E = \frac{V_A}{d} = \frac{397.8\,V}{9\,mm} \approx 44 \frac{KV}{m}$

concentric cylinders:



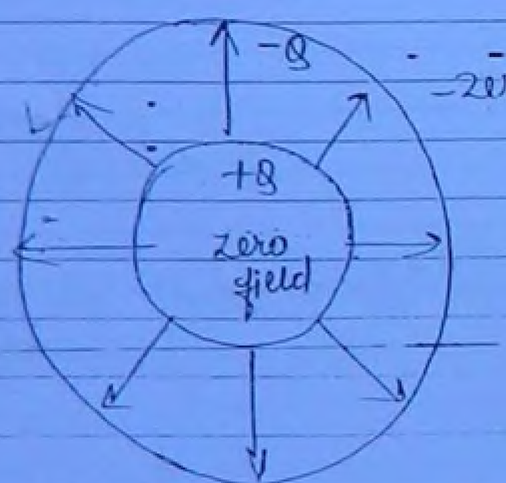
(i) If two concentric cylinders have equal and opposite charge densities that means the charges are unequal on their surfaces. Hence a field or flux leaving exist outside the cylinder also.

ii If two concentric cylinders have equal and opposite charge the densities are unequally spread but net flux outside of cylinder is zero. Hence the field is confined b/w the cylinders

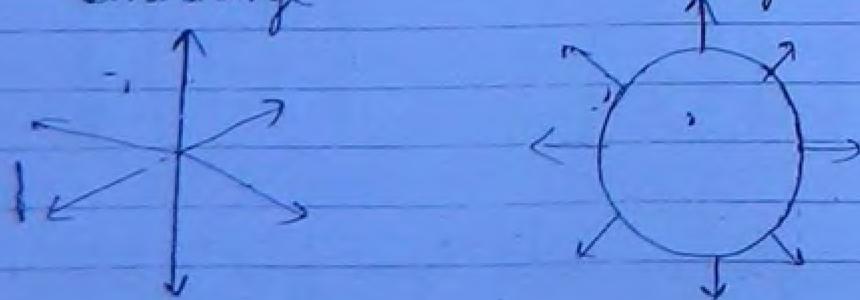
(112)

$$\rho_{s_1} = \frac{Q}{2\pi bh}$$

$$\rho_{s_2} = \frac{-Q}{2\pi ah}$$



A charge on the cylindrical surface has the same radial field divergent from the surface similar to that of a field from a line charge.



$E \propto \frac{1}{\rho}$ $\qquad$ $E \propto \frac{1}{\rho}$

Inductors:

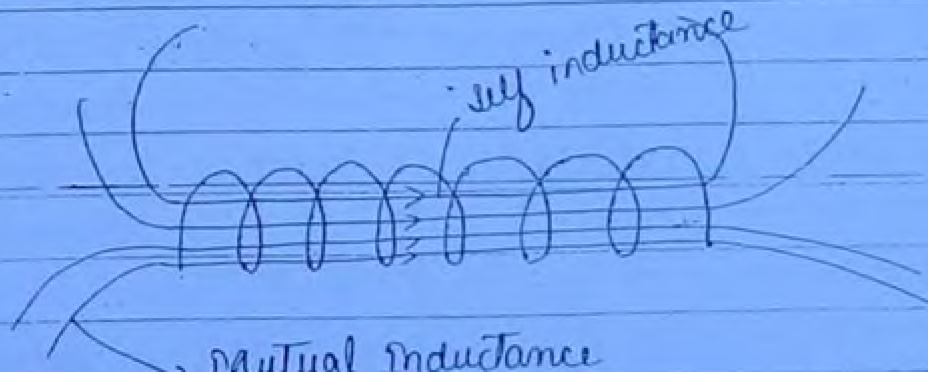
- Ability to hold magnetic field (H) confined into a small region is called as Inductance.

- Henry $L = \dfrac{\int B \cdot ds}{\oint H \cdot dl} = \dfrac{\mu \int H \cdot ds}{\oint H \cdot dl} = \dfrac{\psi_m}{I} = \text{flux}/\text{current}$

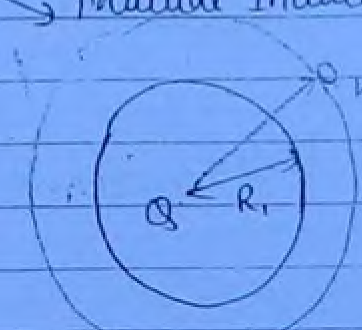
Inductance is a measure of the confined flux and the current utilize for this confinement.

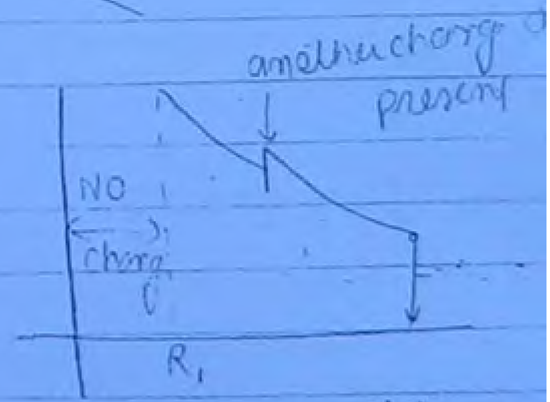
- Geometries :- Solenoids
  - concentric cylinders
  - Toroids.

$\left.\right\}$ All the have some or other circular dielectric



47 W.B





- The graph shows that the field is zero upto R_1 distance since there is no charge enclosed upto R_1 so that charge is Hollow sphere of charge Q_1 and radius R_1

51. $J \cdot A = \dfrac{Amp}{m^2} \times \dfrac{weber}{m}$

$\dfrac{weber}{m} = \dfrac{Joule}{Amp\text{-}m}$

$$\frac{\text{weber}}{m} = \frac{W}{I\partial l}$$

$$J \cdot A = \frac{Amp}{m^2} \times \frac{\text{weber}}{m} = \frac{Amp}{m^2} \times \frac{Joule}{Amp\text{-}m} = \frac{Joule}{m^3}$$

Summary 1

Scalar function $\xrightarrow{\nabla - \text{gradient}}$ Vector
Intensity per m

eg. voltage V $\xrightarrow{\nabla \cdot V}$ E $\frac{\text{volts}}{m}$

vector function $\xrightarrow{\nabla \times \text{curl}}$ vector function
Intensity per m — Density per m^3.

eg. Intensity

Vector function $\longrightarrow$ Scalar function

Density $\longrightarrow$ (per m^3) volume

c/m^2 flux density $\xrightarrow{\nabla \cdot D}$ ρ_v (c/m^3)

summary 2. (115)

Maxwell's Equations:

Integral form	Point form	Not maxwell's Eqn. open integrals.
1. $\oint D \cdot ds = Q$	1 $\nabla \cdot D = \rho_v$	1. $\int D \cdot ds = \psi_e$ = colomb
2 $\oint E \cdot dl = 0$ ✓	2. $\nabla \times E = 0$	2. $-\int E \cdot dl = V$ = volts
3. $\oint B \cdot ds = 0$	3 $\nabla \cdot B = 0$	3. $\int B \cdot ds = \psi_m$ = weber
4. $\oint H \cdot dl = I$ ✓	4. $\nabla \times H = J$	4. $\int H \cdot dl = V_m$ = amps

Note: - Rate of change of magnetic flux with time is EMF voltage.
Rate of change of electric flux with the time is current.

$\frac{\text{weber}}{\text{sec}}$ = volts [Faraday's law]

$\frac{\text{colomb}}{\text{sec}}$ = amps

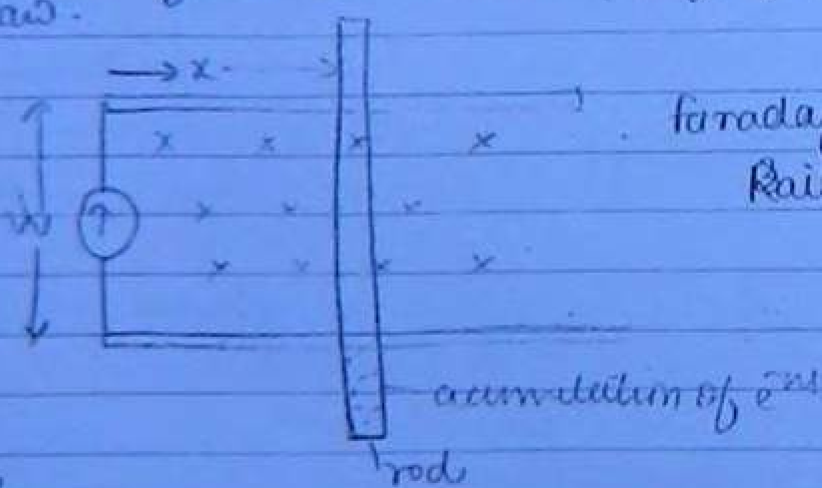
Time varying fields & Maxwell's Eq^n (116)

- Maxwell's I^st and III^rd eq^n i.e surface integrals are unmodified and are consistent for time varying fields also. However the line integral are modified.

Faraday's Law and Maxwell's II Eq^n

$$\oint E \cdot dl = 0 \quad \& \quad \nabla \times E = 0$$

Faraday's law states that the EMF is produced even in a closed conductor and the magnetic flux through the area of the conductor changes with the time. ~~Produce voltage opposes the changing flux this is Lenz law.~~



Faraday's sliding Rail experiment.

$$\text{Hence} \quad \boxed{V = \oint E \cdot dl = -\frac{d\psi}{dt}}$$

$$V = -\frac{d}{dt}(B \cdot A) \qquad (F_y = q(V_z \times B_z)$$

117

$$= -BW\frac{dx}{dt}$$

$$V = -B \cdot W \cdot V$$

$$V = \oint E \cdot dl = \frac{-d}{dt}\int B \cdot dl$$

$$\int(\nabla \times E) dl = \oint E \cdot dl = \int -\frac{\partial B}{\partial t} \cdot dl$$

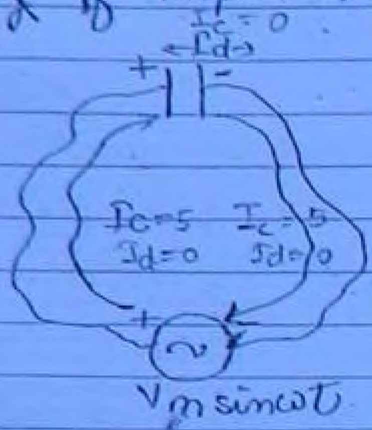
$$\oint E \cdot dl = \int -\frac{\partial B}{\partial t} \cdot ds \quad \& \quad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$ Hence modified

Note: Potential at a point is unique at a time but it can change at various time. Hence the modification

Inconsistency of Ampere's law | Maxwell's IV Eqⁿ.

118



$$I = C\frac{dv}{dt}$$

$$I = C \cdot \omega \cdot V_m \sin(\omega t + 90^\circ)$$

$$V_m \sin\omega t = \left(\frac{1}{j\omega C}\right) I \qquad \{e^{j90} = j\}$$

$$V = IX.$$

It is clear from the eqⁿ that a current flows in the wire and on the place of the capacitor but there is no current b/w capacitor plates hence ampere's law fails to explain the closed nature of current in the ckt

Maxwell added a term I_d or J_d which is said to flow b/w the capacitor plates making the ckt closed

Applying continuity Eqⁿ.

$$\nabla \cdot J_d = \frac{\partial \rho_v}{\partial t}$$

$$\nabla \cdot J_d = \frac{\partial}{\partial t}(\nabla \cdot D)$$

$$\nabla \cdot J_d = \nabla \cdot \frac{\partial D}{\partial t}$$

$\frac{320}{\sqrt{2}} = 230V$

$$\oint H \cdot dl = I_c + I_d$$

$$\nabla \times H = J_c + J_d$$

$$I_d = \int \frac{\partial D}{\partial t} \cdot dl \qquad J_d = \frac{\partial D}{\partial t}$$

119

The term J_d is called as displacement current density and it accepted as a current format but it is not due to a moving electron it is due to a time varying electric flux line.

Physical interpretation of flow in a capacitor.

When a time varying voltage is given to the capacitor the plates are alternately charge and discharge in accordance to the polarity changes. This is itself a continous process and hence the wires always have a J_c or I_c

Between the plates of the capacitor there is an electric flux changing due to charge and discharge of the plates. This is a form of current this is J_d or I_d.

Mathematically

$$|J_c| = |J_d|$$

In this example

Summary:

1. $$\nabla \times H = \frac{\partial D}{\partial t} + J_c$$

$$= \epsilon \frac{\partial E}{\partial t} + \sigma E \text{ (Time varying electric field)}$$

2. $$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t}$$