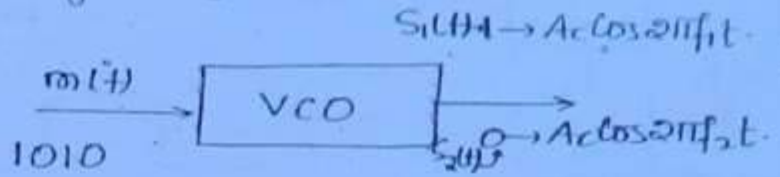
18.11

FREQUENCY SHIFT KEYING (FSK):

In this Binary 1 is represented by high freq

In this Binary 1 is represented by high $freq^n$ carrier & Binary 0 by low $freq^n$ carrier.

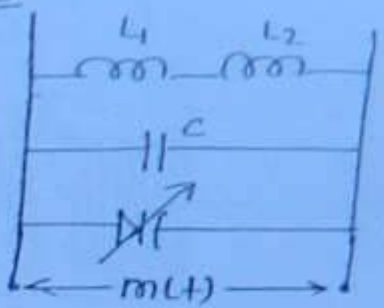
939

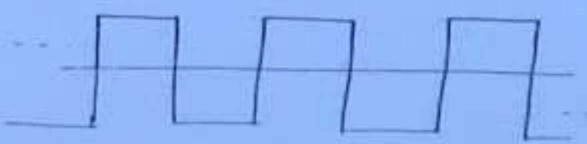


NRZ ⇒ 1 = +ve

0 = −ve

VCO



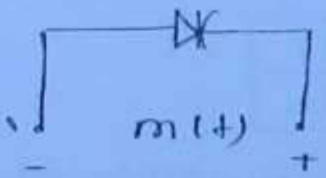


$$\boxed{f_1 = \frac{1}{2\pi\sqrt{(L_1+L_2)(C+C')}}}$$

i) Tx of 1:

m(t) is +ve

then



Varactor diode connected in Reverse mode.

And, as $C' \propto 1/\omega$

So, in R-B width of depletion layer is high hence C' is less

So f_1 is high.

$$\boxed{\underset{(as\ \omega\uparrow)}{RB\uparrow \rightarrow C'\downarrow} \rightarrow f_1\uparrow = f_1}$$

ii) Tx of 0:

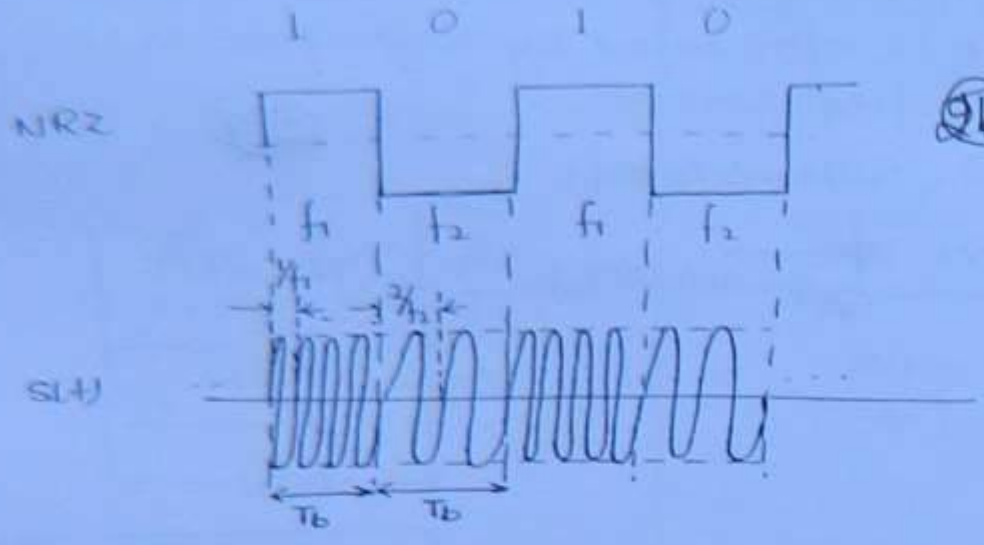
m(t) is −ve

$$\boxed{\underset{(as\ \omega\downarrow)}{F\text{-}B\uparrow \rightarrow C'\uparrow} \rightarrow f_i\downarrow = f_2}$$

$\Rightarrow \boxed{f_1 >>> f_2}$

As $f_1 > f_2$, then also both the frequencies should be in the Range of MHz.

* Graphical Interpretation:-



2146

$T_b = 4/f_1$; $T_b = 2/f_2$

So, In general,

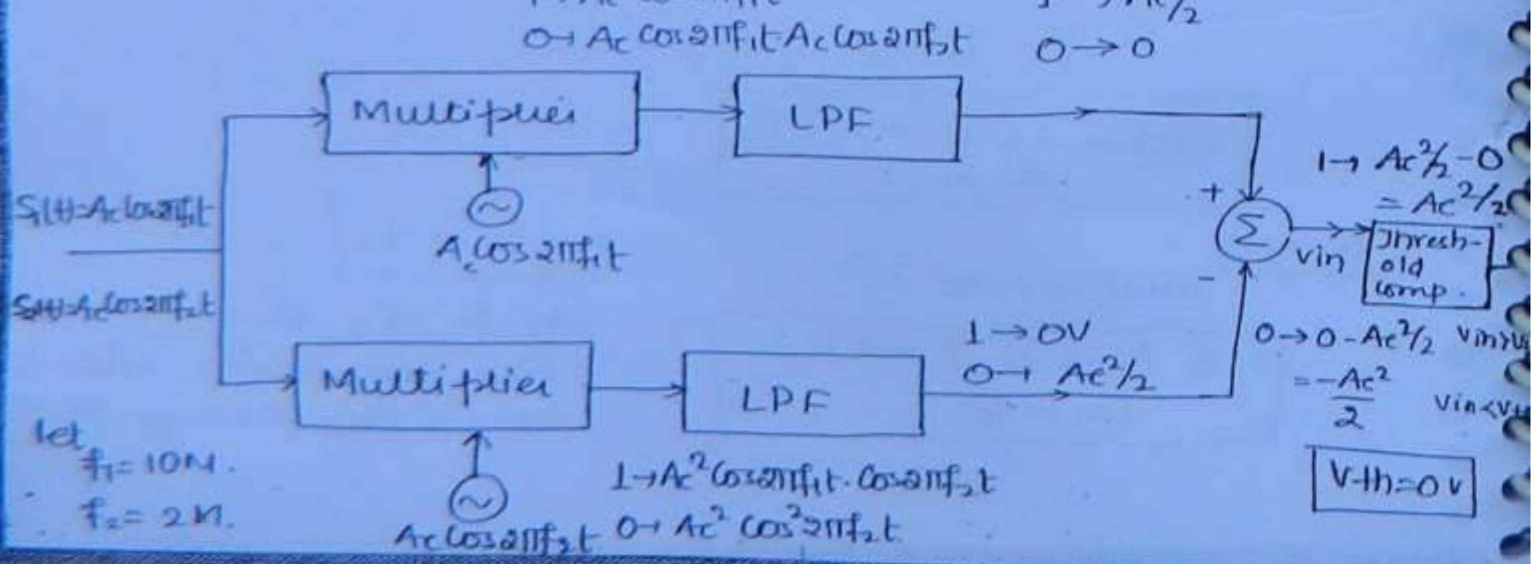
$T_b = \frac{n_1}{f_1}$; $T_b = \frac{n_2}{f_2}$

$\boxed{f_1 = \frac{n_1}{T_b}}$; $\boxed{f_2 = \frac{n_2}{T_b}}$

f_1 & f_2 should be Integer multiple of the Bit Rate

ie $\boxed{f_1 = n_1 T_b}$; $\boxed{f_2 = n_2 T_b}$

* FSK RECIEVER:



$\boxed{V_{th} = 0V}$

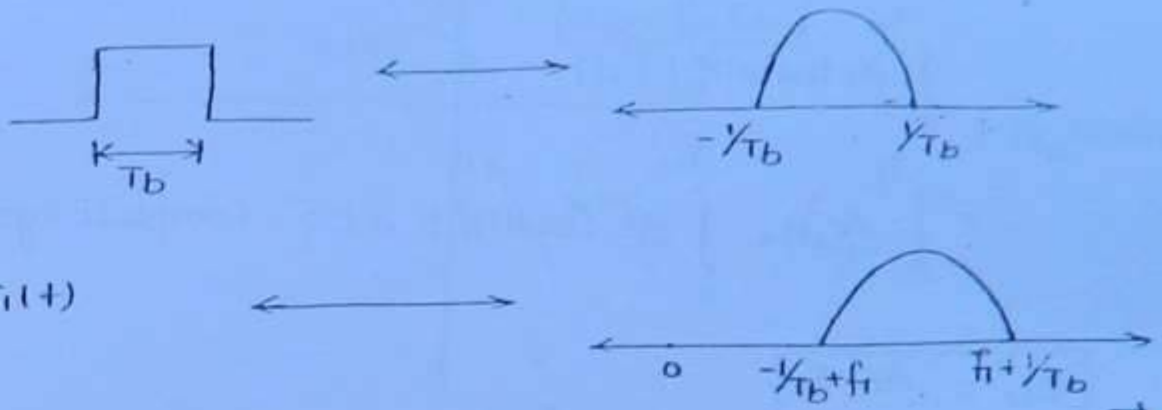
let $f_1 = 10M$.
$f_2 = 2M$.

Note

× The demodulation of FSK is affected by QNI

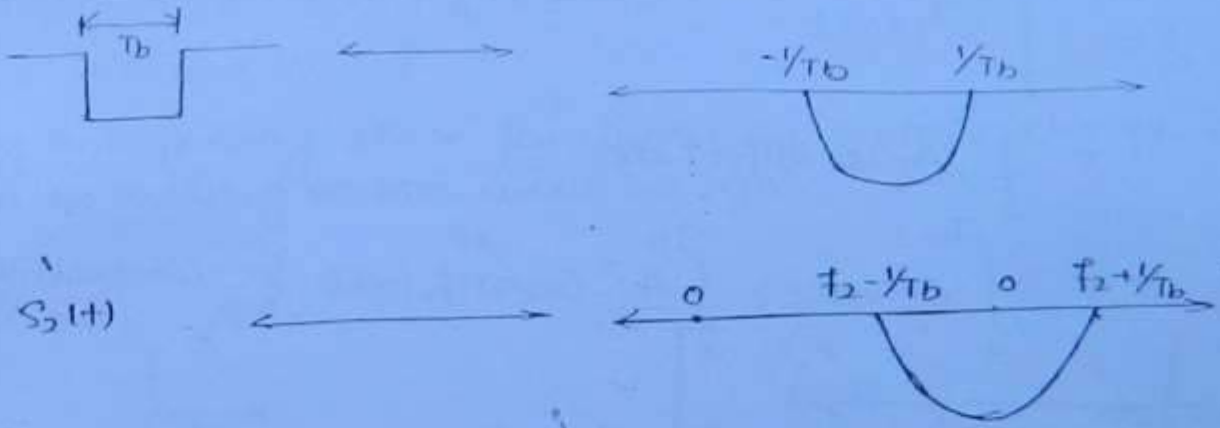
× Transmission B.W:

1. Tx of 1:



$\Rightarrow B.W = 2/T_b$

2. Tx of 0:



$\Rightarrow B.W = 2/T_b$

So,

the FSK $B.W = (f_1 + 1/T_b) - (f_2 - 1/T_b)$ {highest +ve freq^n - lowest +ve freq^n}.

$$\boxed{B.W\text{-}FSK = f_1 - f_2 + 2R_b}$$

Note:

× FSK needs high Transmission B.W compared to ASK and PSK. (drawback of FSK).

* Energy per bit:

* Tx of 1:

$$E_b = \int_0^{T_b} S_1^2(t)\,dt$$

$$= \int_0^{T_b} (A_c \cos 2\pi f_1 t)^2\,dt$$

$$= \int_0^{T_b} \frac{A_c^2}{2} dt + \int_0^{T_b} \frac{A_c^2}{2} \cancel{\cos 4\pi f_1 t\, dt}^{\,0} \quad \{\because \text{Complete cycle} = \text{Area} =$$

$$\boxed{E_b = \frac{A_c^2}{2} T_b}$$

* Tx of 0:

$$E_b = \int_0^{T_b} S_2^2(t)\,dt$$

$$= \int_0^{T_b} (A_c \cos 2\pi f_2 t)^2\,dt$$

$$= \int_0^{T_b} \frac{A_c^2}{2} dt + \int_0^{T_b} \frac{A_c^2}{2} \cancel{\cos 4\pi f_2 t\, dt}^{\,0} \quad \{\because \text{complete cycle} = \text{Area} = 0\}$$

$$\boxed{E_b = \frac{A_c^2}{2} T_b}$$

Note:

* Transmitter Energy Requirements of PSK and FSK wi be the same.

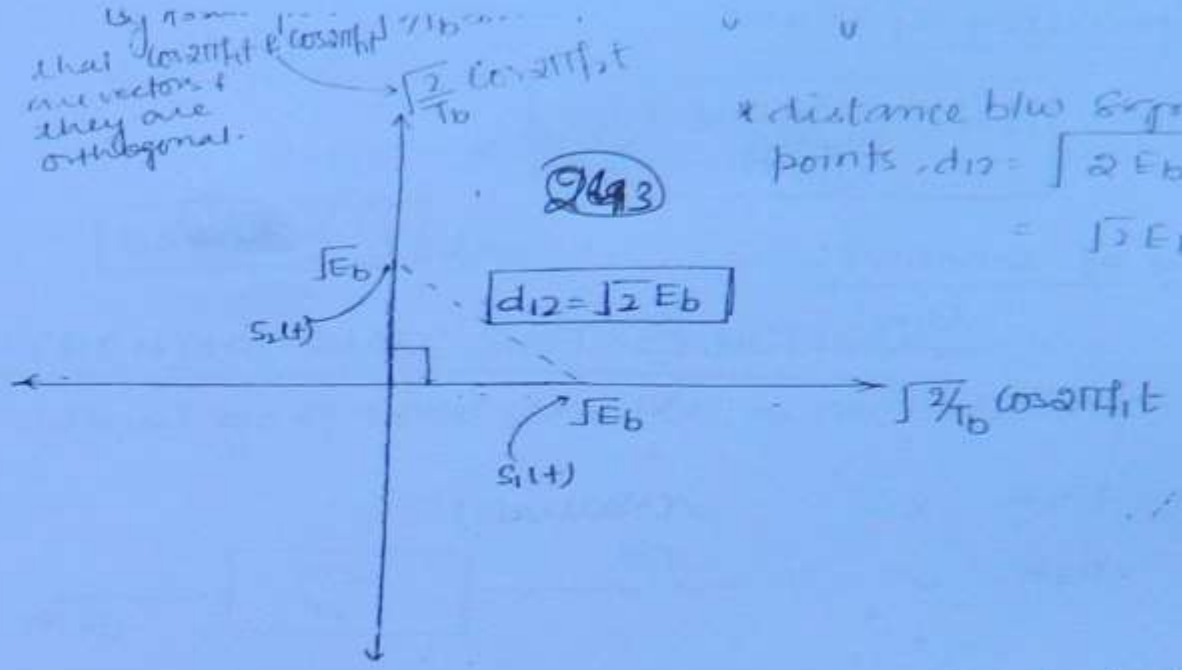
* CONSTELLATION DIAGRAM:

$$1 \rightarrow S_1(t) = A_c \cos 2\pi f_1 t = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t$$

$$0 \rightarrow S_2(t) = A_c \cos 2\pi f_2 t = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_2 t$$

So,

$$\left.\begin{aligned} S_1(t) &= \sqrt{E_b} \cdot \sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t \\ S_2(t) &= \sqrt{E_b} \cdot \sqrt{2/T_b} \cos 2\pi f_2 t \end{aligned}\right\} \text{In terms of Normalised functions.}$$



* distance b/w signalling points, $d_{12} = \sqrt{2E_b}$

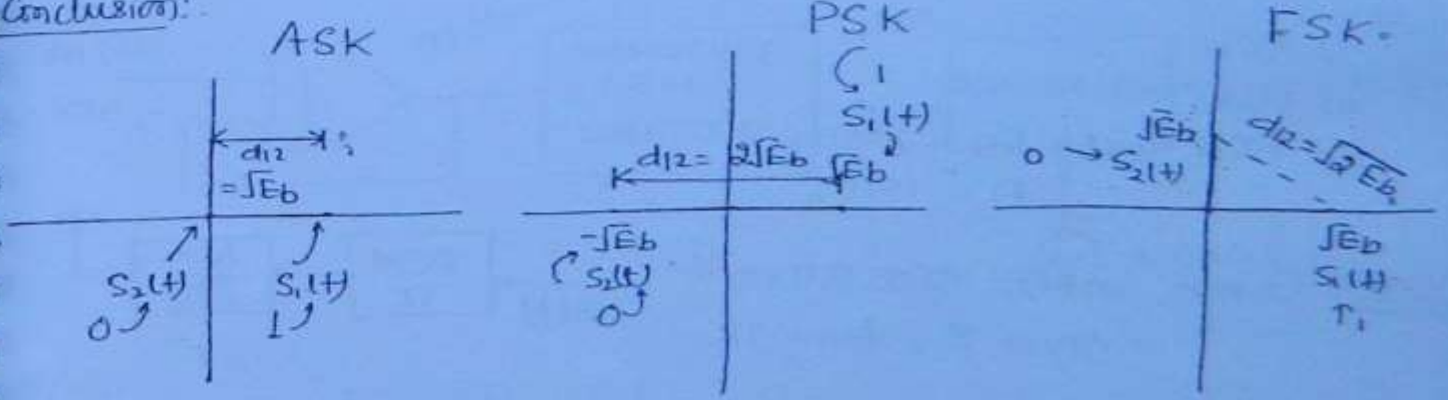
$= \sqrt{2}E_b$

we can't locate $\cos 2\pi f_2 t$ as all the axis corresponds to f_1 freqⁿ.

* $\sqrt{\frac{2}{T_b}}\cos 2\pi f_1 t$ and $\sqrt{\frac{2}{T_b}}\cos 2\pi f_2 t$ are othogonal functions in the interval $(0, T_b)$.

By interpreting these functions as vectors, the phase angle b/w Resulting vectors will be 90°.

Conclusion:



In a constellation diagram, if the dist. b/w signalling points is less; then Pe will be more, and vice versa.

Pe depends upon the dist b/w signalling pts.

so $\boxed{\text{Pe : ASK > FSK > PSK}}$

Comparison of B.W :

$$\boxed{B.W :- ASK \overset{}{<} FSK \quad \& \; PSK}$$

Usage of schemes:

	B.W	Pe
ASK	✓	X
FSK	X	✓(Moderate)
PSK	✓	✓

Note:
PSK is much preferred Signalling scheme compa to ASK and FSK

Q1 A msg signal of $8\cos 8\pi\times10^3 t$ is given to 10 bit PCM system. The Resulting PCM signal is transmitted through free space, by using Band Pass modulation scheme. Find the Tx signal B.W if modulation scheme is

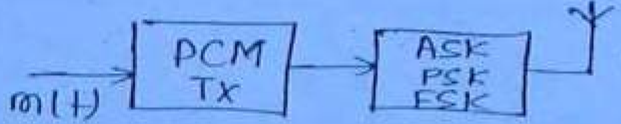
a) ASK

b) PSK

c) FSK with $F_H = 2MHz$.
$F_L = 1MHz$.

Soln: Given, $m(t) = 8\cos 8\pi\times10^3 t$

$A_m = 8$; $f_m = 4K$.



$n = 10$.

∴ Sampling Rate is not given

So, $f_s = NR = 2f_m = 8K$

So, $R_b = nf_s = 10\times8K$
$= 80Kbps$.

So, For ASK , $\boxed{B.W = 2R_b = 160K}$ Ans ; $\boxed{\text{For PSK } B.W = 160K}$ Ans

For FSK,

$B.W = f_H - f_L + 2R_b$

$B.W = (2-1)M + 160K$

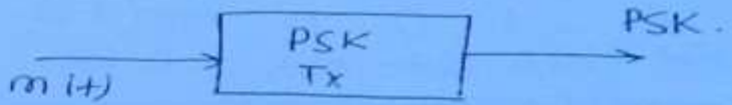
$\boxed{B.W = 1.16M}$ Ans

2/5

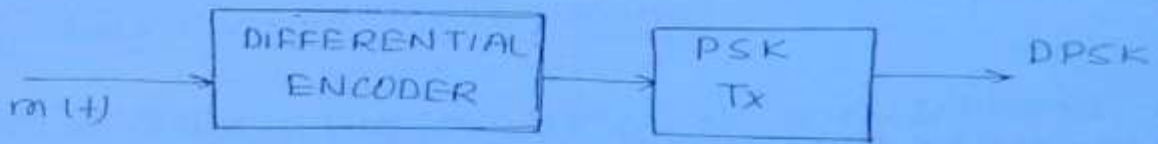
* DIFFERENTIAL PHASE SHIFT KEYING (DPSK):

* The advantage of DPSK over PSK is no QNE.

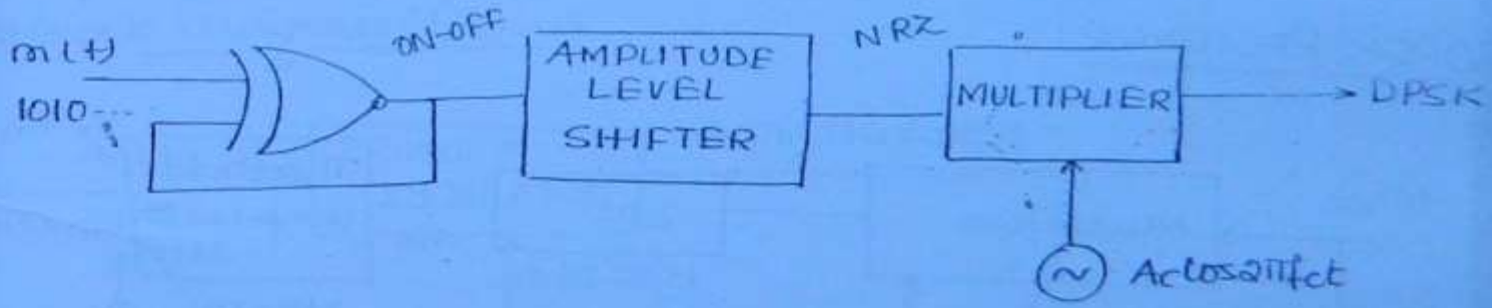
PSK:

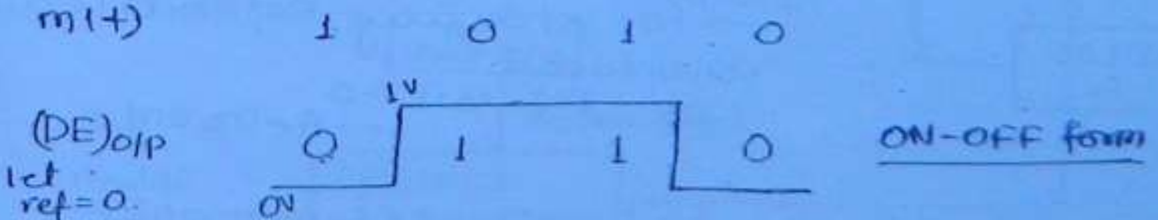


DPSK:

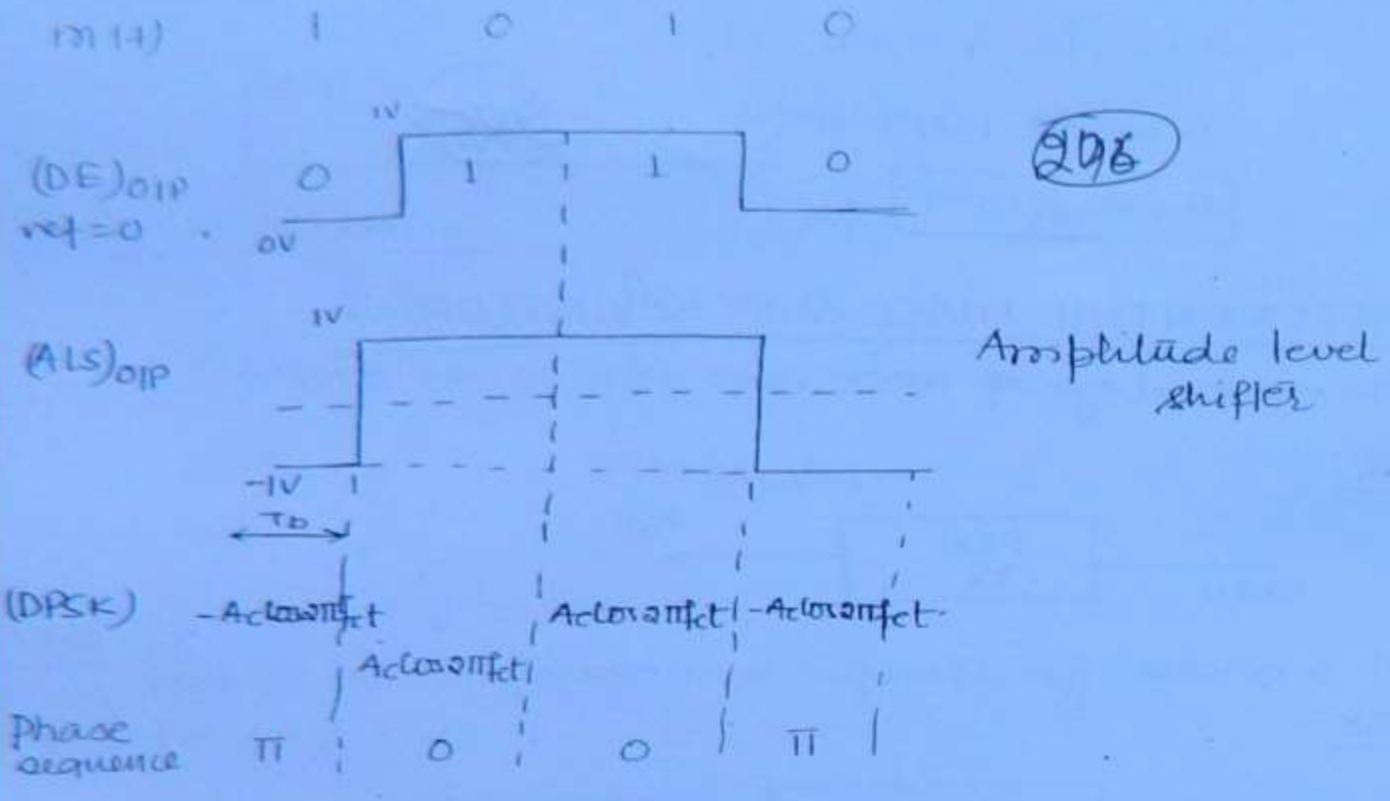


* Internal cktary:





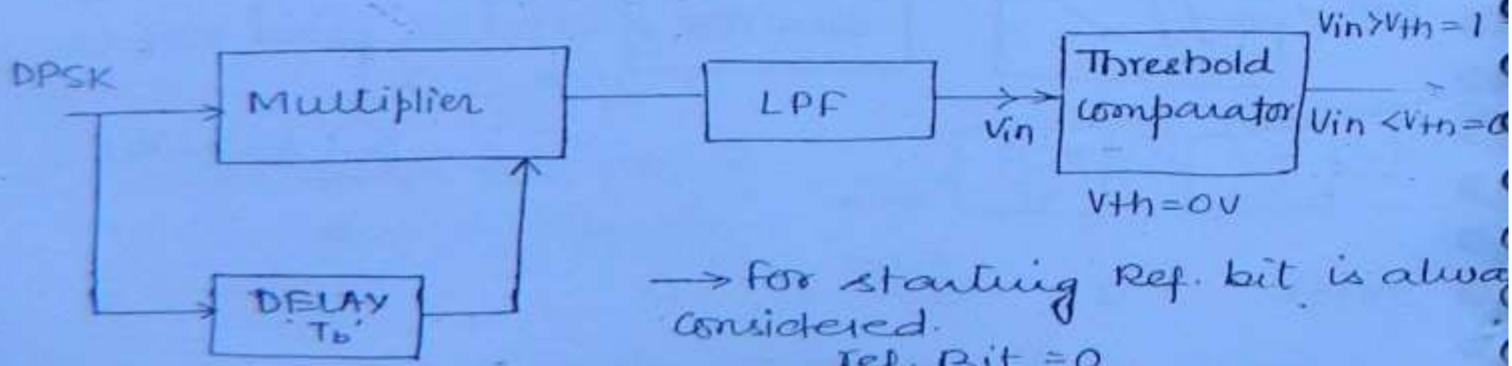
Note! The o/p of Differential encoder is in ON-OFF form & the Input of PSK Tx should be NRZ form. Hence it needs to be converted. It is done by the Amplitude level shifter.



Note:

m(t) will be given and the ref. bit will also be given and the phase sequence of Resulting DPSK will be asked.

DPSK RECIEVER:



→ for starting Ref. bit is alwa
considered.

Ref. Bit = 0
$= -A_c \cos 2\pi f_c t$

*Analysis:

DPSK →	$-A_c \cos 2\pi f_c t$	$A_c \cos 2\pi f_c t$	$A_c \cos 2\pi f_c t$	$-A_c \cos 2\pi f_c t$
(Mul)O/P →	$A_c^2 \cos^2 2\pi f_c t$	$-A_c^2 \cos^2 2\pi f_c t$	$A_c^2 \cos 2\pi f_c t$	$-A_c^2 \cos^2 2\pi f_c t$
(LPF)O/P →	$A_c^2/2$	$-A_c^2/2$	$A_c^2/2$	$-A_c^2/2$
Final O/P →	1	0	1	0

Q) A Binary signal of [illegible] by the DPSK transmitter, reference bit is 1 find phase sequence of the Resulting DPSK signal

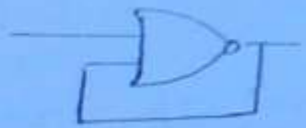
a) $0\pi00$

b) $\pi0\pi\pi$

c) $00\pi0$

d) $\pi\pi0\pi$

[illegible]



Solⁿ: $m(t) = 0 \quad 1 \quad 0 \quad 0$

(D·E) o/p $\quad 0 \quad 0 \quad 1 \quad 0$

ref = 1 $\quad -A_c\cos\ldots \quad -A_c\cos\ldots \quad A_c\cos\ldots \quad -A_c\cos\ldots$

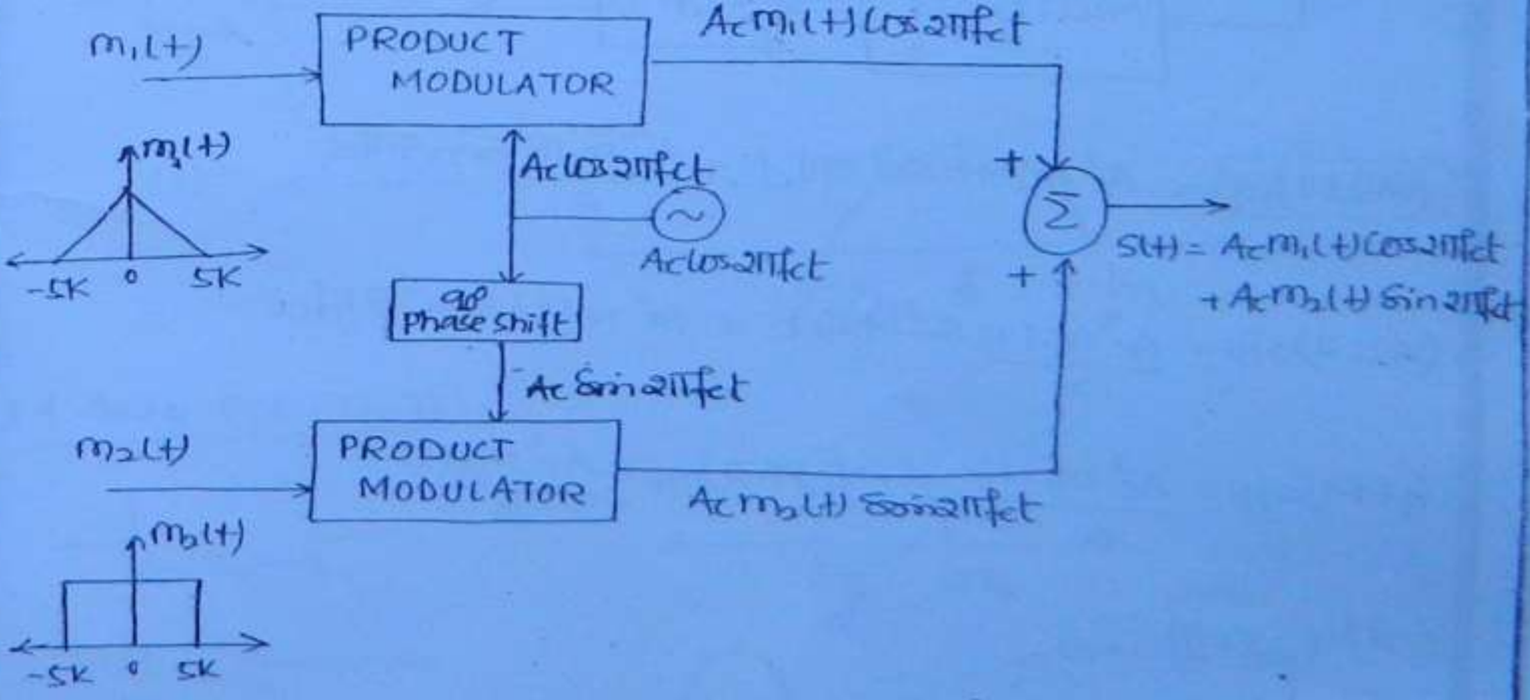
ϕ sequence	π	π^0	0	π

Ans

* Quadrature Carrier Multiplexing:

* By using this 2 signals will be multiplexed, where the corresponding carriers will have same freqⁿ, and having 90° phase shift b/w them.

* Block diagram: (Tx):



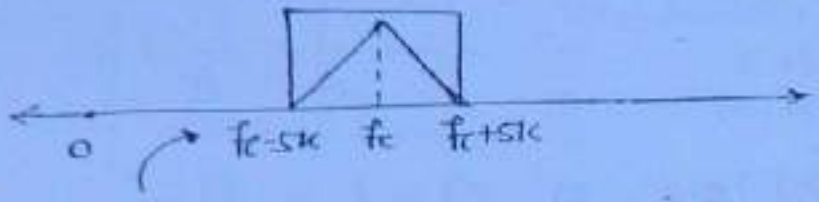
$M_1(t) = A_c m_1(t) \cos 2\pi f_c t \longleftrightarrow \frac{A_c}{2} \left\{ m_1(f - f_c) + m_1(f + f_c) \right\}$

$M_2(t) = A_c m_2(t) \sin 2\pi f_c t \longleftrightarrow \frac{A_c}{2j} \left\{ m_2(f - f_c) - m_2(f + f_c) \right\}$

~~284~~ 248

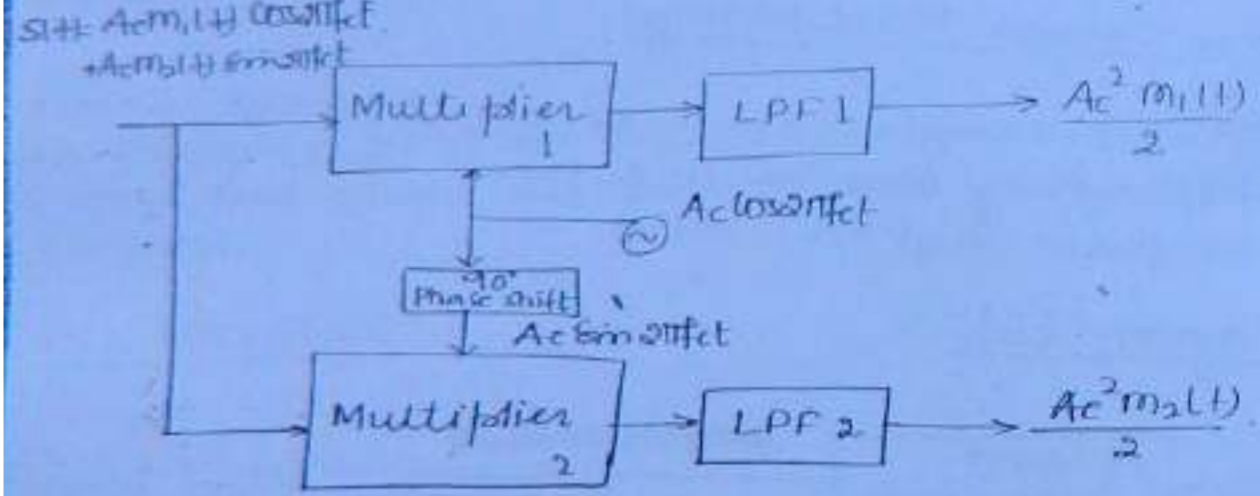
So,

$s(t) \longleftrightarrow$



No interference since carriers are quadrature to each other.

*Reciever Block diagram:



$(Mul\ 1)\ o/p = A_c^2 m_1(t) \cos^2 2\pi f_c t + \frac{A_c^2 m_2(t)}{2} \sin 4\pi f_c t$

$(Mul\ 2)\ o/p = \frac{A_c^2 m_1(t)}{2} \sin 4\pi f_c t + A_c^2 m_2(t) \sin^2 2\pi f_c t$

$(LPF1)\ o/p = \frac{A_c^2 m_1(t)}{2}$; $(LPF2)\ o/p = \frac{A_c^2 m_2(t)}{2}$

* M-ARRAY SIGNALLING :

* In ASK, PSK & FSK, one bit is transmitted at a time
ie N=1

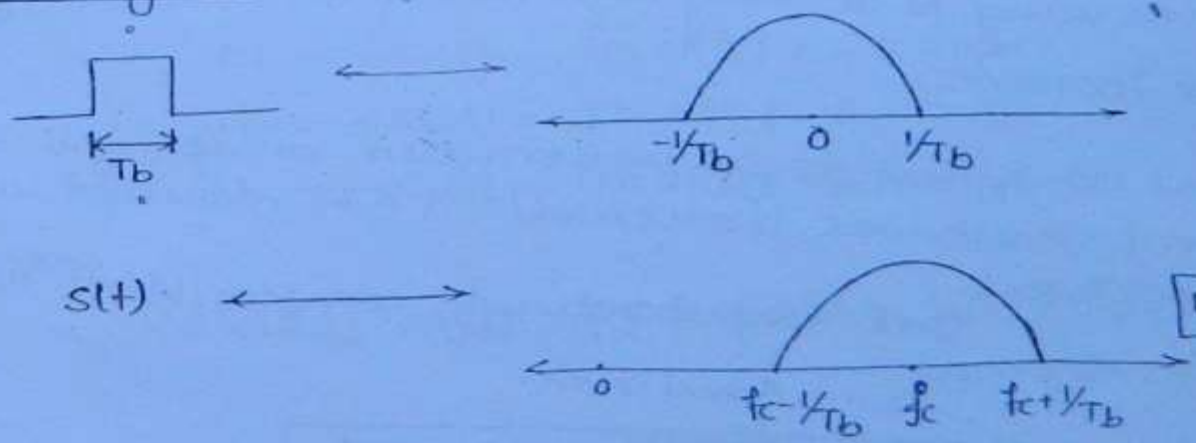
No.of symbols possible ⇒ M=2 ⇒ 0;1 229

* 2 No. of symbols are possible, hence ASK, PSK & FSK are called as Binary signalling schemes or 2 Array signalling schemes.

* ASK ⟶ BASK.
PSK ⟶ BPSK.
FSK ⟶ BFSK.

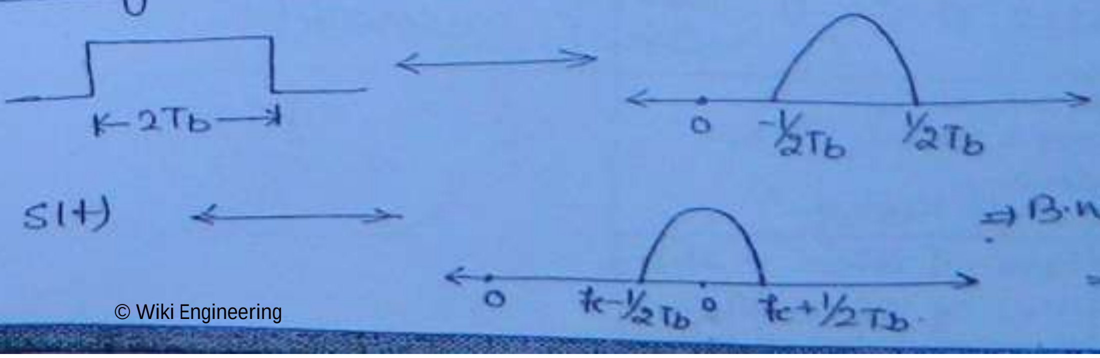
* For 4 Array PSK:
$M = 4.$
$N = 2. \{ 2^{\textcircled{2}N} = \textcircled{4}_{M} \}.$
Hence 2 bits are transmitted at a time $\boxed{(N=2)}$.
So, no. of symbols possible are $\boxed{M=4}$

*2- Array PSK(N=1):



$\boxed{B \cdot W = 2R_b}$

*4 Array PSK (N=2):-



$\Rightarrow B \cdot W = \left(f_c + \frac{1}{2T_b}\right) - \left(f_c - \frac{1}{2T_b}\right)$

$= 2 \times \frac{1}{2T_b} = \boxed{R_b = BW}$

* 8 Array PSK

$$B.W = \frac{2}{3} T_b$$

$$\boxed{B.W = 0.6 T_b}$$

* Conclusion:

* By increasing the No. of bits to be transmitted in specific time instant, then the B.W decreases

$$\boxed{N \uparrow \longrightarrow B.W \downarrow}$$

* If increasing N to very high value the complexity of Tx & Rx increases.

* As the no. of bits to be transmitted in specific time instant increases, transmission B.W required will be decreases. But correspondingly complexity of the system increases.

w.r.t Actual carrier

* $\boxed{\text{Phase shift in M-Array PSK} = \Phi = 2\pi/M}$

a) for $M=2 \Rightarrow \Phi = \pi$ $\Rightarrow 1; S_1(t) = A_c \cos 2\pi f_c t$ ⟩ π
$0; S_2(t) = -A_c \cos 2\pi f_c t$

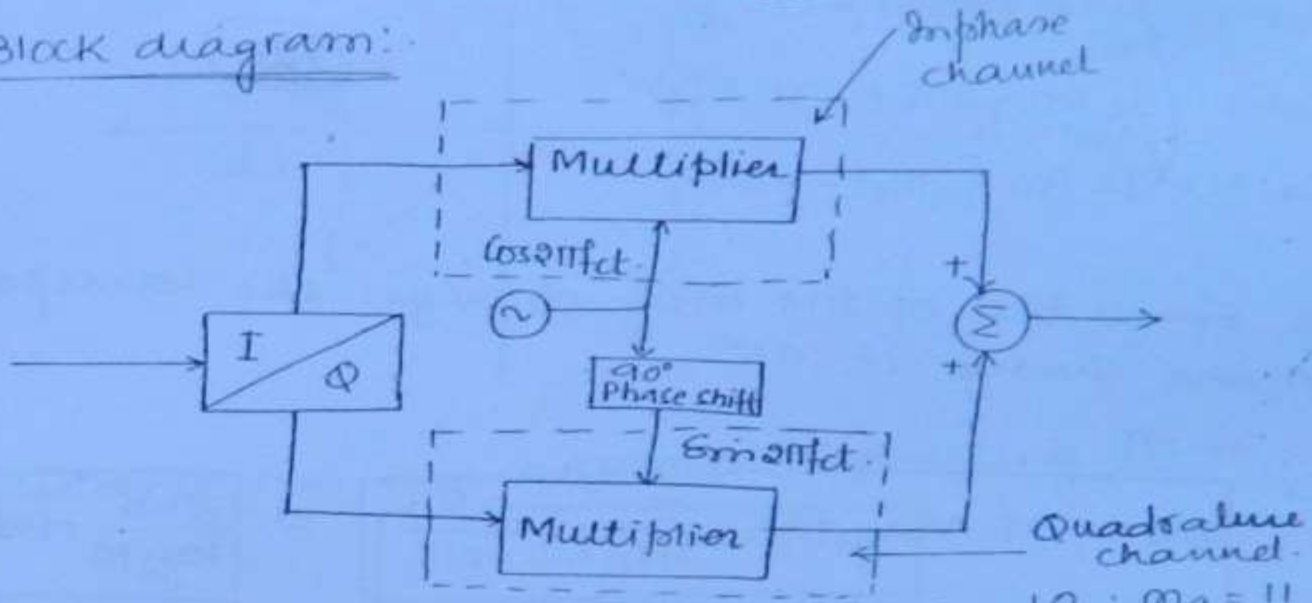
b) for $M=4 \Rightarrow \Phi = \pi/2$
4 Array PSK = QPSK. ⇒
Quadrature phase shift Keying

$\rightarrow$ $\boxed{\begin{array}{l} B.W \text{ of BPSK} = 2R_b \\ B.W \text{ of QPSK} = R_b. \end{array}}$

*QPSK Transmitter:

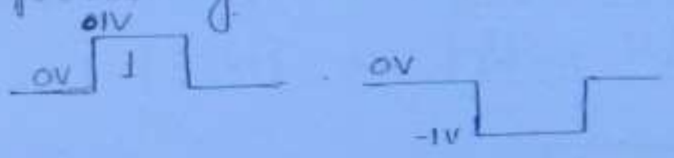
QPSK is 4 Array PSK

*Block diagram:



$m_1 = 00$; $m_2 = 01$; $m_3 = 10$; $m_4 = 11$

*NRZ signalling mechanism is used.



1 → +ve
0 → -ve

$00 \longrightarrow s_1(t) = -\cos 2\pi f_c t - \sin 2\pi f_c t$

$01 \longrightarrow s_2(t) = -\cos 2\pi f_c t + \sin 2\pi f_c t$

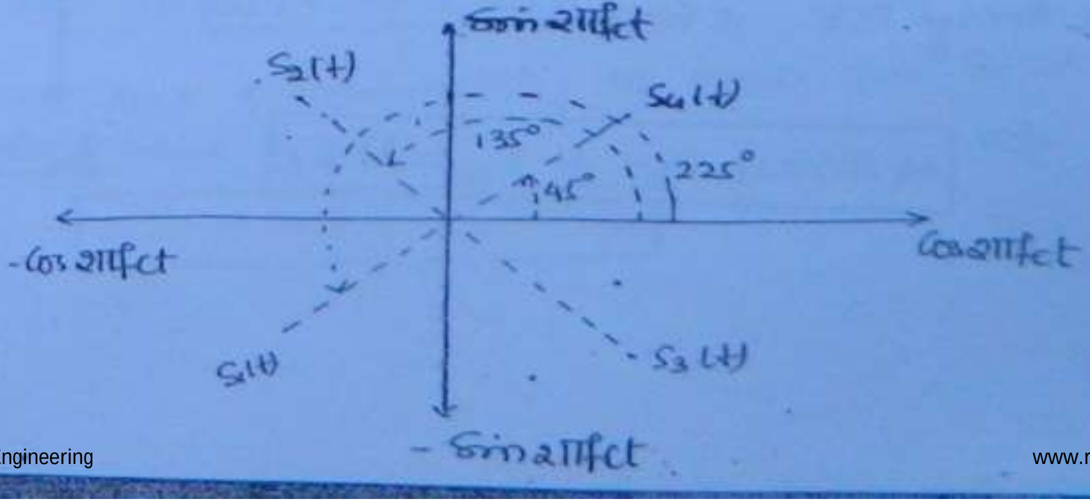
$10 \longrightarrow s_3(t) = \cos 2\pi f_c t - \sin 2\pi f_c t$

$11 \longrightarrow s_4(t) = \cos 2\pi f_c t + \sin 2\pi f_c t$

Now, as

$$A\cos 2\pi f_0 t + B\sin 2\pi f_0 t = \sqrt{A^2+B^2}\cos\{2\pi f_0 t + \Phi\}$$

$$\Phi = -\tan^{-1}(B/A).$$



So,

00 $S_1(t) = \sqrt{2}\cos\{2\pi f_c t + 2[illegible]^\circ\}$

01 $S_2(t) = \sqrt{2}\cos\{2\pi f_c t + 135^\circ\}$

10 $S_3(t) = \sqrt{9}\cos\{2\pi f_c t - 315^\circ\}$

11 $S_4(t) = \sqrt{2}\cos\{2\pi f_c t - 45^\circ\}$.

90° 90° 90° 90°

* As single bit of the msg changes the correspondi phase change is 90°.

Note:

$$\boxed{\text{B.W of M-Array PSK} = \frac{2}{NT_b} = \frac{2R_b}{N}} \Rightarrow \boxed{\frac{2R_b}{\log_2 M} = BW}$$

1. For 2 Array PSK, B.W = $2R_b$. $\{N=1\}$
2. For 4 Array PSK, B.W = R_b $\{N=2\}$

As. $\boxed{M = 2^N}$

only Es } No Calc No PSD

*CONSTELLATION DIAGRAM:

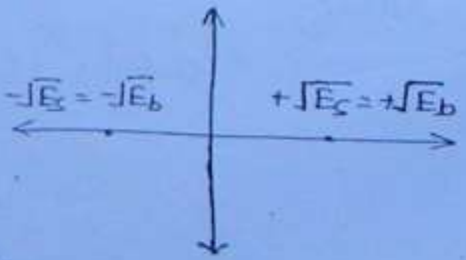
*For M Array PSK; distance of Each of the Signalling pt from the origin = $\sqrt{E_s}$. ← Symbol energy

* ~~for 2 Array PSK ⇒ $\sqrt{E_s} = \sqrt{E_b}$.~~

*For 2 Array PSK ⇒ $E_s = E_b$

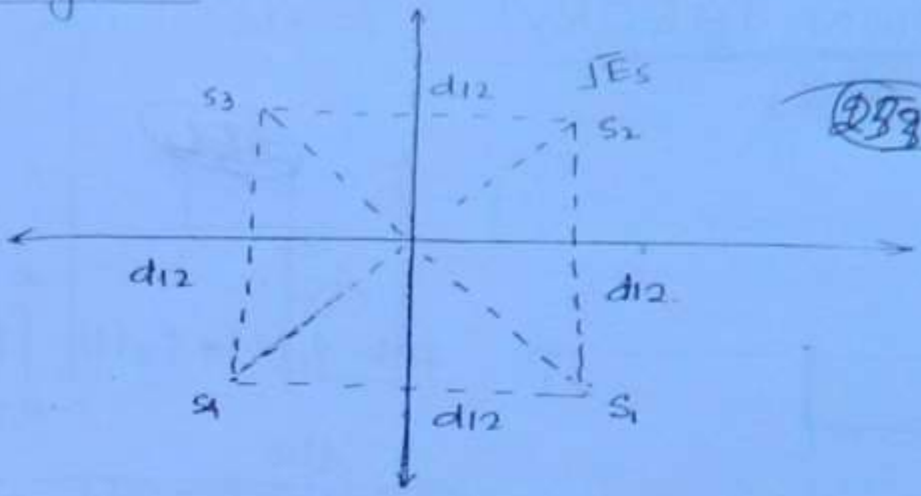
*For 4 Array PSK ⇒ $E_s = 2E_b$.

* So, for $\boxed{\text{N Array PSK} \Rightarrow E_s = NE_b}$



"2-Array PSK"

4 Array PSK:



* distance b/w two adjacent signalling pts is

$$\boxed{d_{12} = 2\sqrt{E_s}\,\sin \pi/M}$$

* For 2 Array PSK ; $d_{12} = 2\sqrt{E_b}\cdot \sin \pi/2$

$$\boxed{d_{12} = 2\sqrt{E_b}}$$

* For 4 Array PSK ; $d_{12} = 2\sqrt{E_s}\,\sin \pi/M$

$$= 2\sqrt{2E_b}\cdot \sin \pi/4$$

$$= 2\sqrt{2E_b}\cdot 1/\sqrt{2}$$

$$\boxed{d_{12} = 2\sqrt{E_b}}$$

Conclusion:

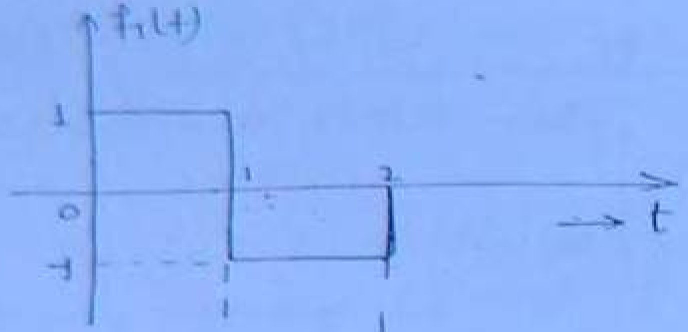
* Since, the distance of adjacent signalling pts is the same. ie $\boxed{d_{12} = 2\sqrt{E_b}}$

Hence, the probability of error for BPSK and QPSK are same

$$\boxed{P_e(BPSK) = P_e(QPSK)}$$

* INFORMATION THEORY:

* Analysis:



As,

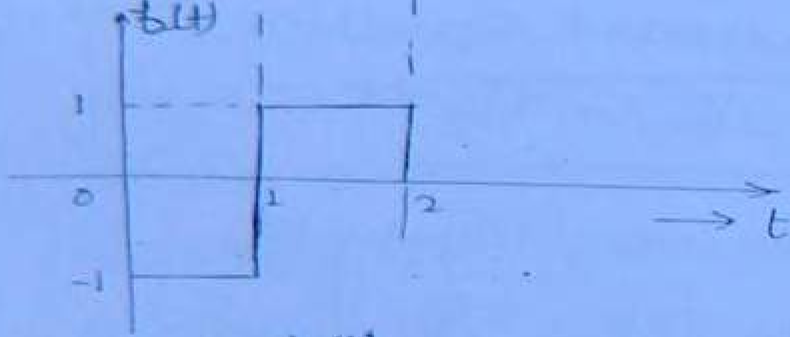
$$f(t) = f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) * f_2(t$$

*** Also,

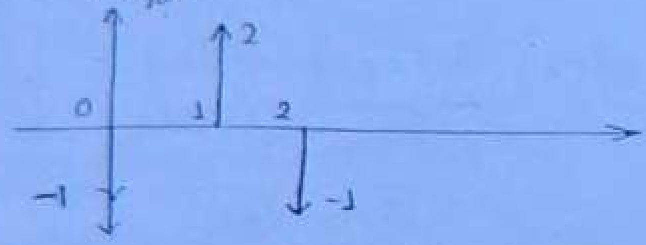
$$\boxed{\frac{d}{dt}\{f_1(t) * f_2(t)\} = f_1(t) * \left\{\frac{d}{dt} f_2(t)\right\}}$$

So,

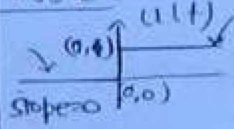
$$f_1(t) * f_2(t) = \int_{-\infty}^{t} f_1(t) * \frac{d}{dt} f_2$$



Now, $d/dt\, f_2(t)$



Note:



$$\frac{d}{dt} u(t) = \frac{\text{change y}}{\text{change x}}$$

$= 1/0 = \infty$

$d/dt\, u(t) = \infty,\ t = 0$

$= 0\ ;\ t \neq 0$

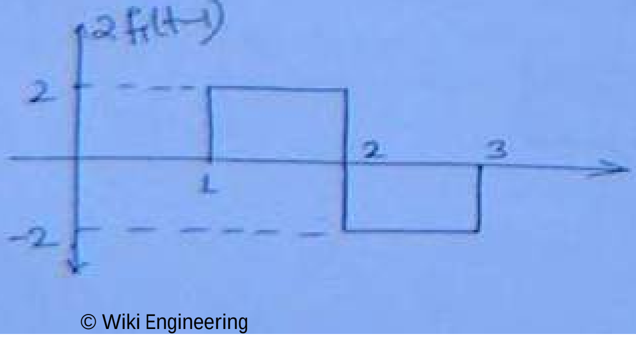
So,

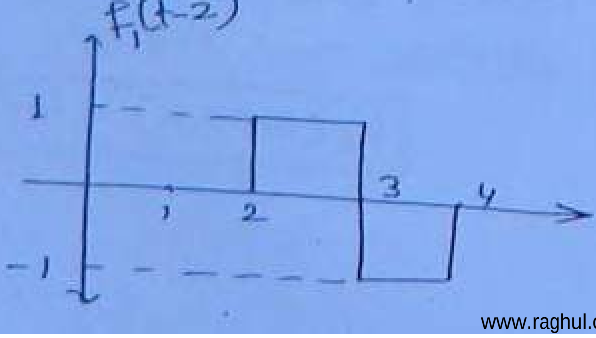
$$\boxed{d/dt\, u(t) = -\delta(t) + 2\delta(t-1) - \delta(t-2)}$$

So, $f_1(t) * d/dt\{f_2(t)\} = f_1(t) * \{-\delta(t) + 2\delta(t-1) - \delta(t-2)\}$.

$$\boxed{f_1(t) * \frac{d f_2(t)}{dt} = -f_1(t) + 2f_1(t-1) - f_1(t-2)}$$

Now,





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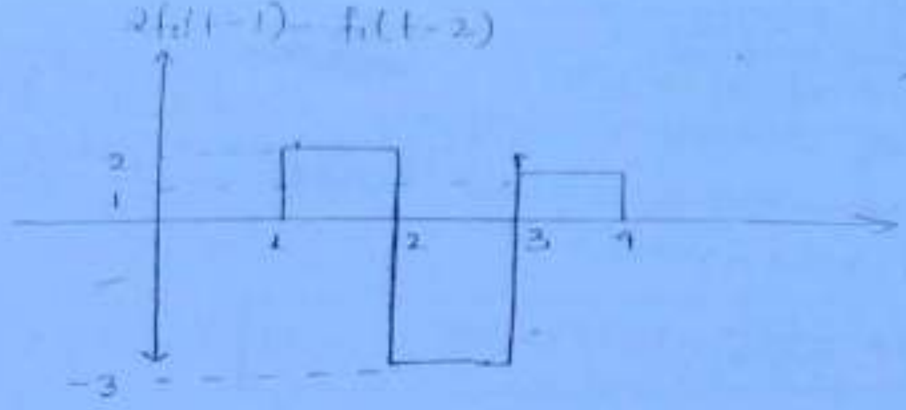
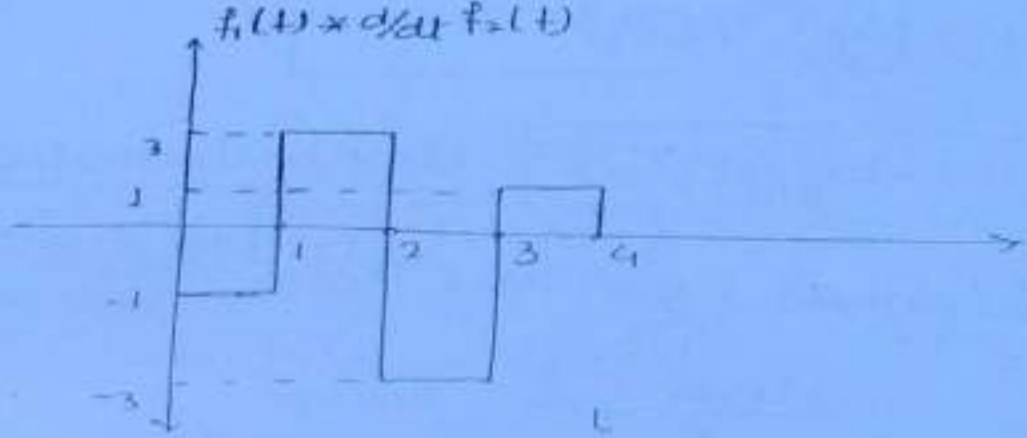


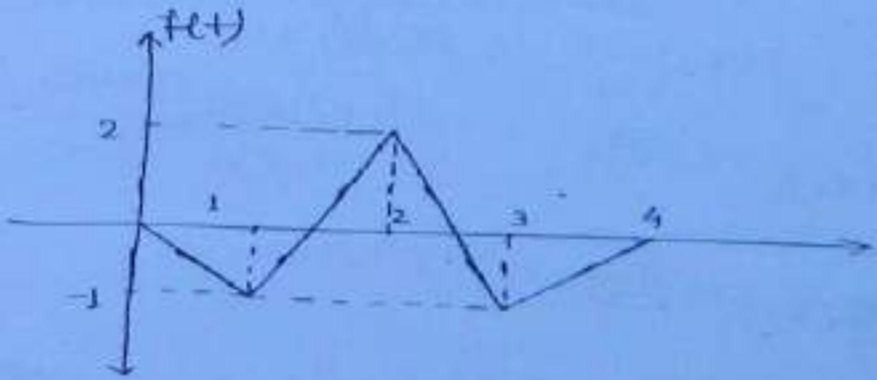


Now, $f(t) = \int_{-\infty}^{t} f_1(t) * d/dt\, f_2(t)$ = Area of the fun

i) At $t = 0.1 \Rightarrow$ Area $= -0.1$

At $t = 0.2 \rightarrow$ Area $= -0.2$

$t = 1 \Rightarrow$ Area $= -1$



Note:

$$\boxed{\begin{array}{l} f(t) = f_1 * f_2 \\ A_1 \cdot A_2 = A_1 * A_2 \end{array}}$$ ← Areas.

* INFORMATION THEORY:

* Information means importance
* If the probability of occurence of an event is less then the information associated with that event will be more and vice-versa.

$$I\{x_i\} \propto \frac{1}{P\{x_i\}}.$$

$$I\{x_i\} = \log_b(1/P\{x_i\})$$

or $$\boxed{I(x_i) = -\log_b\{P(x_i)\}}$$

* Units of $I\{x_i\}$ depends upon the base chosen
ie

b	Units
2	bits
e	nat
10	decit

Q1. A source is generating 3 possible symbols with probabilities of 1/4, 1/2, 1/4. find the information associated with each of the symbol.

Soln:- Given, $P(x_1) = 1/4$; $P(x_2) = 1/2$; $P(x_3) = 1/4$

$$I(x_1) = \frac{1}{\log_2\{P(x_1)\}} = +\log_2 4 = +2 \text{ bits}$$

$$I(x_2) = \frac{1}{\log_2\{P(x_2)\}} = \log_2 2 = 1 \text{ bit}$$

$$I(x_3) = \frac{1}{\log_2\{P(x_3)\}} = \log_2 4 = 2 \text{ bits.}$$

Note:-
The prob. of occurrance of x_2 is high so the information associated with x_2 will be less.

*Average Information or Entropy (H)

* Units of $\boxed{H \text{ is bits/symbol}}$

* Mathematically it is given as:- (287)

$$H = \sum_i I\{x_i\} \cdot P(x_i)$$

$$\boxed{H = \sum_i p(x_i) \cdot \log_2 1/P(x_i)}$$

$$\boxed{H = -\sum_i p(x_i) \log_2 P(x_i)}$$

*Information Rate (R) :-

* Units of $\boxed{R \text{ is bits/sec}}$

- Now,

$$R = \frac{\text{bits}}{\text{symbol}} \times \frac{\text{Symbol}}{\text{sec}}$$

Symbol (r) Rate

So, $\boxed{R = H \times r}$ $\Rightarrow$ $\boxed{\text{Information Rate} = \text{Symbol Rate} \times \text{Entropy}}$

Q. A source is generating 4 possible symbols with the probabilities of 1/8, 1/8, 1/4, 1/2.
Find Entropy and Information Rate if the source is generating 1symbol/msec.

Soln: Given,

$P(x_1) = 1/8$; $P(x_3) = 1/4$

$P(x_2) = 1/8$; $P(x_4) = 1/2$.

Symbol Rate = 1symb/msec

r = 1000 Symbol/sec

Now,

$$H = \sum_{i=1}^{4} P(x_i) \log_2 1/P(x_i) = \frac{1}{8}\log_2 8 + \frac{1}{8}\log_2 8 + \frac{1}{4}\log_2 4 + \frac{1}{2}\log_2 2$$

$$\boxed{H = 1.75 \text{ bits/symbol}}$$

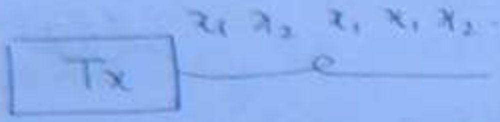
So, $\boxed{R = 1.75 \times 1000 = 1.75 \text{ Kbps}}$ Ans.

note

* Entropy is measure of uncertainty

* Analysis:

1.



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Case 1:

$P(x_1) = P(x_2) = 1/2$

$H = \sum_{i=1}^{2} P(x_i) \log_2 1/P(x_i)$

$= \frac{1}{2}\log_2 2 + 1/2 \log_2 2$

$\boxed{H = 1 \text{ bits/symbol} = H_{max}}$

when the prob are equal.

Case 2:

$P(x_1) = 1 ; P(x_2) = 0$

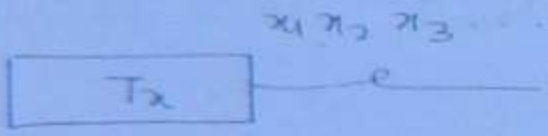
$H = -\sum_{i=1}^{2} P(x_i) \log_2 P(x_i)$

$= 0 + 0$

$\boxed{H = 0 \text{ bits/symbol} = H_{min}}$

when the prob. of one is 1 & other 0.

2.



Case 1:

$P(x_1) = P(x_2) = P(x_3) = 1/3$

$H = \sum_{i=1}^{3} P(x_i) \log_2 \frac{1}{P(x_i)}$

$H_{max} = \frac{1}{3}\log_2 3 + 1/3 \log_2 3 + \frac{1}{3}\log_2 3$

$\boxed{H_{max} = \log_2 3 \text{ bits/symbol}}$

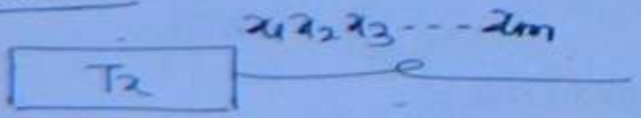
Case 2:

$P(x_1) = 1 ; P(x_2) = 0 = P(x_3)$

$H_{min} = \sum_{i=1}^{3} P(x_i) \log_2 \frac{1}{P(x_i)}$

$\boxed{H_{min} = 0 \text{ bits/symbol}}$

Conclusion:-



$P(x_1) = P(x_2) = P(x_3) = \ldots P(x_m) = 1/M$

$\boxed{H_{max} = \log_2 M \text{ bits/symbol}}$; $H_{min} = 0$ bits/symbol.

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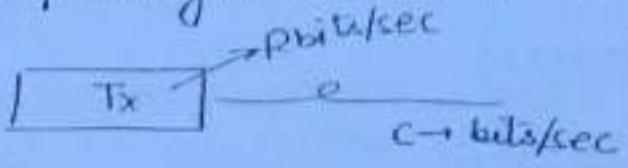
*I[illegible] ∴ If all the symbols are having equal prob of [illegible] then Entropy will be maxm.

*CHANNEL CAPACITY:

~~185~~ 257

*It specifies the no of bits allowed by the channel in 1 sec

*Channel capacity; C = bits/sec



Hence,

$$\boxed{C \geq R \leftarrow \text{No Information loss}}$$

*SHANON - HARTLEY LAW:

It gives the Relation b/w channel capacity (C) and its Bandwidth (B.W)

Mathematically,

$$\boxed{C = B\log_2(1+S/N)} \qquad (S/N)_{dB} = 10\log_{10}(S/N)$$

Normal S/N (not in dB)

where,

C = Channel capacity (bits/sec)

B = channel B.W (Hz).

S = Signal power expected at channel o/p.

N = Noise power.

$(S/N)_{dB}$	(S/N)
1) 10 dB	10
2) 20 dB	100
3) 15 dB	$10^{1.5}$

Q. For a channel of B.W = 4KHz
(S/N) = 15 dB
Find the channel capacity

Soln: As (S/N) = 15dB

So, $(S/N) = 10^{1.5} = 31.6$

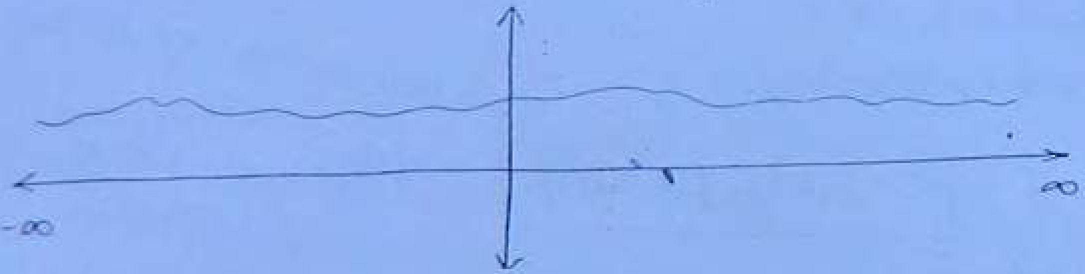
So, $C = B\log_2\{1+S/N\}$.

$= 4\log_2\{1+31.6\}$

$\boxed{C = 20.1\ \text{Kbps}}$ Ans

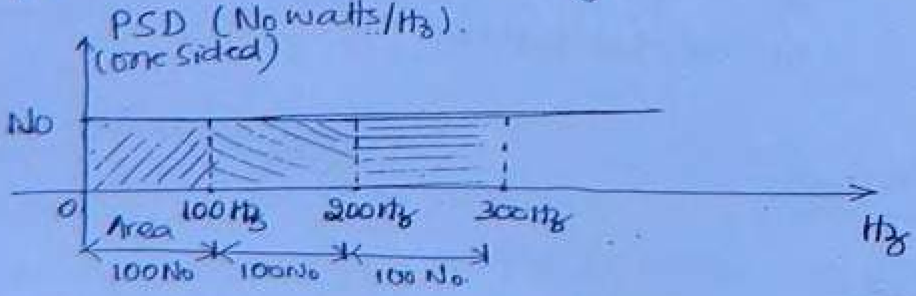
*Capacity of AWGN (additive white Gaussion Noise) channel :-

* white Noise has the frequency spectrum as following-



It covers the all frequency component

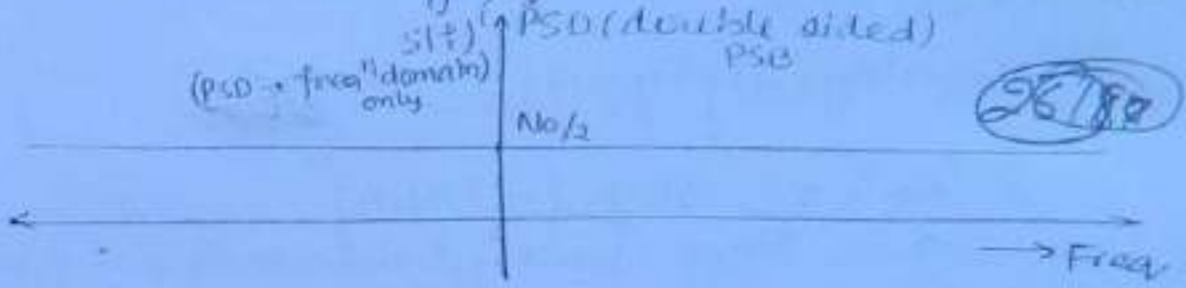
* The PSD of the white Noise is given as:-



* Freq^n b/w 0 to 100Hz will be affected by 100No watt of powe

* Freq^n b/w 100 to 200Hz will be affected by 100No watt of powe

* If the -ve frequency is also considered then;



* Regarding white Noise, its power is given as:

$$N(\text{watts}) = \frac{\text{watts}}{Hz} \times Hz$$

$$\boxed{N = N_o \times B} \text{ watts}$$

* Default power spectral density is one sided PSD.

Note:-

* Each of the frequency component transmitted through a channel is affected by same amount of white Noise power.

* The channel BW is given as:

$$\uparrow C = B \uparrow \log_2 \{1 + S/N\}.$$

(linear)

So, for a AWGN channel.

$$\uparrow C = B \uparrow \log \log_2 \{1 + S/N_oB\}.$$

(non linear)

Conclusion:

* For AWGN channel as B → ∞ channel capacity becomes.

$$\boxed{C_\infty = 1.44\, S/N_o}$$

Proof: As we know that

$$C = B\log_2\{1 + S/N_0B\}$$

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$$C = \frac{N_0}{S} \times \frac{S}{N_0} B\log_2\{1 + S/N_0B\}$$

let $\frac{N_0B}{S} = x$

as $B \to \infty \Rightarrow x \to \infty$

So,

$$C_\infty = \frac{S}{N_0} \lim_{x\to\infty} x \log_2(1 + 1/x)$$

$$C_\infty = \frac{S}{N_0} \log_e 2$$

v imp
***

$$\boxed{C_\infty = 1.44\, S/N_0}$$

Q. For AWGN of having BW 4KHz, two sided Noise PSD i given by:- 10^{-12} watts/Hz. Find the channel capacity Required to get signal power of 0.1 mW at the O/P o the channel.

Soln: Given, BW = 4KHz

$$\frac{N_0}{2} = 10^{-12} \Rightarrow N_0 = 2\times10^{-12}$$

So, $N_0B = 2\times10^{-12} \times 4\times10^{3}$

$= 8\times10^{-9}$ watts

NOW,

$$C = B\log_2\{1 + S/N_0B\}$$

$$= 4\times10^{3}\log_2\left\{1 + \frac{0.1\times10^{-3}}{8\times10^{-9}}\right\}$$

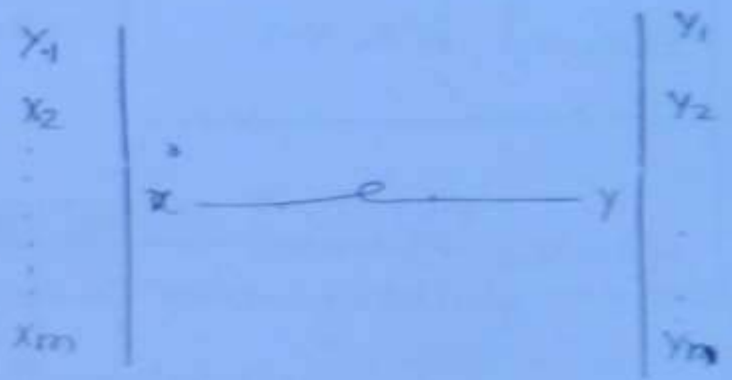
~~C = 54.44 Kbps~~ Ans

$\boxed{C = 54.44 \text{ Kbps}}$ Ans

* CHANNEL TRANSITION MATRIX OR CONDITIONAL PROB MATRIX

$P(0/1) \longrightarrow$ Probability that Txd 1 is Rxd as 0 [illegible]

$P(1/0) \longrightarrow$ Prob. that Txd 0 is Rxd as 1. 263



So,

$$[P(Y/X)] = \begin{bmatrix} P(y_1/x_1) & P(y_2/x_1) & \cdots & P(y_n/x_1) \\ P(y_1/x_2) & P(y_2/x_2) & \cdots & P(y_n/x_2) \\ \vdots & \vdots & & \vdots \\ P(y_1/x_m) & P(y_2/x_m) & & P(y_n/x_m) \end{bmatrix}_{m\times n}$$

So,

$$\boxed{\sum_{j=1}^{n} P(y_j/x_i) = 1 \quad ; \text{ for any value of } i}$$

$$P(y_1/x_1) + P(y_2/x_1) + \cdots P(y_n/x_1) = 1$$

* Sum of the elements in each Row of Channel x^tion matrix will be equal to 1.

* $P[X] = [P(x_1) \ P(x_2) \cdots\cdots P(x_m)]_{1\times m}$

Input matrix

$$\boxed{\underset{\text{output matrix}}{[P(y)]} = [P(y_1) \quad P(y_2) \qquad P(y_n)]_{1\times n}}$$

So, $$\boxed{[P(y)]_{1\times n} = [P(x)]_{1\times m} \, [P(y/x)]_{m\times n}}$$

* Channel Matrix $= [P(x,y)] = \begin{bmatrix} P(x_1,y_1) & P(x_1,y_2) & \cdots & P(x_1,y_n) \\ P(x_2,y_1) & P(x_2,y_2) & \cdots & P(x_2,y_n) \\ \vdots & & & \\ \vdots & & & \\ P(x_m,y_1) & P(x_m,y_2) & \cdots & P(x_m,y_n) \end{bmatrix}_{m}$

Note:

$P(x_1,y_1)$ = Probability that when x_1 is generated & Tx and to be Recieved as y_1.

Now $$\boxed{[P(x,y)]_{m\times n} = [P(x)]_{m\times m}\text{diagonal} \times [P(y/x)]_{m\times n}}$$

$\therefore$ $[P(x)]$ diagonal $= \begin{bmatrix} P(x_1) & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & P(x_2) & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & P(x_3) & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & & & & & \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & P(x_m) \end{bmatrix}_{m\times m}$

*Binary Symmetric Channel:

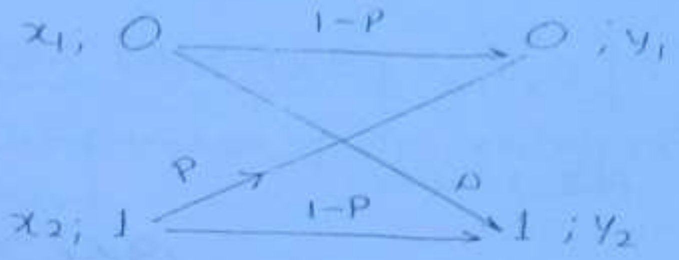
* Tx generating 0 & 1

* Rx recieving 0 & 1

* So, for Binary Symmetric channel.

$$\boxed{P(1/0) = P(0/1)}$$

$$\underline{P_{e0}} = \underline{P_{e1}}.$$



$$[P(Y/X)] = \begin{bmatrix} P(y_1/x_1) & P(y_2/x_1) \\ P(y_1/x_2) & P(y_2/x_2) \end{bmatrix}$$

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So
$$\boxed{[P(Y/X)] = \begin{Bmatrix} 1-P & P \\ P & 1-P \end{Bmatrix}}$$

*CONDITIONAL ENTROPY:

- * It specifies uncertainty about Reciever w.r.t Transmitter.
- * Mathematically,

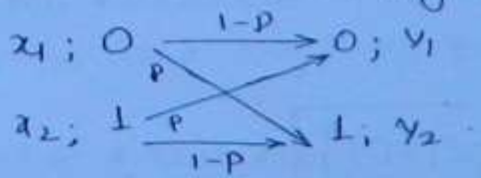
$$\boxed{H(Y/X) = -\sum_{i=1}^{m}\sum_{j=1}^{n} P(x_i, y_j)\log_2 P(y_j/x_i)}$$

let,

1) $P(y_1/x_1) = 0.9$; $P(y_2/x_1) = 0.1$; $P(y_1/x_2) = 1$; $P(y_2/x_2) = 0$.
   H(Y/X) less ← Since P(Y/X) is high.
2) $P(y_1/x_1) = 0.4$; $P(y_2/x_1) = 0.6$; $P(y_1/x_2) = 0.5$; $P(y_2/x_2) = 0.5$
   H(Y/X) high ← Since P(Y/X) is low.

* H(Y/X) = height of uncertainty for Rx w.r.t Tx.

Q. Find conditional entropy for Binary symmetric channel.



Soln:

So, $H(Y/X) = -\sum_{i=1}^{2}\sum_{j=1}^{2} P(x_i, y_j) \cdot \log_2 P(y_j/x_i)$

Now
$$P(Y/X) = \begin{bmatrix} P(y_1/x_1) & P(y_2/x_1 \\ P(y_2/x_2) & P(y_2/x_2) \end{bmatrix} = \begin{bmatrix} 1-P & P \\ P & 1-P \end{bmatrix}.$$

Now,

$$P(x,y) = [P(x)]_{diag} \cdot [P(y/x)]$$

let, $P(x_1) = \alpha$; $P(x_2) = 1-\alpha$

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So, $[P(x)] = [\alpha \quad 1-\alpha]$

$$[P(x)]_{diag} = \begin{bmatrix} \alpha & 0 \\ 0 & 1-\alpha \end{bmatrix}$$

So,

$$P(x,y) = \begin{bmatrix} \alpha & 0 \\ 0 & 1-\alpha \end{bmatrix} \begin{bmatrix} 1-P & P \\ P & 1-P \end{bmatrix}$$

$$P(x,y) = \begin{bmatrix} \alpha(1-P) & \alpha P \\ (1-\alpha)P & (1-\alpha)(1-P) \end{bmatrix} = \begin{bmatrix} P(x_1,y_1) & P(x_1,y_2) \\ P(x_2,y_1) & P(x_2,y_2) \end{bmatrix}$$

Now,

$$H(Y/X) = -\sum_{i=1}^{2} \sum_{j=1}^{2} P(x_i, y_i) \times P \log_2 P(y_j/x_i)$$

$$= -\left\{ P(x_1,y_1) \log_2 P(y_1/x_1) + P(x_2,y_2) \log_2 P(y_2/x_1) + P(x_2,y_1) \log_2 P(y_1/x_2) + P(x_2,y_2) \log_2 P(y_2/x_2) \right\}$$

So,

$$H(Y/X) = -\left\{ \alpha(1-P)\log_2(1-P) + \alpha P \log_2 P + (1-\alpha)P \log_2 P + (1-\alpha)(1-P)\log_2(1-P) \right\}.$$

$$\boxed{H(Y/X) = -\left\{ P\log_2 P + (1-P)\log_2(1-P) \right\}}$$

So, $$\boxed{H(Y/X) = P\log_2 1/P + (1-P)\log_2 1/(1-P)}$$

* RANDOM VARIABLES:

* It is the process of Assigning no to the outcome of an experiment.

* let, 2 coins are tosed, hence the outcomes are:

{HH, HT, TH, TT.} = [S]

All these outcomes are taken under the variable called as sample space variable.

* under some specific condition, the sample space variable is transformed into Random variable.

S	x (Random variable)
H H	2
H T	1
T H	1
T T	0

Correspond to no. of Heads

* when, the Random variable takes the discrete variable then it is called as (discrete Random variable).

* For a variable to be considered as Random variable the criteria is that the variable should be undeterministic in nature.

Note:

1. A Random variable 'x' is defined as it specifying no. of heads in the exp. of tossing a coin twice.

So,

Sample variable {S}	Random variable, x = {x}.
H H	2
H T	1
T H	1
T T	0

2. If Random variable takes discrete set of values then it is called as Discrete Random variable.

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3. The above is discrete Random variable.

4. If Random variable takes continuous set of values then it is called as continuous Random variable.

5. Random variable which is specifying temp. in a Room from 6 AM to 6 PM corresponds to continuous Random variable.

*PROBABILITY MASS FUNCTION:

It specifies probability of a Random variable taking each of its possible values.

Q. Plot Probability Mass Function for a Random variable which is specifying no. of heads in the expt of Tossing a coin twice

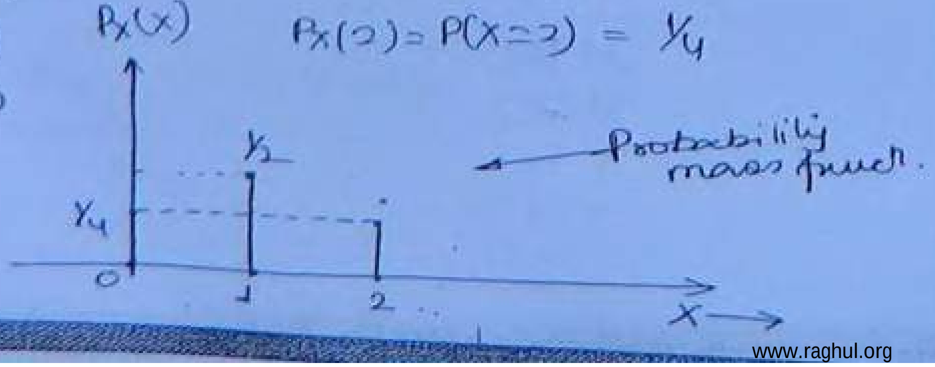
Soln:- $P_X(x_i) = P(X = x_i)$.

S	$X = \{x_i\}$
HH	2
HT	1
TH	1
TT	0

So, $P_X(0) = P(X=0) = 1/4$

$P_X(1) = P(X=1) = 1/2$

$P_X(2) = P(X=2) = 1/4$



Properties of [illegible]

1) $0 \leq P_x(x_i) < 1$

2) $\sum_i P_x(x_i) = 1$

Note:

Prob. Mass Func^n (PMF) is used to specify discrete Random variable.

*CUMULATIVE PROBABILITY DISTRIBUTION FUNCTION(-CDF):-

* Standard notation is given as $F_X(x) = P(X \leq x)$

* It specifies Probability of Random variable (X) taking the values upto 'x'.

Q Construct CDF for the above discrete Random variable.

Sol^n:

S	X
HH	2
HT	1
TH	1
TT	0

Now,

$X = \{x\} =$	0	1	2
$P_x(x) =$	1/4	1/2	1/4

Now, $F_X(-1) = P(X \leq -1) = 0$.

$F_X(0) = P(X \leq 0) = 1/4$.

$F_X(0.5) = P(X \leq 0.5) = 1/4$.

$F_X(1) = P(X \leq 1) = 1/4 + 1/2 = 3/4 . \{P_x(0) + P_x(1)\}$.

$F_X(1.9) = P(X \leq 1.9) = 3/4$.

$F_X(2) = P(X \leq 2) = 1$.

$F_X(10) = P(X \leq 10) = 1$.

∴ the plot is given as.

<u>Note:</u> CDF of discrete Random variable is always staircase funch.

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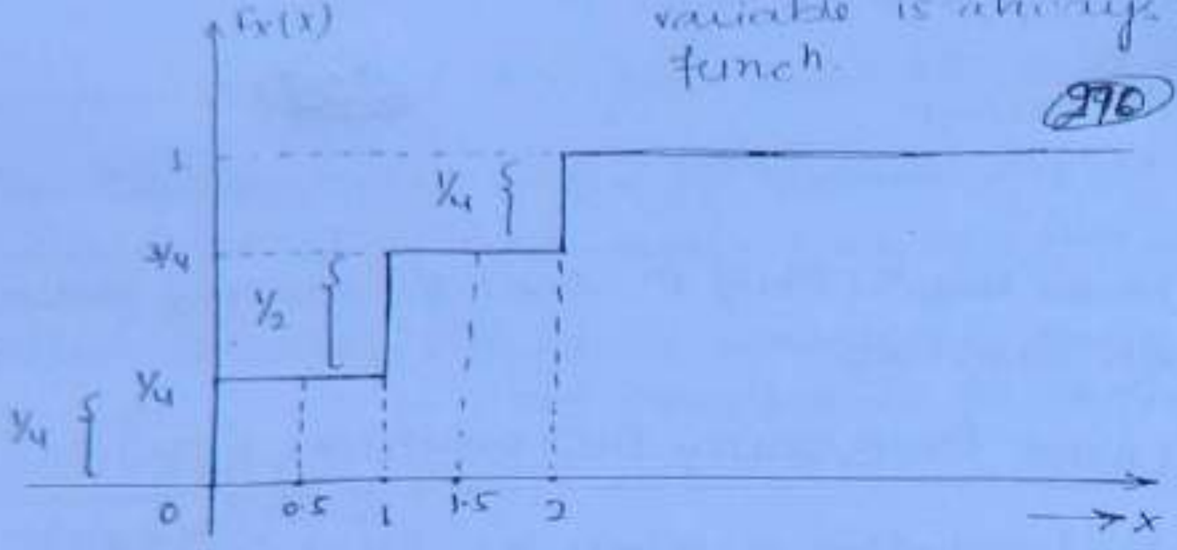


$P(x=1) = 1/2$ {Jump offered.}

$P(x=2) = 1/4$ {Jump offered.}

Also,

$$\boxed{F_x(x) = \frac{1}{4}u(x) + \frac{1}{2}u(x-1) + \frac{1}{4}u(x-2)}$$

<u>Note:</u>

Fx(x) of a discrete Random variable will be a staircase function

Q

$X=\{x_i\}$	-1	0	1	2
$P_X(x_i)$	0.3	$0.2K$	0.4	0.1

i) $K = ?$

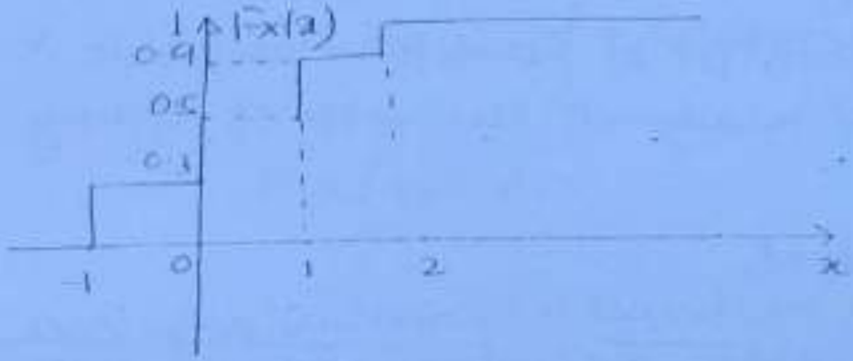
ii) Plot $F_X(x)$

Solⁿ: As $\sum_{i=} P_X(x_i) = 1$

$$0.3 + 0.2K + 0.4 + 0.1 = 1$$

$$\boxed{K = 1}$$ Ans

ii)



* Properties of $F_X(x)$:

i) $F_X(-\infty) = P(X \leq -\infty) = 0$.

ii) $F_X(\infty) = P(X \leq \infty) = 1$

iii) $P(X \leq x) = F_X(x)$.

iv) $P(x_1 \leq X \leq x_2) = F_X(x_2) - F_X(x_1)$
$P(X \leq x_2) - P(X \leq x_1)$.

v) $P(X > x) = 1 - F_X(x)$.

*** * PROBABILITY DENSITY FUNCTION (PDF):

* Denoted by $f_X(x)$.

* It is generally used to specify continuous Random variable.

* The Relation b/w PDF and distribution function is give

as:

$$f_x(x) = d/dx\, F_x(x)$$

$$\boxed{F_X(x) = \int_{-\infty}^{x} f_x(x)dx = P(X \le x)}$$

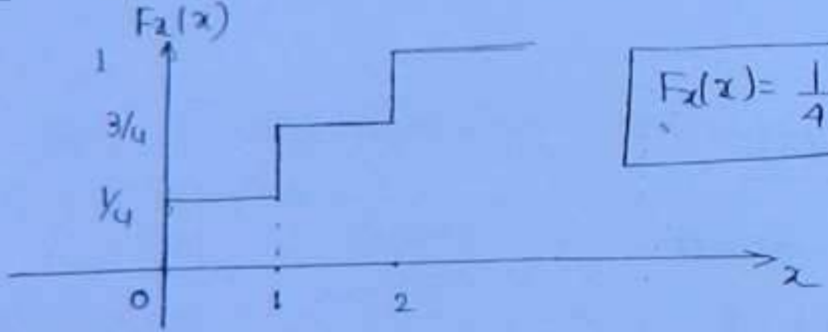
Now,

$$\boxed{P(x_1 \le x \le x_2) = \int_{x_1}^{x_2} f_x(x)dx}$$

$$\boxed{P(X \ge x) = \int_{x}^{\infty} f_x(x)dx.}$$

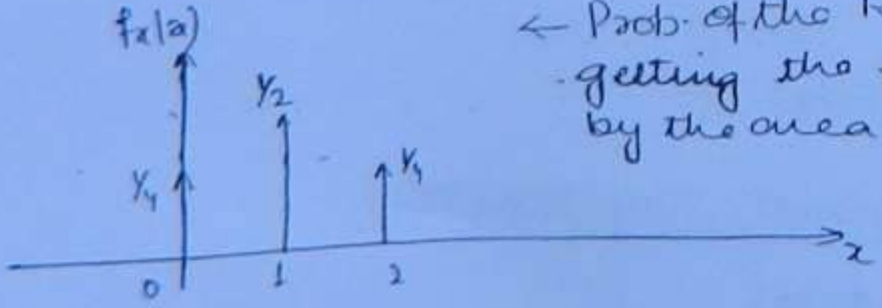
Q. Plot density function for a Random variable X which is specifying no: of heads in the expt of tossing a coin twice.

Soln: As we know that,



$$\boxed{F_x(x) = \frac{1}{4}u(x) + \frac{1}{2}u(x-1) + \frac{1}{4}u(x}$$

So, $f_x(x) = d/dx\, F_x(x)$



← Prob. of the Random variable getting the prob. can be given by the area of the PDF.

So, $$\boxed{f_x(x) = 1/4\,\delta(x) + 1/2\,\delta(x-1) + 1/4\,\delta(x-2)}$$

Conclusion: Density function of discrete Random variable will be in terms of impulse function.

The Prob Mass function is given as.

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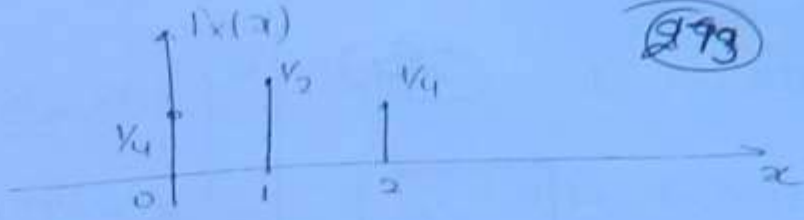
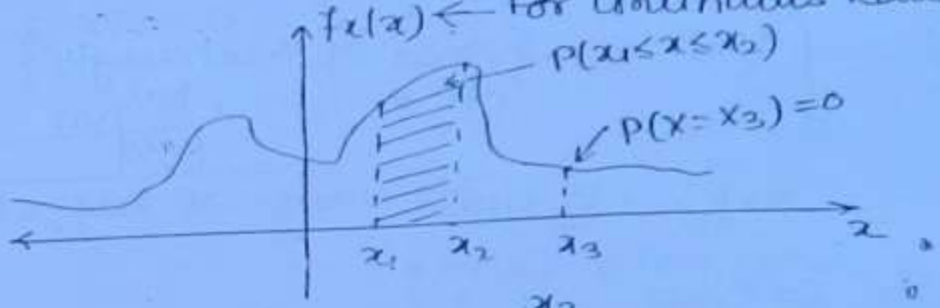


$P(X=0) = 1/4$

$P(X=1) = 1/2$

$P(X=2) = 1/4$.

Note:



← For Continuous Random variable.

$$P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} f_x(x)\,dx = P(x_1 < X \le x_2)$$

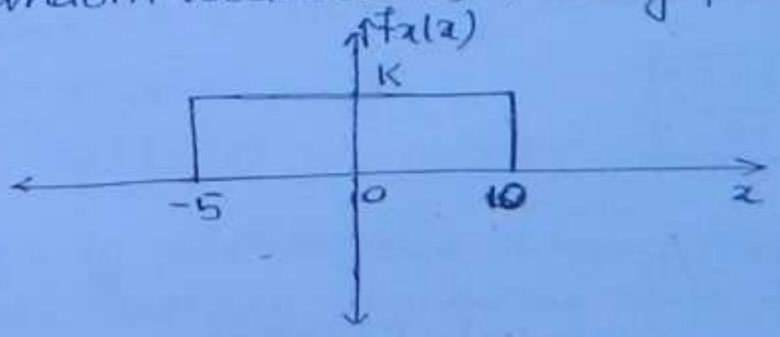
$$P(X = x_3) = 0$$

* The Prob of a continuous Random variable taking a specific single value will be zero.

Now

$$\boxed{P(-\infty \le X \le \infty) = \int_{-\infty}^{\infty} f_x(x)\,dx = 1}$$

Q. For a Random variable X; density funcn was given below



a) find K value.

b) $P(-5 \le X \le 10)$.

c) $P(-5 \le X \le 5)$.

d) Plot $f_x(x)$.

Solⁿ: It corresponds to the continuous Random variable.

Now as $\int_{-\infty}^{\infty} f_x(x)\,dx = 1$. 2014

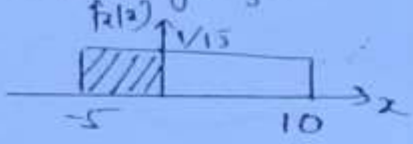
So, $15K = 1 \Rightarrow \boxed{K = 1/15}$

Also, $\int_{-5}^{10} K\,dx = Kx\Big|_{-5}^{10} = 1$

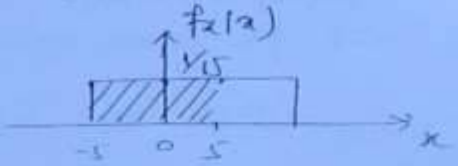
$15 \cdot K = 1 \Rightarrow K = 1/15$

b) $P(-5 \le X \le 0) = \int_{-5}^{0} f_x(x)\,dx$ {Area of Rectangle}.

$= 5 \times 1/15 = 1/3$ units



c) $P(-5 \le x \le 5) = \int_{-5}^{5} f_x(x)\,dx = 10 \times 1/15 = 2/3$ units



d) $F_x(x) = \int_{-\infty}^{x} f_x(x)\,dx$,

$= \int_{-5}^{x} 1/15\,dx = \frac{1}{15} \times x\Big|_{-5}^{x}$

$$\boxed{F_x(x) = \frac{(x+5)}{15}}$$

Now, $F_X(x) = (x+5)/15$

So, $F_X(-10) = 0$ $\{\because F_X(x) = P(X \le x)\}$.

So, $F_X(x) = \int_{-\infty}^{-10} f_x(x)\,dx = 0$.

$$F_x(15) = \int_{-\infty} f_x(x)\,dx = 1$$

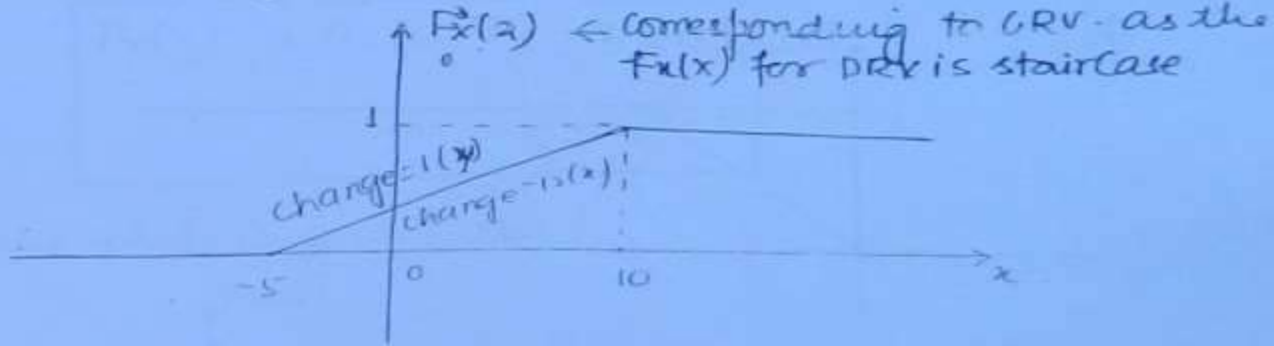
Conclusion:-

$$F_x(x) = \frac{x+5}{15} \quad ; -5 \le x \le 10$$

$$F_x(x) = 0 \quad ; \quad x < -5$$

$$1 \quad ; \quad x > 10.$$

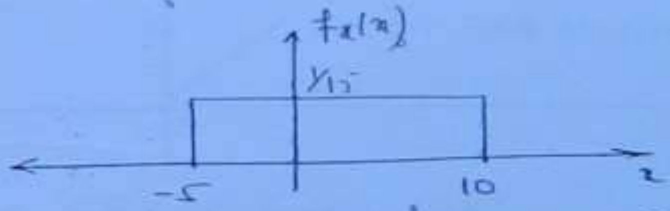
So, the plot is given as:-



← corresponding to CRV as the $F_x(x)$ for DRV is staircase

Now
$$\frac{d}{dx} F_x(x) = f_x(x) \quad \left\{ \because \frac{d}{dx} = \text{slope} \right\}.$$

So,

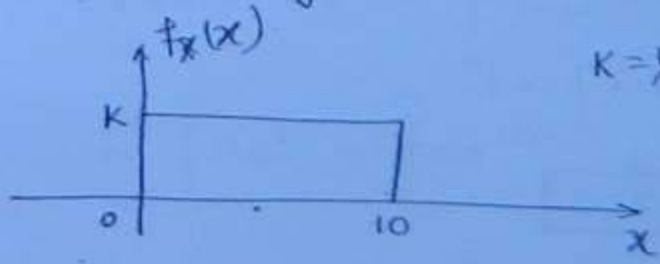


Q2. A continuous R.V x; uniformly distributed in the interval 0 to 10. Plot, a) $f_x(x)$.

b) $PF_x(x)$

Soln: uniformly distributed means that the Prob. of variable taking values at diff. instant is equal.

So,



$K = 1/10$; since Area = 1.

So, $F_X(x) \quad \int_{-\infty}^{x} f_X(x)\,dx$

$$= \int_{0}^{x} f_X(x)\,dx$$

$$= \int_{0}^{x} \frac{1}{10}\,dx$$

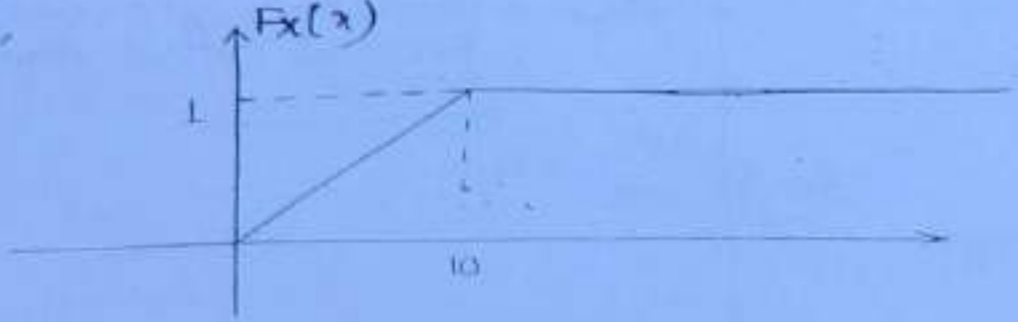
$$\boxed{F_X(x) = x/10}$$

$F_X(x) = 0 \;;\; x < 0$

$= x/10 \;;\; 0 \le x \le 10$

$= 1 \;;\; x > 10$

So,



Q. for a CRV; given

$$f_X(x) = a e^{-bx} \;;\; x \geq 0$$

i) Find Relation b/w a & b.

ii) Plot $f_X(x)$.

Soln: As, $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$$\int_{0}^{\infty} f_X(x)\,dx = 1 = \int_{0}^{\infty} a e^{-bx}\,dx = 1$$

$$= -\frac{a}{b} e^{-bx}\Big|_{0}^{\infty} = 1$$

$$= -\frac{a}{b}\{0-1\} = 1$$

$$\Rightarrow \boxed{a = b}$$

So, $f_X(x) = ae^{-ax} \; , x \geq$

Now, $F_X(x) = \int_{-\infty}^{x} f_X(x) \cdot dx$

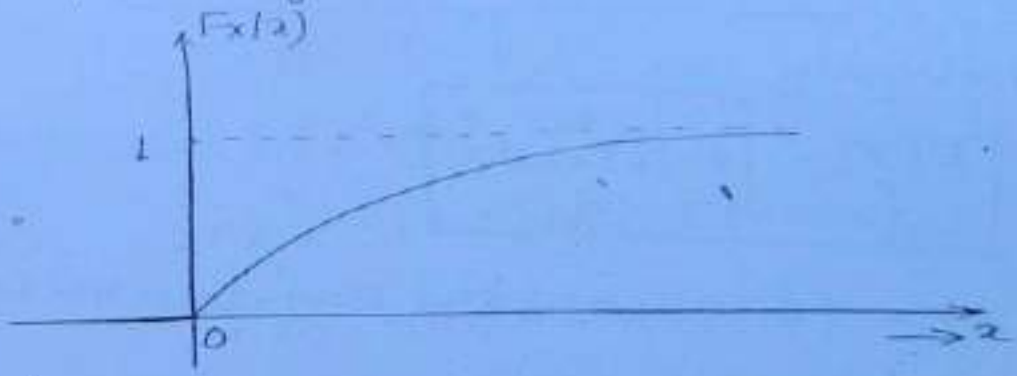
$$= \int_{0}^{x} ae^{-ax} \, dx$$

$$= -\frac{a}{a} e^{-ax} \Big|_{-0}^{x}$$

$$= -1 \{(e^{-ax} - 1)\}$$

$$\boxed{\begin{array}{ll} F_X(x) = (1-e^{-ax}) \; ; & x \geq 0 \\ \qquad 0 & ; x < 0 \end{array}}$$

So, the plot is given as:



Q. For a CRV, X. Given that

$$f_X(x) = ae^{-b|x|}$$

Find the Relation b/w a & b.

Soln.. Given that,

$$f_X(x) = ae^{-b|x|}$$

as, $\int_{-\infty}^{\infty} f_X(x) = 1 = \int_{-\infty}^{0} ae^{-b(-x)} + \int_{0}^{\infty} ae^{-b \cdot x} = 1$

$$= \int_{-\infty}^{0} ae^{bx} + \int_{0}^{\infty} ae^{-bx} = 1$$

$$\frac{a}{b} e^{+bx}\Big|_{-\infty}^{0} - \frac{a}{b} e^{-bx}\Big|_{0}^{\infty} = 1$$

$$= \frac{a}{b}\{1-0\} - \frac{a}{b}\{0-1\} = 1$$

$$= a/b + a/b = 1$$

$$\boxed{2a = b}$$ Ans

So, $\boxed{F_X(x) = a e^{-2a|x|}}$

Statistical Averages of Random Variable!

a) MEAN:

$$\boxed{\text{Mean }[X] = \text{Expectation, } E[X] = \bar{X} = m_1}$$

Mathematically,

$$\boxed{E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx}$$

Mean is the d-c value of the Random variable.

) MEAN SQUARE VALUE (MSQ):

$$\boxed{msq[X] = E[X^2] = \overline{X^2} = m_2}$$

Mathematically,

$$\boxed{E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx}$$

× It gives the total power of the Random variable.

c) VARIANCE (σ^2)

as

$$E[K] = K$$

$$\boxed{E[K] = \int_{-\infty}^{\infty} K \cdot f_X(x)\,dx = K}$$

and, $E[Kx] = \int_{-\infty}^{\infty} K \cdot x \, f_X(x)\,dx$

$$\boxed{E[KX] = K\,E[X]}$$

And, $\boxed{E[X_1 + X_2] = E[X_1] + E[X_2]}$

So the variance is defined as:-

$$\sigma^2 = E\left[(X - \bar{X})^2\right]$$

$$= E\left[(X - m_1)^2\right]$$

$$= E\left[X^2 + m_1^2 - 2Xm_1\right]$$

$$= E[X^2] + E[m_1^2] - E[2Xm_1]$$

$$= m_2 + m_1^2 - 2m_1 E[X]$$

$$= m_2 + m_1^2 - 2m_1^2$$

$$\boxed{\sigma^2 = m_2 - m_1^2}$$

So, σ^2 = total power - d.c power

$\boxed{\sigma^2 = \text{A.C power of Random variable}}$

d) STANDARD DEVIATION:

As

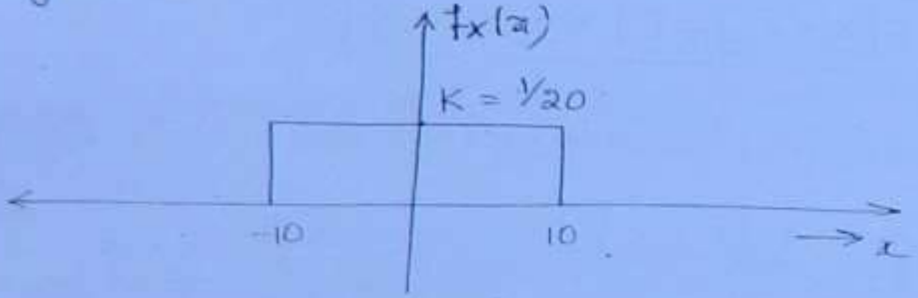
$\sigma^2 = $ variance

So $\boxed{\sigma = \sqrt{\text{variance}}}$

σ = AC component of Random variable.

Q. A continuous Random variable is uniformly distributed in the interval $(-10, 10)$. Find all of its statistical averages.

Soln:-



a) Mean, $m_1 = \int_{-\infty}^{\infty} x \, f_x(x) \, dx$

$$= \int_{-10}^{10} x \cdot \frac{1}{20} \, dx$$

$$= \frac{1}{20} \int_{-10}^{10} x \, dx = \frac{1}{20} \times \frac{x^2}{2} \Big|_{-10}^{10}$$

$$\frac{1}{40} \times 0$$

$\boxed{m_1 = 0}$ Ans

Conclusion:

If the density function is symmetric about the vertical Axis passing through the origin, then the Mean value of the ~~function~~ R.V is 0.

b) mean Square value. $m_2 = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$

$$= \int_{-10}^{10} x^2 \tfrac{1}{20}\, dx$$

$$= \frac{x^3}{3}\Big|_{-10}^{10} \times \tfrac{1}{20}$$

$$= \frac{1}{60} \times 2000$$

$$\boxed{m_2 = 33.33} \text{ Ans}$$

c) Variance, $\sigma^2 = m_2 - m_1^2$

$$= 33.33 - 0$$

$$\boxed{\sigma^2 = 33.33} \text{ Ans}$$

$\left\{ \because \sigma^2 = m_2 - m_1^2 \right.$

$\because m_1 = 0$

so, $\boxed{\sigma^2 = m_2}$ ***

d) Standard deviation $= \sqrt{\text{variance}}$

$$= \sqrt{33.33}$$

$$\boxed{\sigma = 5.77} \text{ Ans}$$

Q2. For a CRV, X; given

$$f_X(x) = \lambda e^{-\lambda x}; \; x \geq 0$$

Find all of its statistical averages.

Sol^n: i) Mean $= \int_{-\infty}^{\infty} x f_X(x)\, dx =$

$$= \int_{0}^{\infty} x \cdot \lambda e^{-\lambda x} \cdot dx$$

$$= \lambda \int_{0}^{\infty} \underset{u}{x} \cdot \underset{v}{e^{-\lambda x}}\, dx \quad \left\{ \because \int uv\, dx = u \int v\, dx - \int u' \int v \right.$$

$$= \lambda \left\{ x \cdot \frac{e^{-\lambda x}}{-\lambda} \Big|_{0}^{\infty} - \int_{0}^{\infty} 1 \cdot \frac{e^{-\lambda x}}{-\lambda}\, dx \right.$$

$$= \lambda \left\{ -x(0-0) + \frac{1}{\lambda^2}\left[e^{-\infty} - e^{0} \right] \right\} \Rightarrow \boxed{m_1 = 1/\lambda}$$

2) Mean Square value

$$msq = \int_{-\infty}^{\infty} x^2 \, f_x(x) \, dx$$

$$\boxed{msq = 2/\lambda^2}$$ Ans

3) $\sigma^2 = m_2 - m_1^2$

$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$$

$$\boxed{\sigma^2 = 1/\lambda^2}$$ Ans

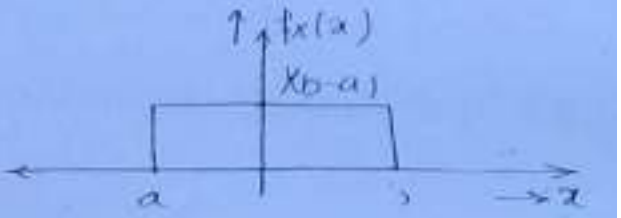
4) Standard deviation $= \sqrt{\sigma^2}$

$$\boxed{S.D = 1/\lambda}$$ Ans

* UNIFORM PROBABILITY DENSITY FUNCTION:

It is defined as:

$$f_x(x) = \frac{1}{(b-a)} \quad ; \quad a \le x \le b$$



Q. A CRV; is possessing uniform density function specified above. Find all of its statistical averages.

Solⁿ: As we know that,

$$Mean = \int_{-\infty}^{\infty} x \, f_x(x) \, dx$$

$$= \int_{a}^{b} x \cdot 1/(b-a) \, dx$$

$$= \frac{1}{b-a} \int_{a}^{b} x \, dx$$

$$= \frac{1}{(b-a)} \left. \frac{x^2}{2} \right|_{a}^{b}$$

$$= \frac{1}{2(b-a)} \left[b^2 - a^2 \right] \Rightarrow \boxed{m_1 = \frac{b+a}{2}}$$

2) Mean Square Value. msq $\int_{-\infty}^{\infty} x^2 f_x(x)\,dx$

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$$= \int_a^b a^2 \frac{1}{(b-a)}\,dx$$

$$= \frac{1}{3(b-a)}\left[x^3\right]_a^b$$

$$= \frac{b^3-a^3}{3(b-a)}$$

$$\boxed{msq = \frac{b^2+a^2+ab}{3}}$$

3) NOW, variance $\sigma^2 = m_2 - m_1^2$

$$= \frac{b^2+a^2+ab}{3} - \frac{(a+b)^2}{4}$$

$$\boxed{\sigma^2 = \frac{(a-b)^2}{12}}$$

4) Standard daviation $= \sqrt{\sigma}$

$$\boxed{S.D = \frac{(a-b)}{2\sqrt{3}}}$$

* GAUSSIAN DENSITY FUNCTION:

It is given as:

$$\boxed{f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-a)^2}{2\sigma^2}}}$$

For, $x = a$

$f_x(x)$ will be max^m.

Hence the plot is given as:-

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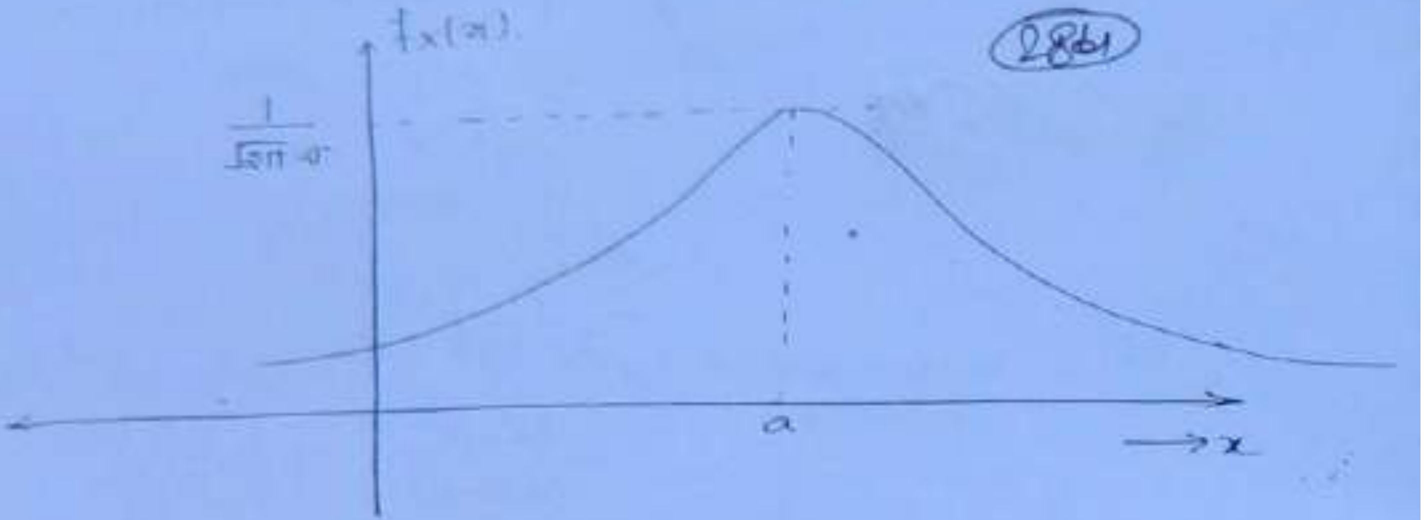


a) Mean. $m_1 = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx$

$$= \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-a)^2}{2\sigma^2}}\, dx$$

After Analysis.

$$\boxed{m_1 = a}$$

b) Mean Square value, $msq = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$

$$\boxed{msq = \sigma^2 + a^2}$$

c) Variance, $\sigma^2 = m_2 - m_1^2$

$$= \sigma^2 + a^2 - a^2$$

$$\boxed{\text{variance} = \sigma^2}$$

d) Standard deviation

$$\boxed{SD = \sigma}$$

*_Analysis:_

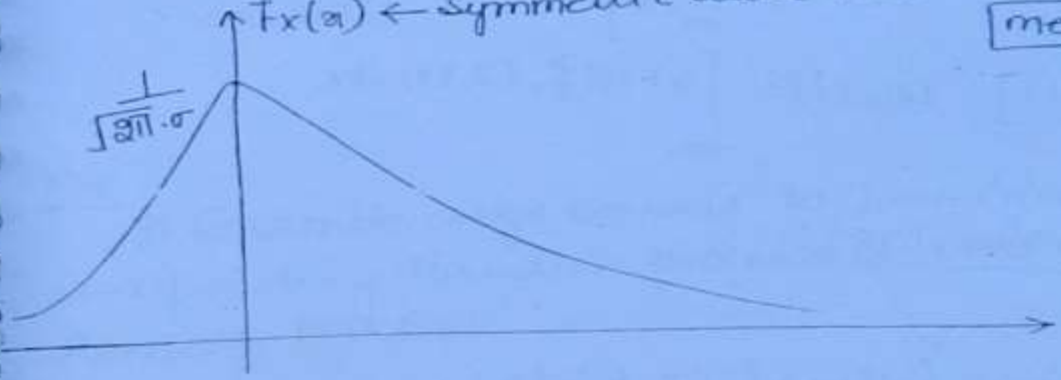
i) If mean = 0 = m₁ = 0

So, $a = 0$

Then, $f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$ | max at $x = \underline{0}$

So, the plot is given as!.

$f_x(a)$ ← Symmetric about Vertical Axis, hence $\boxed{\text{mean} = 0}$



Note:

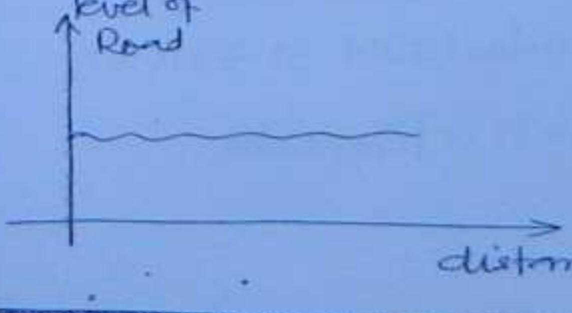
* while Noise possess Gaussian density function, so it is also called as the Gaussian Noise.

only ES subj numerical

*RANDOM PROCESS:-

* Random variable as a function of time is called as the RANDOM PROCESS.

RANDOM VARIABLE	RANDOM PROCESS
1. denoted as 'x'	1. denoted as x(t).
2. $F_x(a) = P(x \le a)$	2. $F_x(x,t) = P(x(t) \le x)$
3. $f_x(a) = \frac{d}{dx} F_x(a)$	3. $f_x(x,t) = \frac{d}{dx} F_x(x,t)$.



*Statistical Averages of Random Process:

1) ENSEMBLE AVERAGES:-

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The Averages calculated on a group of Similar Random processes are called as Ensemble Avgs.

** depends on density function.

** a) Ensemble mean:

Given as:

$$E[x(t)] = m_1(t) = \int_{-\infty}^{\infty} x(t) f_x(x,t)\, dx$$

b) Mean Square value, msq:

Given as:

$$m_2(t) = \int_{-\infty}^{\infty} x^2(t) f_x(x,t)\, dx$$

c) Auto Correlation Function:

Given as:

$$*** \boxed{R(\tau) = E[x(t)\, x(t-\tau)]}$$

2) TIME AVERAGES:

* The Statistical averages computed on a Random process on a time basis is called as Time Averages

*** Time Averages are independent of density function.

a) Time Mean:

Given as:

$$\langle x(t) \rangle = \int_{t_1}^{t_2} x(t)\, dt$$

b) Mean square value, msq:

Given as:

$$\langle x^2(t)\rangle = \int_{t_1}^{t_2} x^2(t)\,dt$$

c) Auto Correlation Function:

Given as:

$$\langle x(t)x(t-\tau)\rangle = \int_{t_1}^{t_2} x(t)x(t-\tau)dt$$

Note:

If Ensemble avgs equals to Time Averages, then the corresponding Random process is said to be "ERGODIC RANDOM PROCESS".

* If only means are same, then it is said to be ERGODIC IN MEAN/MSQ/AUTOCORRELATION.

* STRICT SENCE STATIONARY RANDOM PROCESS:

* If Prob. density function of Random Process is Independent of time, then it is said to be "SSSRP"

So, $\boxed{f_X(x,t) = f_X(x, t+\Delta t)}$

* WIDE SENSE STATIONARY RANDOM PROCESS:

* A Random process is said to be WSSRP, if it satisfies the following

a) Mean should be constant, Independent of t

b) ACF ie $R(\tau)$ should be function of only τ.

$$R(\tau) = E[x(t)\,x(t-\tau)].$$

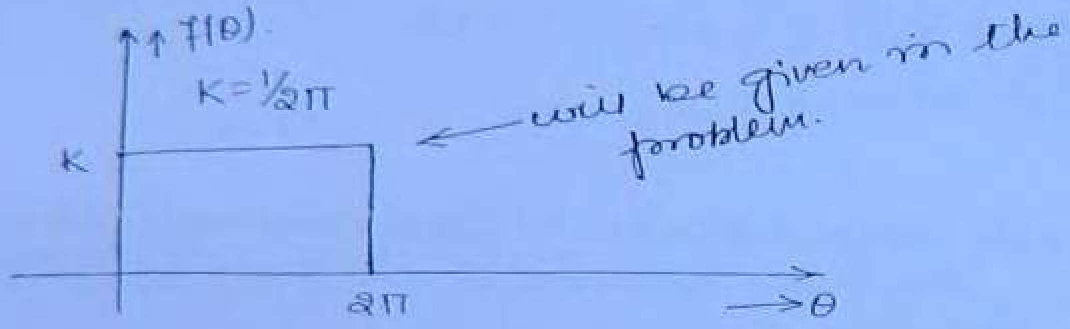
Q A Random process is given by

$$x(t) = A\cos\{\omega_0 t + \theta\}$$

where A & ω_0 are constants and θ is Random variable; which is uniformly distributed in the interval $(0, 2\pi)$.

Find whether the given RP is WSS or not?

Solⁿ:



will be given in the problem.

Now, mean of Funcⁿ = $m_1(t) = \int_{-\infty}^{\infty} x(t) \cdot f(\theta)\, d\theta$

$$= \int_0^{2\pi} A\cos\{\omega_0 t + \theta\} \cdot \frac{1}{2\pi}\, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} A\cos\{\omega_0 t + \theta\}\, d\theta$$

∴ θ is Random variable, all other parameters are considered as considered including t

$$m_1(t) = \frac{1}{2\pi}\int_0^{2\pi} A\cos\omega_0 t \cos\theta\, d\theta - \frac{1}{2\pi}\int_0^{2\pi} A\sin\omega_0 t \sin\theta\, d\theta$$

$$= \frac{A\cos\omega_0 t}{2\pi}\int_0^{2\pi} \overset{0}{\cancel{\cos\theta\, d\theta}} - \frac{A\sin\omega_0 t}{2\pi}\int_0^{2\pi} \overset{0}{\cancel{\sin\theta\, d\theta}}$$

So, $\boxed{m_1(t) = 0 = \text{Constant}}$

Also, ACF $= R(\tau) = E[x(t)x(t-\tau)]$

$$= E[A\cos(\omega_0 t + \theta) \cdot A\cos\{\omega_0(t-\tau) + \theta\}]$$

$$= E[A\cos(\omega_0 t + \theta) \cdot A\cos(\omega_0 t - \omega_0\tau + \theta)]$$

$$R(\tau) = E\left[\frac{A^2}{2}\cos(2\omega_0 t - \omega_0\tau + 2\theta) + \frac{A^2}{2}\cos\omega_0\tau\right]$$

$$R(\tau) = E\left[\frac{A^2}{2}\cos(2\omega_0 t - \omega_0\tau + 2\theta)\right] + E\left[A^2/2\cos\omega_0\tau\right]$$

Now As $E[K] = K$

So, $E[A^2/2\cos\omega_0\tau] = A^2/2\cos\omega_0\tau$ {∵ Integration done w.r.t θ}.

Now,

$$E\left[A^2/2\cos(2\omega_0 t - \omega_0\tau + 2\theta)\right] = \int_0^{2\pi} \frac{A^2}{2}\cos(\underbrace{2\omega_0 t - \omega_0\tau}_{A} + \underbrace{2\theta}_{B}) \cdot 1/2\pi \, d\theta.$$

$$\Rightarrow \boxed{E\left[A^2/2\cos(2\omega_0 t - \omega_0\tau + 2\theta)\right] = 0.}$$

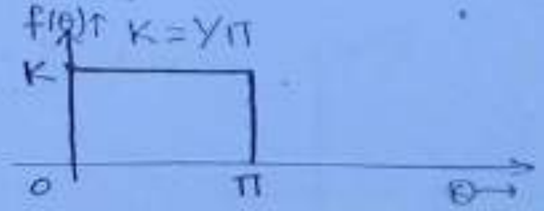
So, $\boxed{ACF = R(\tau) = \frac{A^2}{2}\cos\omega_0\tau}$

Hence, it is function of only τ.

Hence, the given function is WSSRP Ans.

Q. Repeat Above if θ is uniformly distributed in the Interval $(0, \pi)$.

Solⁿ:-



Now mean of funcⁿ = $m(t) = \int_{-\infty}^{\infty} x(t) f(\theta) d\theta$

$$= \int_0^{\pi} 1/\pi \, A\cos(\omega_0 t + \theta) \, d\theta$$

$$= \frac{A}{\pi}\int_0^{\pi} \cos(\omega_0 t + \theta) d\theta$$

$$m_1(t) = \frac{A}{\pi}\int_{0}\left[\cos\omega_0 t\cos\theta - \sin\omega_0 t\sin\theta\right]d\theta$$

$$= \frac{A}{\pi}\cos\omega_0 t\int_0^{\pi}\cos\theta\, d\theta - \frac{A\overset{\sin\omega_0 t}{\cancel{\cos\omega_0 t}}}{\pi}\int_0^{\pi}\sin\theta\, d\theta$$

$$= \frac{A}{\pi}\cos\omega_0 t\sin\theta\Big|_0^{\pi} - \frac{A\overset{\sin\omega_0 t}{\cancel{\cos\omega_0 t}}}{\pi}\times -\cos\theta\Big|_0^{\pi}$$

$$= \frac{A}{\pi}\cos\omega_0 t\times(0-0) + \frac{A\overset{\sin\omega_0 t}{\cancel{\cos\omega_0 t}}}{\pi}(\cos\pi - \cos 0)$$

$$= \frac{A\overset{\sin\omega_0 t}{\cancel{\cos\omega_0 t}}}{\pi}(-1-1)$$

$$\boxed{m_1(t) = \frac{-2A\sin\omega_0 t}{\pi}}$$

$m_1(t)$ is not constant, but is a function of t

Hence given RP is not WSS RP.

*CONVOLUTION AND CORRELATION

1. Convolution is used to find the Response of the system
2. Correlation is used to find the Similarity b/w the signals.

Now,
Mathematically, convolution is given as:

$$\boxed{x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau}$$

* Cross-Correlation means correlation of 2 functions.

Hence
X-Correlation is mathematically given as.

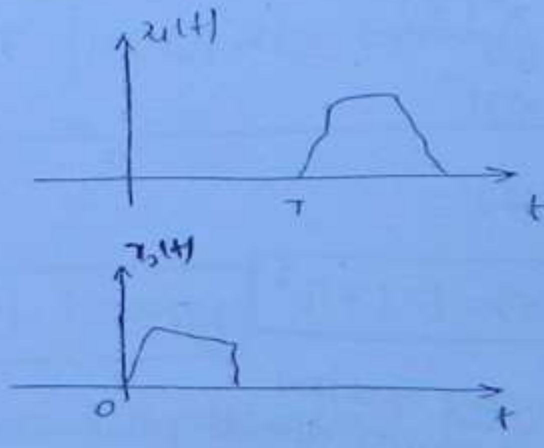
$$\boxed{R_{12}(\tau) = \int_{-\infty}^{\infty} x_1(t)\, x_2(t-\tau)\, dt}$$

Now, For the 2 Signals to be equal, we have!

$$\int_{-\infty}^{\infty} x_1(t) \cdot x_2(t)\, dt = X(t)$$

If the value of $X(t) = 0$, hence, the signals are atmost dissimilar.

But let



The value of $\int_{-\infty}^{\infty} x_1(t) \cdot x_2(t)$ is Zero in this case, but some Similarity is present. Hence we delay the 2nd Signal by τ and for the value of τ for which max^m area is overlapped, it is said to be the similar value. ie

$$R_{12} = \int_{-\infty}^{\infty} x_1(t)\, x_2(t-\overset{\text{Searching/Scanning parameter}}{\tau})\, d\tau$$

Now

$$R_{12}(\tau) = \int_{-\infty}^{\infty} x_1(t)\, x_2(t-\tau) \cdot d\tau$$

$$\boxed{R_{12}(\tau) = x_1(t) * x_2(-t) \,/_{t \to \tau}}$$

*Auto correlation of a signal means finding the similarity of a signal with its shifted version.

Hence Mathematically,

$$ACF[x(t)] = R(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t-\tau)\, d\tau$$

So, $\boxed{R(\tau) = x(t) * x(-t)}$

So by Fourier xform we get:

$$\boxed{R(\tau) \longleftrightarrow X(f) \cdot X(-f)}$$

Note:

Now, if $x(t)$ is Real, then

$$\boxed{X^*(f) = X(-f)}$$

So,

$$R(\tau) \longleftrightarrow X(f) \cdot X^*(f)$$

$$R(\tau) \longleftrightarrow |X(f)|^2$$

$$R(\tau) \longleftrightarrow S(f)$$

where, $\boxed{S(f) = |X(f)|^2}$.

Energy spectral density of $x(t)$.

So,

Conclusion:

$$\boxed{\text{Fourier } x^{\text{form}} \text{ of ACF} = \text{ESD}}$$

And, $\boxed{IFT[S(f)] = R(\tau)}$

Now,

$$\int_{-\infty}^{\infty} S(f) e^{j2\pi f\tau} df = R(\tau)$$

put $\tau = 0$.

So, $$\boxed{\int_{-\infty}^{\infty} S(f)\, df = R(0) \quad ---(1)}$$ ← Autocorrelation function at origin will give the Area of ESD.

ote:

* $R(\tau)$ is max^m at $\tau = 0$.
and as the value of τ is increasing, the similarit
is decreasing and $R(\tau)$ is decreasing.

Now,

$$R(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t-\tau)\, dt$$

at $\tau = 0$

$$\boxed{R(0) = \int_{-\infty}^{\infty} x^2(t)\, dt = \text{Energy}[x(t)]} \quad ---(2)$$

So, from (1) & (2) we get:

$$\boxed{\int_{-\infty}^{\infty} S(f)\, df = \int_{-\infty}^{\infty} |X(f)|^2 df = \text{Energy}[x(t)]}$$ ← PARSEVAL'S THEOREM.

Conclusion:

$$\boxed{\text{Area}[ESD] = \text{Energy}}$$

The above discussion is valid only for the Energy Signal. If the Signal is power signal, we have to generalise the discussion by Average Auto-correlation function.

* AVG AUTO-CORRELATION FUNCTION :-

let
Periodic signal by $x_T(t)$.

* All periodic Signal are Power Signals; but Reverse is not true.

Now,
$$R(\tau) = \int_{-\infty}^{\infty} x_T(t) \cdot x_T(t-\tau)\,dt$$

at $\tau = 0$
$$\boxed{R(0) = \int_{-\infty}^{\infty} x_T^2(t)\,dt = \text{Energy of Signal} = \infty}$$
← It fails as value of ACF $= \infty$.

* Autocorrelation function is max^{m} at $\tau = 0$, but the value should be some finite value.

* But for above case at $\tau = 0$, $R(0) = \infty$.
Hence, the formula for ACF of periodic signal fails for the analysis of Power/Periodic Signals

* ACF shouldn't be ∞. So, it is failed for Power Signals

* For Power Signals, avg auto correlation func^{n} will be defined.

So,
$$\tilde{R}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x_T(t)\, x_T(t-\tau)\,dt \Rightarrow \lim_{T\to\infty} \frac{1}{T} [x_T(t) * x_T(-t)]$$

$$\boxed{\tilde{R}(0) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x_T^2(t)\,dt = \text{Power of } x_T(t)} \quad \text{--- (3)}$$

taking F.T on both sides we get :-

$$\tilde{R}(\tau) \Leftrightarrow \lim_{T\to\infty} \frac{1}{T} |X_T(f)|^2$$

$$\tilde{R}(\tau) \longleftrightarrow \lim_{T\to\infty} \frac{1}{T} \overset{\text{ESD.}}{S_T(f)}$$

$\because$ Energy of Power Signal over entire range $= \infty$.
But for 1 period, E = finite value

So, $\tilde{R}(\tau) \longleftrightarrow S(f)$

Now,

(295)

$$IFT[S(f)] = \tilde{R}(\tau)$$

So, $$\int_{-\infty}^{\infty} S(f) e^{j2\pi f\tau} df = \tilde{R}(\tau)$$

$\tau \to 0$

$$\boxed{\int_{-\infty}^{\infty} S(f)\, df = \tilde{R}(0)} \quad \text{- - - - (4)}$$

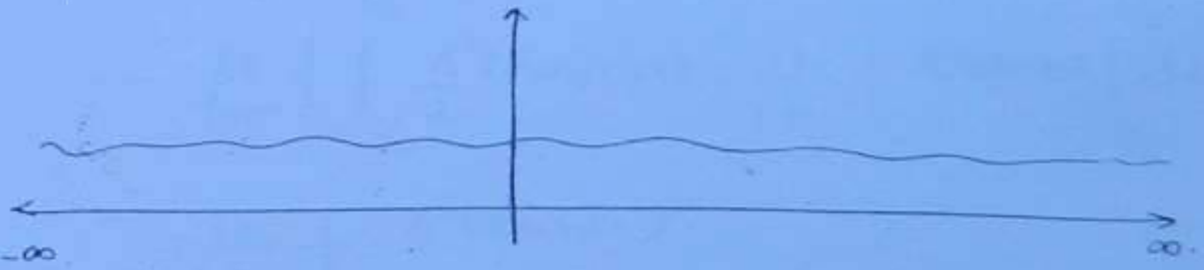
Conclusion:

So, from eqn (3) & (4) we get:-

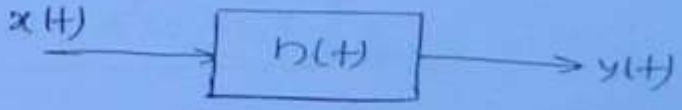
xxx $\boxed{\text{Area}[PSD] = \text{Power}}$

Note:

while Noise corresponds to power signal; as it occupies the spectrum from $-\infty$ to $+\infty$



Note:



$$Y(f) = H(f) \cdot X(f)$$

$$|Y(f)|^2 = |H(f)|^2 |X(f)|^2$$

$$\boxed{(ESD)_{o/p} = |H(f)|^2 \cdot (ESD)_{i/p}}$$

Also,

$$|Y(f)|^2 = |H(f)|^2 |X(f)|^2$$

$$\frac{|Y(f)|^2}{T} = \frac{|H(f)|^2 \, |X(f)|^2}{T}$$

$$\underset{T \to \infty}{lt} \frac{|Y(f)|^2}{T} = |H(f)|^2 \underset{T \to \infty}{lt} \frac{|X(f)|^2}{T}$$

$$\boxed{(PSD)_{o/p} = |H(f)|^2 (PSD)_{i/p}}$$

Q1. Given,

$x(t) = A\cos\omega_0 t$

Find

i) ACF

ii) PSD

iii) Power

Sol^n: ∵ $x(t)$ is periodic func^n.

Hence, Avg. ACF has to be calculated

So,

a) $$R(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,x(t-\tau)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} A\cos\omega_0 t \, A\cos(\omega_0 t - \omega_0\tau)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{A^2}{2}\cancelto{0}{\cos(2\omega_0 t - \omega_0\tau)} + \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \cos\omega_0\tau\,dt\,\frac{A^2}{2}$$

∵ Periodic

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{A^2}{2}\cos\omega_0\tau\,dt = \lim_{T\to\infty} \frac{1}{T}\cdot A^2\cos\omega_0\tau \int_{-T/2}^{T/2} dt$$

$$= \lim_{T\to\infty} \frac{1}{\cancel{T}}\cdot A^2\cos\omega_0\tau \cdot \cancel{T}$$

$$\boxed{R(\tau) = A^2\cos\omega_0\tau}$$ Ans

*** Note !

$$\left.\begin{matrix} A\cos(\omega_0 t + \Phi) \\ \text{or} \\ A\sin(\omega_0 t + \Phi) \end{matrix}\right\} \longrightarrow R(\tau) = \frac{A^2}{2}\cos\omega_0\tau.$$

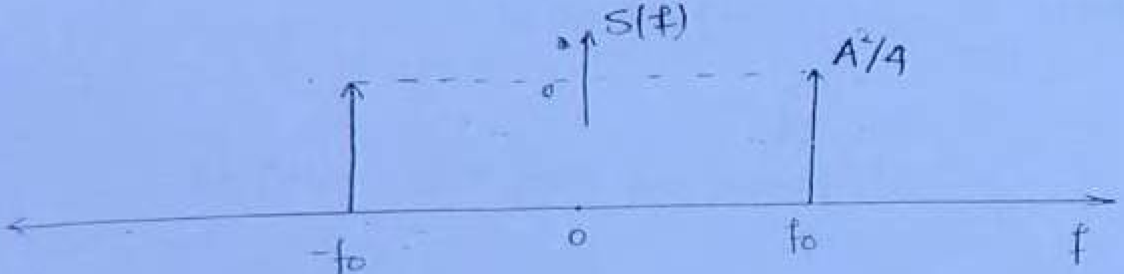
b) $S(f) = FT\,|\,R(\tau)\,|$

$$= FT\left[\frac{A^2}{2}\cos 2\pi f_0 \tau\right]$$

$$= \frac{A^2}{2}\left[\frac{\delta(f+f_0)+\delta(f-f_0)}{2}\right]$$

So, $$\boxed{S(f) = \frac{A^2}{4}\left\{\delta(f+f_0 + \delta(f-f_0)\right\}.}$$ Ans

Plot :



c) Power = Area $[S(f)]$.

$$= \frac{A^2}{4} + \frac{A^2}{4}$$

So, $\boxed{\text{Power} = A^2/2}$ Ans

Also, Power $= R(\tau)\big|_{\tau=0}$

$$= \frac{A^2}{2}\cos 2\pi f_0 \tau\Big|_{\tau=0}$$

$\boxed{\text{Power} = R(0) = A^2/2}$ Ans

Q2. A signal of $x(t) = e^{-2t}u(t)$, is passed through a system given by $H(\omega) = \frac{1}{(j\omega+4)}$. Find Energy spectral density of o/p of the system?

$$x(t) \longrightarrow \boxed{H(\omega)} \longrightarrow y(t)$$

Solⁿ: $(ESD)_{o/p} = (ESD)_{i/p} \cdot |H(\omega)|^2$

Now, $$|Y(\omega)|^2 = |X(\omega)|^2 \cdot |H(\omega)|^2$$

Now, $X(\omega) = \frac{1}{(j\omega+2)}$ $\left\{\because\ e^{-at}u(t) \longleftrightarrow \frac{1}{s+a}\right\}$

$$|X(\omega)| = \frac{1}{\sqrt{\omega^2+4}} \Rightarrow |X(\omega)|^2 = \frac{1}{(\omega^2+4)}$$

$$|H(\omega)| = \frac{1}{\sqrt{\omega^2+16}}$$

$$|H(\omega)|^2 = \frac{1}{\omega^2+16}$$

So, $\boxed{|Y(\omega)|^2 = \frac{1}{(\omega^2+4)(\omega^2+16)}}$ ← ESD of O/P

Ans

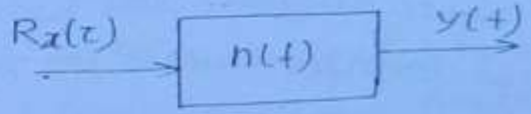
ES

Q3. A Random variable, of having Auto correlation function

$$R(\tau) = e^{-\sigma|\tau|}$$

is passed through a system, whose Impulse Response is given by $h(t) = \frac{\mu}{2} e^{-\mu t} u(t)$.

Find PSD of the O/P Signal?



Soln: Given, $R(\tau) = e^{-\sigma|\tau|}$

$h(t) = \mu/_2 \, e^{-\mu t} u(t)$

Now as

$$(PSD)_{O/P} = (PSD)_{I/P} \cdot |H(\omega)|^2$$

$(PSD)_{I/P} = F.T \text{ of } R_x(\tau)$

$= FT\left[e^{-\sigma|\tau|}\right]$ $\quad \left\{ e^{-a|t|} \Leftrightarrow \frac{2a}{a^2+\omega^2} \right\}$.

$$\boxed{(PSD)_{I/P} = \frac{2\sigma}{\sigma^2+\omega^2}}$$

Now, $H(\omega) \longleftrightarrow \frac{\mu}{2}\left\{\frac{1}{\mu+j\omega}\right\}$.

$$|H(\omega)|^2 \longrightarrow \frac{\mu^2}{4} \times \frac{1}{(\mu^2+\omega^2)}$$

So,

$$(PSD)_{O/P} = \frac{2\sigma}{(\sigma^2+\omega^2)} \times \frac{\mu^2}{4} \times \frac{1}{(\mu^2+\omega^2)}$$

$$\boxed{(PSD)_{O/P} = \frac{\sigma\mu^2}{2(\sigma^2+\omega^2)(\mu^2+\omega^2)}}$$

* NOISE IN ANALOG COMMUNICATION

* Noise may be classified as:
  1) Internal Noise (within the system)
     - → Shot Noise
     - → *** Thermal Noise
     - → Flicker Noise etc
  2) External Noise (external sources).
     - → Automobile Noise.
     - → Atmospheric Noise.
     - → Solar Noise. etc.

* Thermal Noise is also called as:
  - a) White Noise
  - b) Gaussian Noise.
  - b) Johnson Noise.

* Thermal Noise:

* Due to Thermal Agitation, atoms in the electrical components will gain energy, moves in Random passion and collide with each other; heat will be generated. This corresponds to Thermal Noise.

* Each of the frequency component is transmitted through a commn system will be affected by Thermal Noise, so is also called as "WHITE NOISE".

* Thermal Noise Power is given mathematically as:

$$\boxed{\underset{\text{watts}}{N} = \overset{\text{watts/Hz}}{KT}\,\underset{\text{Hz}}{B}\ \text{watt}}$$

where

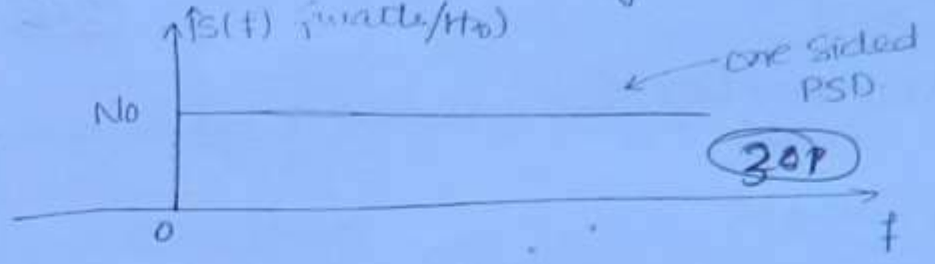
K = Boltzman Const = 1.38×10^{-23} Joules/Kelvin

T = Absolute Temp °K.

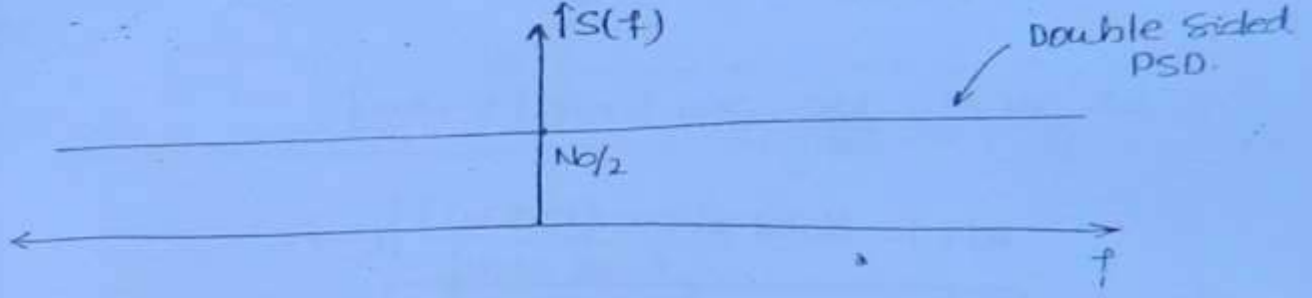
B = Bandwidth.

* $\boxed{KT = N_0 = \text{Power spectral density}}$ (const).

* The plot of PSD is a frequency func which is given as



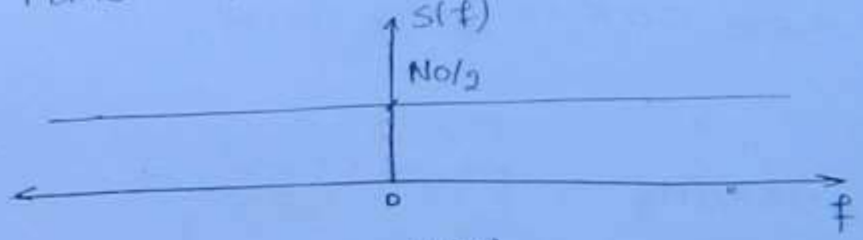
{Area of PSD is given as Power.

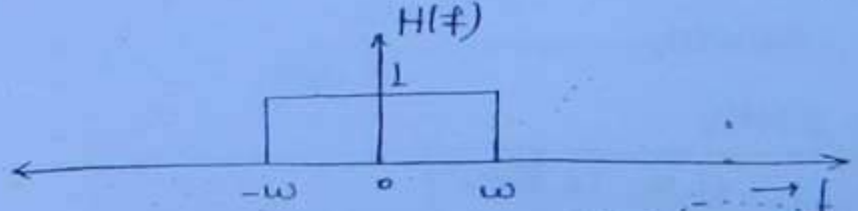


Previous Papers :-

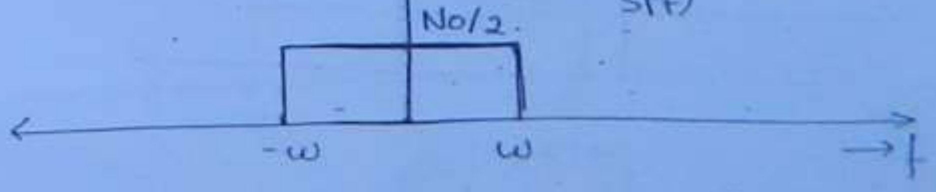
Q1. A white Noise of having 2 sided PSD $N_o/2$ watts/Hz is passed through a LPF whose cut-off frequency (f_c) is ω Hz find O/P white Noise power and corresponding Auto correlation function ?

Soln:





$So(f) = \underbrace{Si(f)}_{S(f)} |H(f)|^2 \rightarrow$ Same as H(f) ... ①



So, O/P Noise power N_o = Area of So(f).

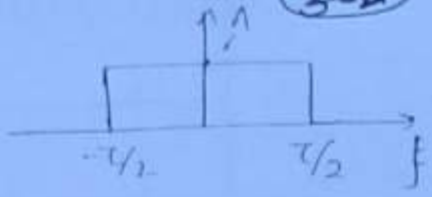
$= 2\omega \times N_o/2$

$$\boxed{\text{O/P Noise power} = N_o\omega \text{ watts}}$$

Autocorrelation func^n = $R(\tau)$ = IFT $[S_o(f)]$

Now,

$A\tau \, Sinc(f\tau) \longleftrightarrow$



comparing we get

$\tau = 2W \; ; \; A = N_o/_2$

So, $R(\tau) = \frac{N_o}{2} \cdot 2W \, Sinc[t \cdot 2W]$

$R(\tau) = N_oW \, Sinc[t \cdot 2W] \Big|_{t \to \tau}$

$\boxed{R(\tau) = N_oW \, Sinc[2W\tau]}$ Ans

Now, $\boxed{R(0) = N_oW = \text{Power}}$ Ans

Q2. A white Noise of having 2 sided PSD 4 Kw/Hz is passed through LPF whose C.O.F is 2KHz. Find O/P white Noise power?

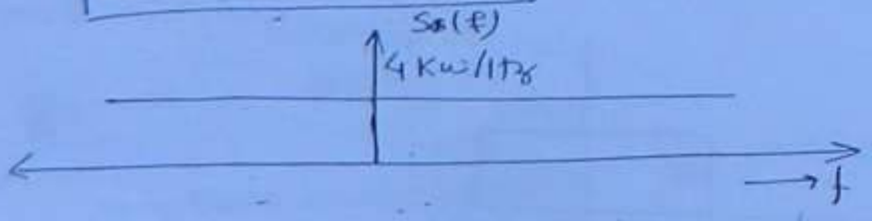
Sol^n: Given,

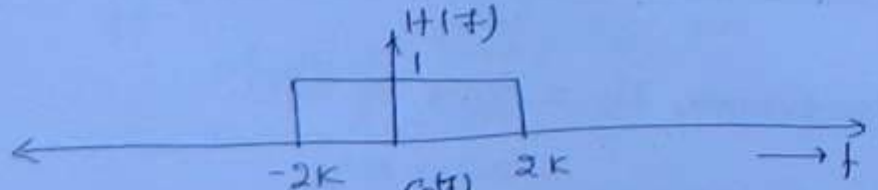
$N_o/_2 = 4 \, Kw/Hz$

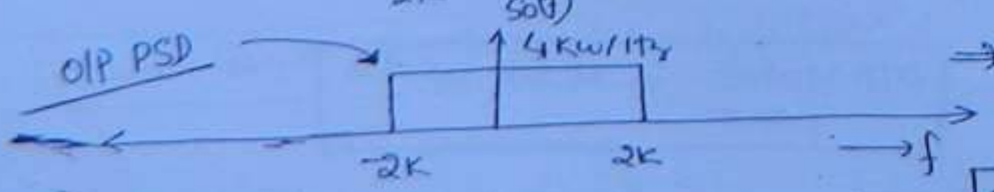
$N_o = 8 \, Kw/Hz$

$B = 2 \, KHz$

So, $\boxed{\text{Power} = N_oB = 16 \, Kw}$







$\Rightarrow N_o = \text{Area}[S_o(f)]$

$= \frac{4Kw}{Hz} \times 2K$

$\boxed{N_o = 16 \, MW}$

Q3 white noise of having psd [illegible] is passed through a syst is specified by

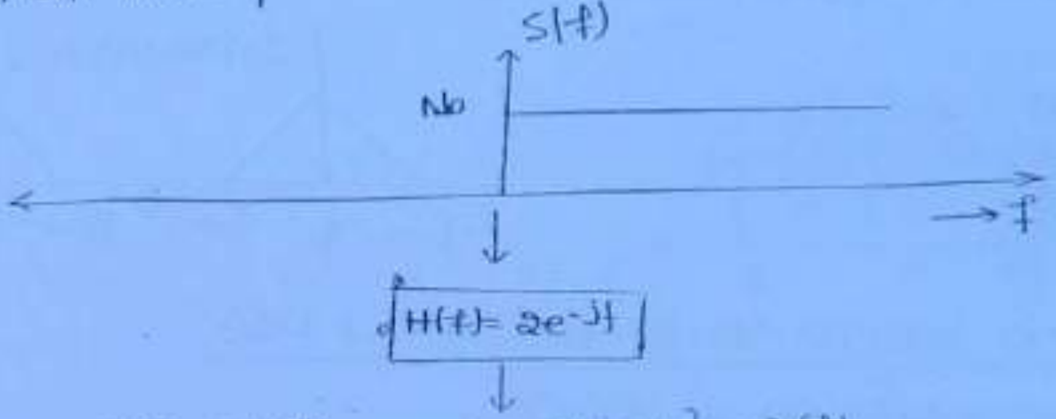
$$H(f) = 2e^{-jf}$$

(3 09)

The Resulting is passed through LPF whose Co(f) is B Hz

Find o/p Noise power

Sol:



PSD at o/p of the System $\longrightarrow S(f)\cdot|H(f)|^2 = S_o(f)$

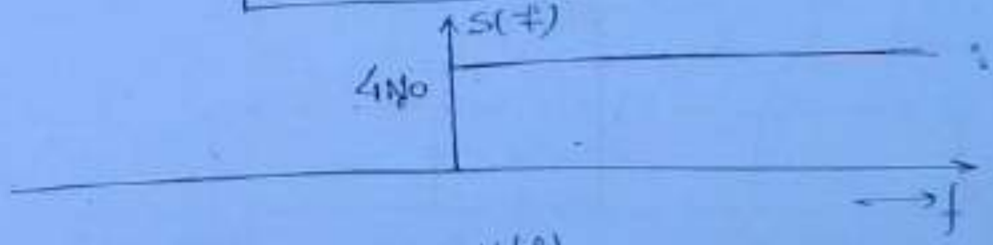
$$H(f) = 2e^{-jf}$$

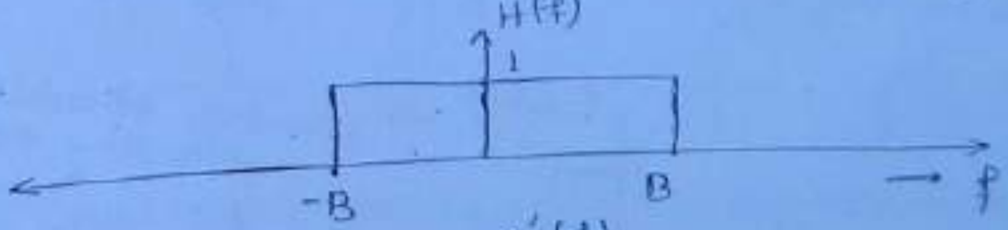
$$|H(f)| = 2\,|e^{-jf}| \to 1$$

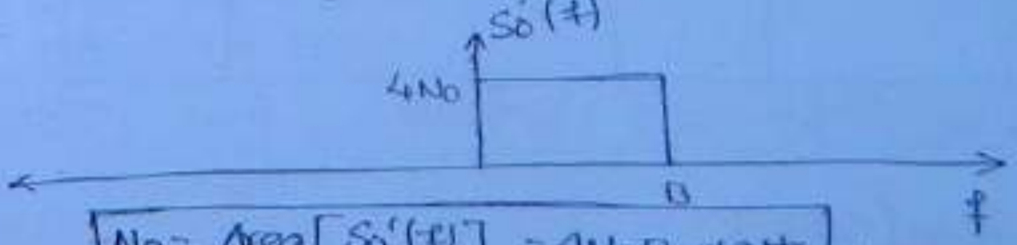
$$|H(f)|^2 = 4.$$

So, $S_o(f) = 4\cdot S(f)$

$$\boxed{S_o(f) = 4N_o}$$



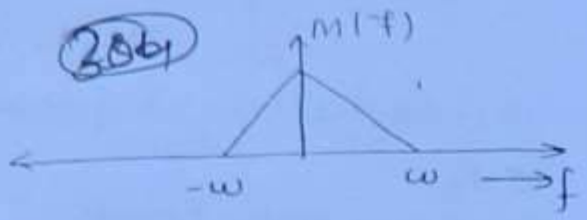




$$\boxed{N_o = \text{Area}[S_o'(f)] = 4N_oB \text{ watts}}$$

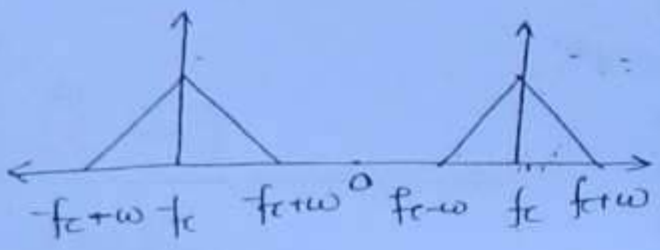
* Analysis (effect of Noise on AM, DSB, SSB)

Assume $m(t)$ ←————→ (30b)



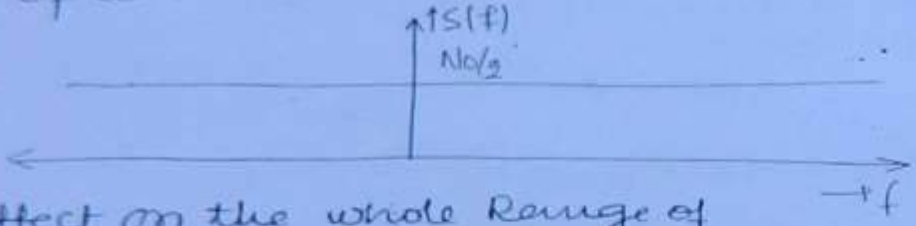
$c(t) = A_c \cos 2\pi f_c t$

$S_{AM}(t)$ ←————→



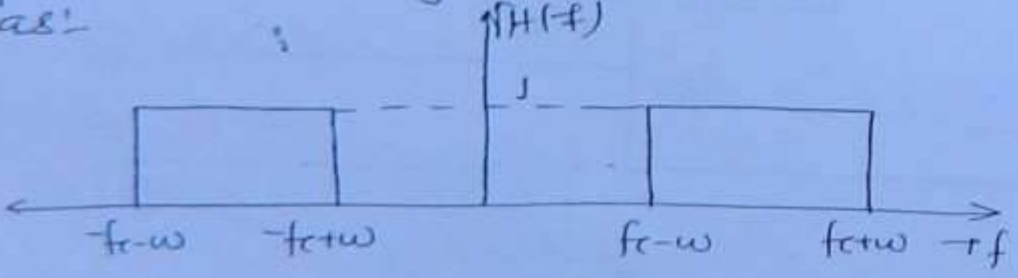
*EFFECT OF WHITE NOISE ON AM & DSB:

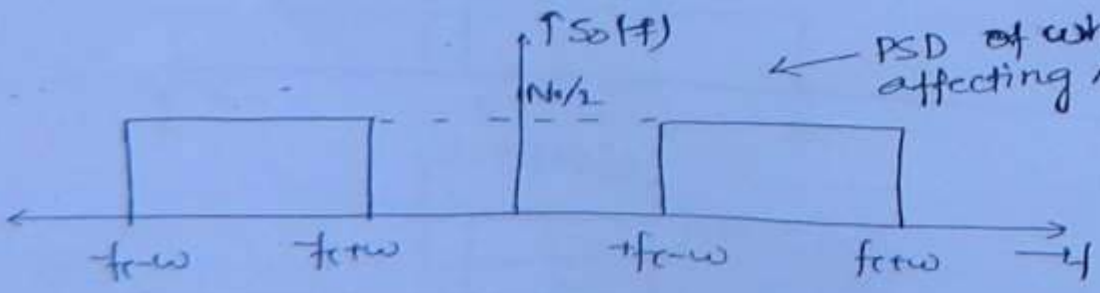
The PSD of while Noise is:



It has effect on the whole Range of frequency

* To visualize the effect on the freq^n components of AM & DSB we pass this through a BPF. So the spectrum is given as:-





PSD of while Noise affecting AM & DSB.

So, $N = \text{Area}[S_o(f)]$

$= 2\omega \times N_0/2 \times 2$ {∵ Two sided}.

So, $\boxed{\text{Noise power, } N = 2N_0W \text{ watts.}}$

EFFECT OF WHITE NOISE ON SSB:

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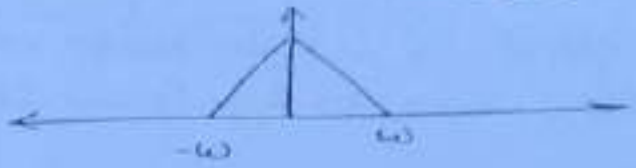
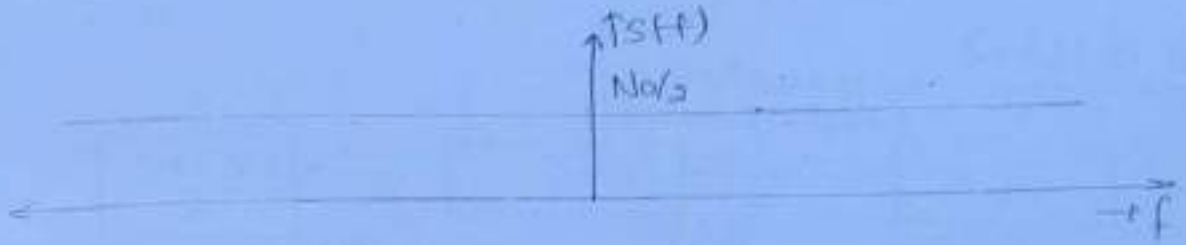
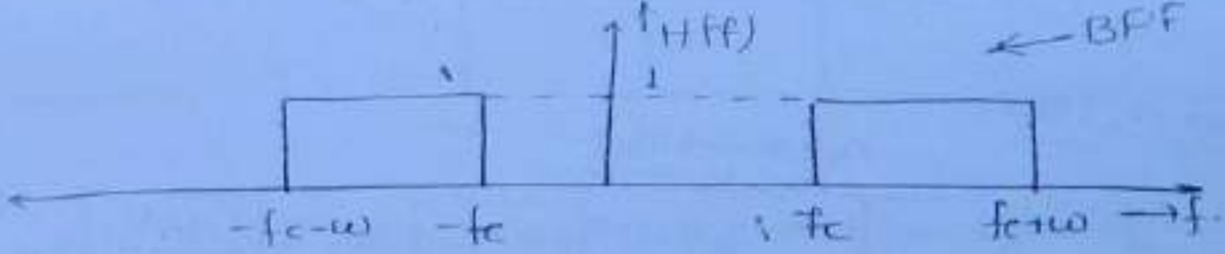
let $m(t) \longleftrightarrow$

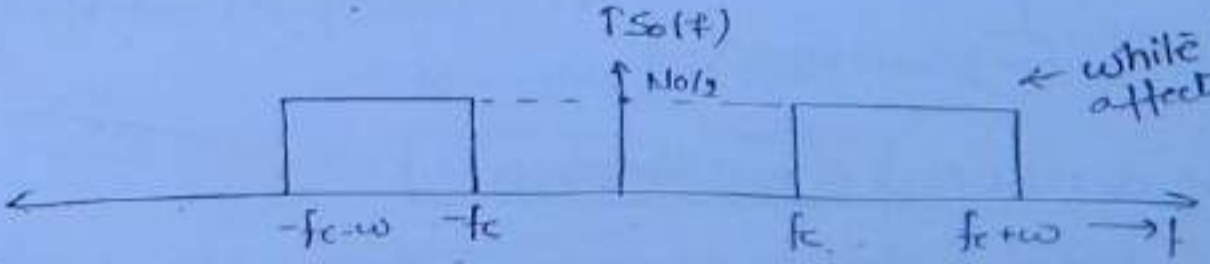


$c(t) = A\cos 2\pi f_c t$

So,
$S_{SSB}(t) \longleftrightarrow$







← white Noise power affecting SSB.

So, white Noise power affecting SSB = $N = W \times N_0/2 + W \times N_0/2$

$$\boxed{N = N_0W \text{ watts}}$$

* NARROW BAND NOISE.

3.06

* When white Noise is passed through BPF the Resu ting is said to be Narrow Band Noise.

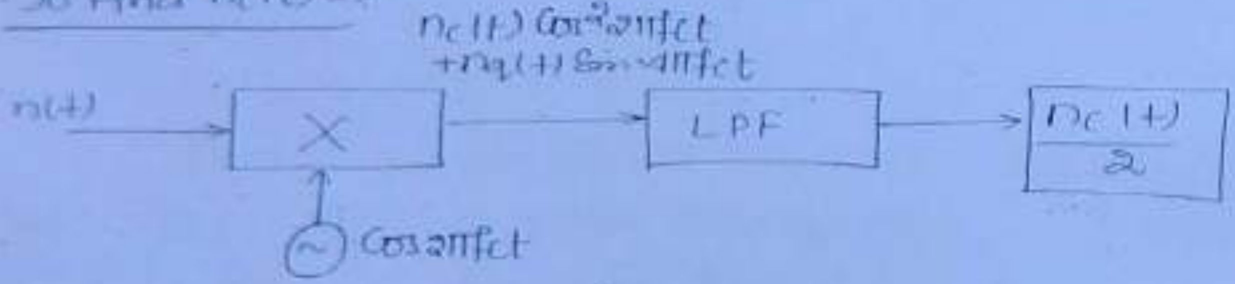
* To find effect of white Noise on AM, DSB and SSB, Narr Band Noise has to be considered.

$$\boxed{n(t) = \text{IFT}\,[S_0(f)]}$$

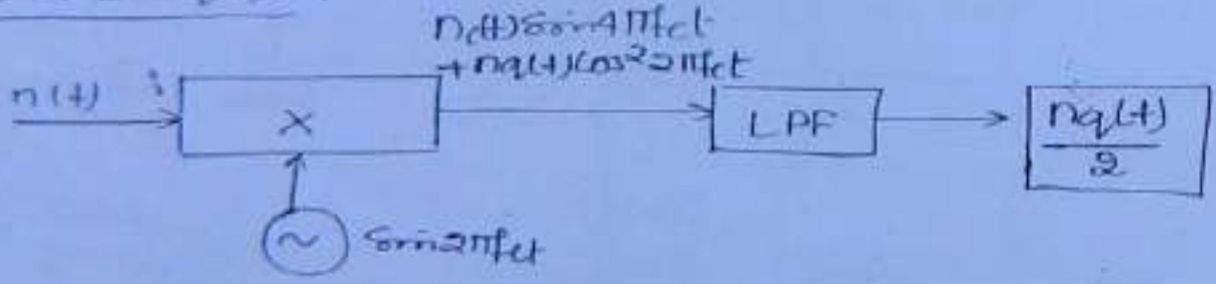
* Analysis:

$$\underset{\substack{\text{time domain} \\ \text{Narrow Band} \\ \text{Noise}}}{n(t)} = \overset{\substack{\text{In phase} \\ \text{component}}}{n_c(t)} \cos 2\pi f_c t + \overset{\substack{\text{quadrature} \\ \text{component}}}{n_q(t)} \sin 2\pi f_c t$$

1. To find $n_c(t) = ?$



2. To find $n_q(t) = ?$



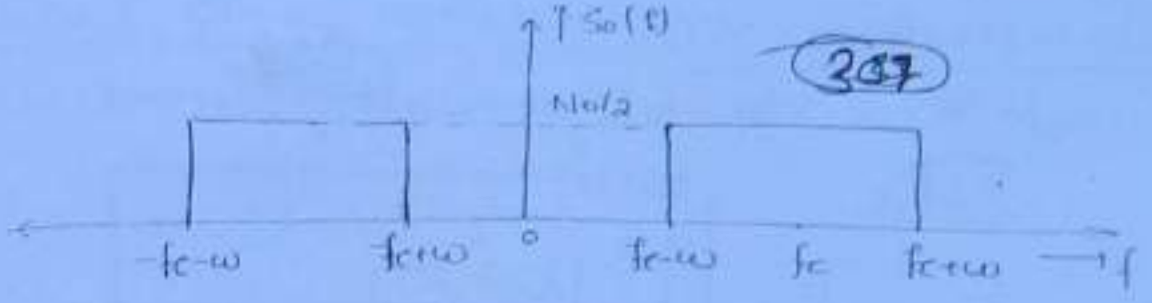
* Effect of $n_c(t)$ & $n_q(t)$ $\overset{\text{PSD.}}{\wedge}$ on AM, DSB:

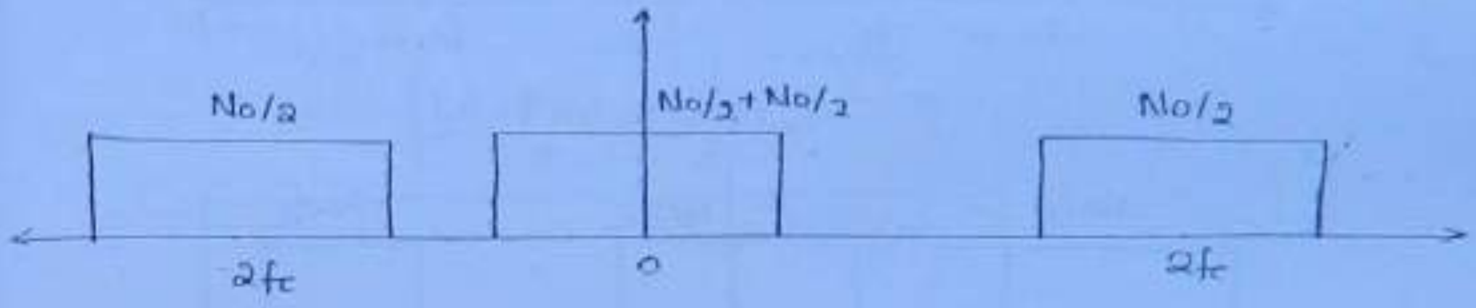
1) by $n_c(t)$:-

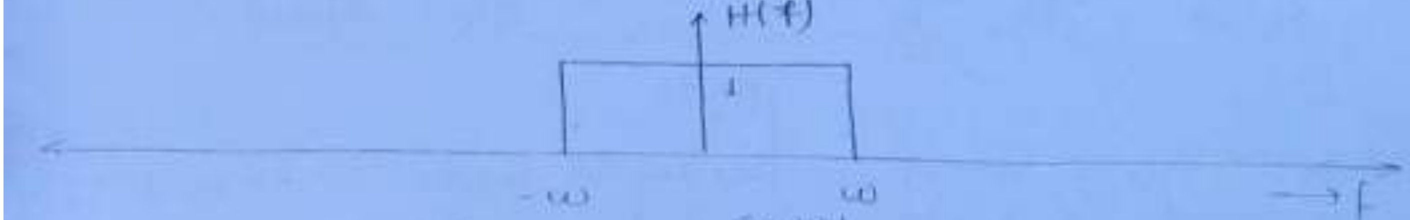
$$n(t) \longleftrightarrow S_0(f)$$

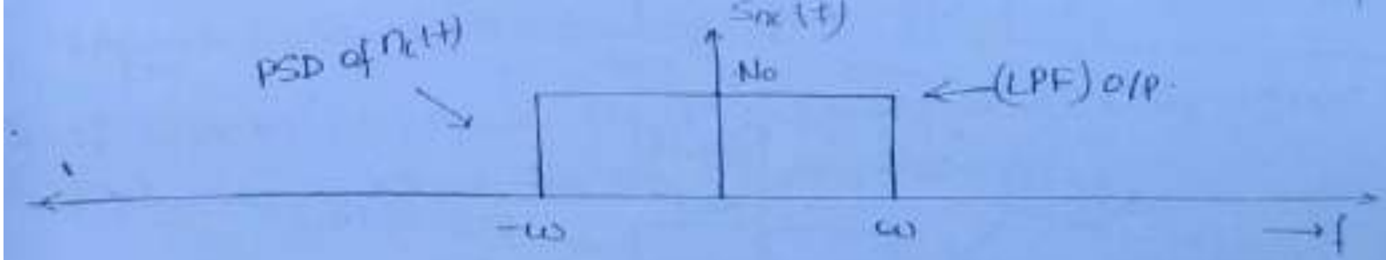
when multiplied by $\cos 2\pi f_c t$, the $S_0(f)$ is shifted left & right by amount f_c

$$S_0\,(O/P)\text{mul} \longrightarrow \frac{S_0(f+f_c) + S_0(f-f_c)}{2}$$









So, while Noise power affecting AM and DSB due to its inphase component; N

$$N = \text{Area}[S_{nc}(f)]$$

$$\boxed{N = 2N_0 W \text{ watts}}$$

* Due to the whole component of while Noise is also

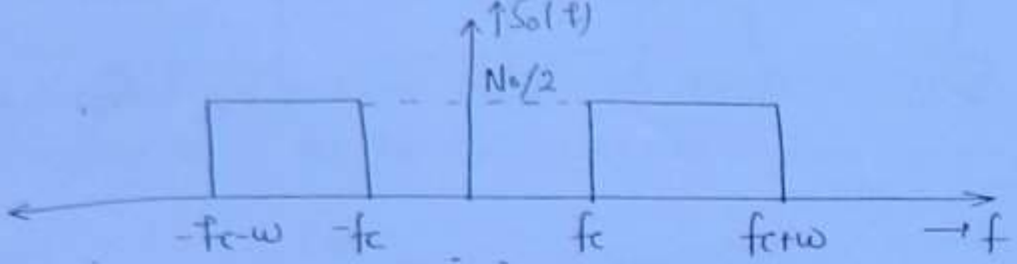
$$N = 2N_0 W.$$

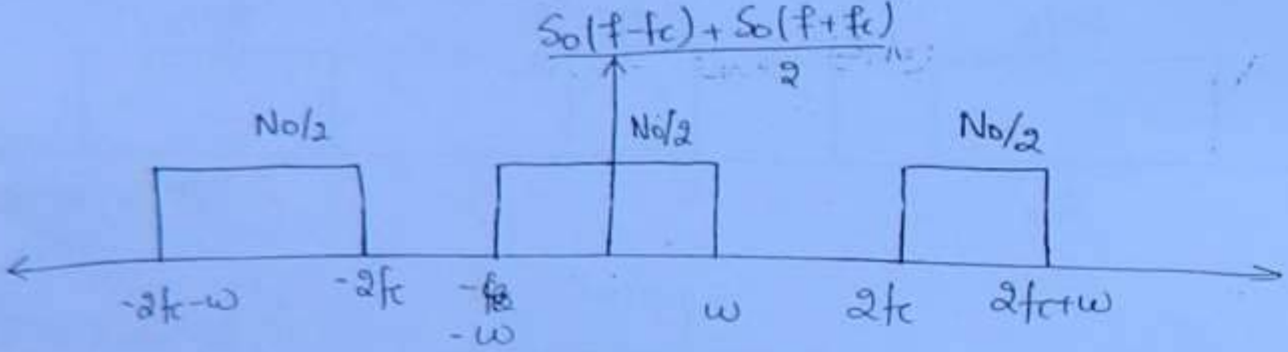
Conclusion:

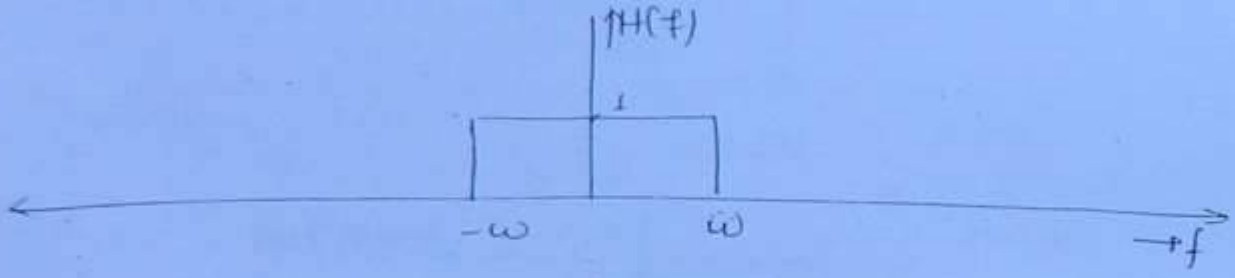
The effect of while Noise on AM and DSB is only due to its inphase components, so that the effect of quadrature component will be null (ie 0).

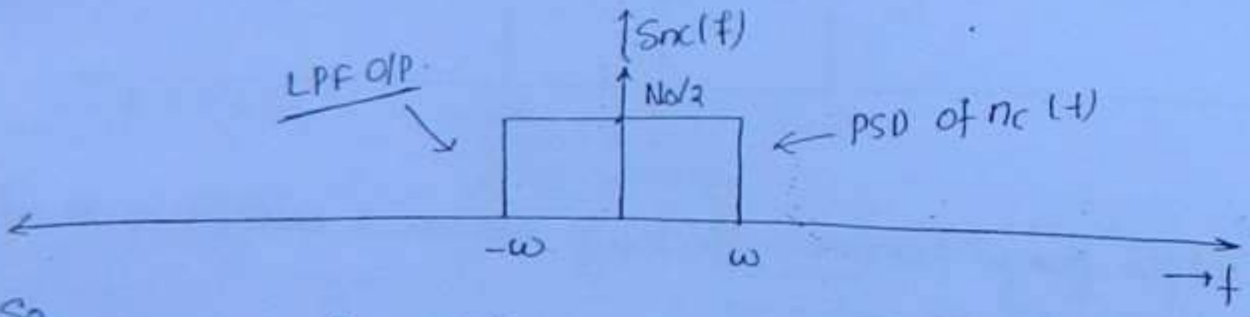
× PSD of n(t) affecting SSB:

× PSD of white Noise affecting SSB is given as:









So,

Power = Area [Snc(f)]

$\boxed{Power = N_0W}$

× Due to Actual white Noise, $\boxed{Power = N_0W}$

Conclusion:

The affect of Quadrature Component of white Noise on SSB will be NULL.

*FIGURE OF MERIT (F.O.M)

Mathematically F.O.M is given as:

$$\boxed{F.O.M = \frac{(S_o/N_o)}{(S_i/N_i)}}$$

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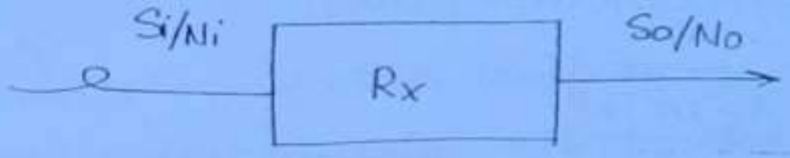


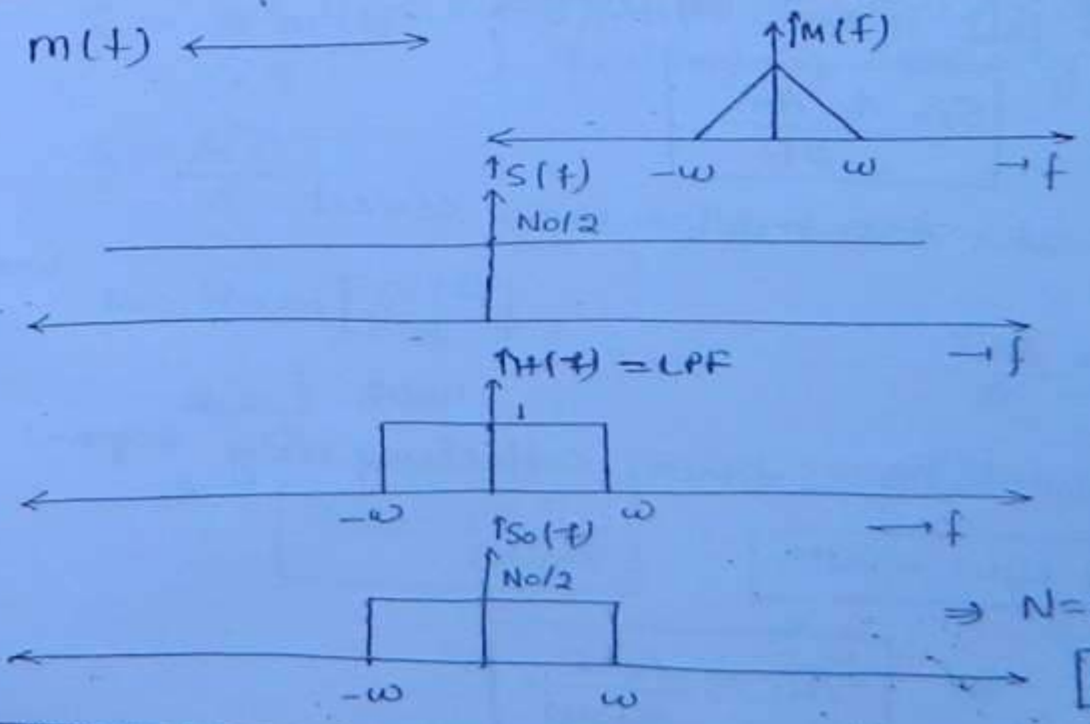
$$\boxed{\begin{array}{l} F.O.M > 1 \Rightarrow S_o/N_o > S_i/N_i \\ F.O.M < 1 \Rightarrow S_o/N_o < S_i/N_i \end{array}}$$

Note:

1. If F.O.M > 1, then Reciever is said to be very much efficient in decreasing the effect of Noise alone.
2. If F.O.M < 1, then Rx is its elf adding some amont of Noise, so that So/No is decreased.

*WHITE NOISE POWER AFFECTING MSG SIGNAL:

$m(t) \longleftrightarrow M(f)$



$\Rightarrow N = 2\omega \times N_o/2$

$\boxed{N = N_oW \text{ watts}}$

* FIGURE OF MERIT OF DSB RX

As,
$S_{DSB}(t) = A_c m(t) \cos 2\pi f_c t$

Power of Signal, $S_i = \frac{A_c^2 m^2(t)}{2R}$

as m(t) is changing, hence, it gives the Instantantaneous power.

So,
$$S_i = \frac{A_c^2 m^2(t)}{2}$$

let, $m^2(t)$ = Intantaneous power of $m(t) = P$

So, $S_i = \frac{A_c^2 P}{2}$ . . . . . . (A)

Now, let $m(t) = A_m \cos 2\pi f_m t$

So, $S_{DSB} = A_c A_m \cos 2\pi f_m t \cos 2\pi f_c t$

So, Power (DSB) = $\frac{A_c^2 A_m^2}{4R}$

Now, P of $m(t) = A_m^2/2R$

& put in eq^n (A) we get:

$$\boxed{S_i = \frac{A_c^2 A_m^2}{2R}}$$

Hence, the Assumption was correct

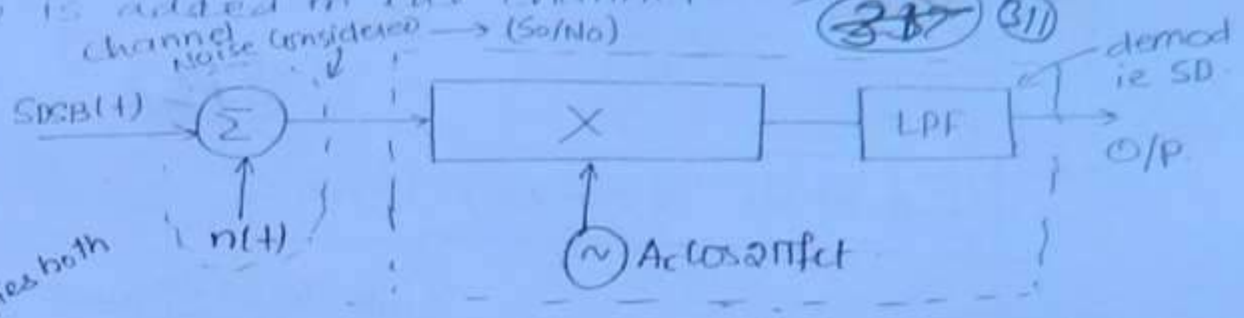
Now
$$S_i = \frac{A_c^2 P}{2}$$

let, N_i = white noise power affecting msg signal

So, $\boxed{N_i = N_0 W \text{ watts.}}$

So, $\boxed{S_i/N_i = \frac{A_c^2 P}{2 N_0 W}}$

* SDSB(t) is transmitted through channel and while Noise is added in the channel



amplifies both S&N.

* The ampl^r and other components donot affect the (S/N) but only demodulator affect the (S/N) Hence, it is taken into consideration only

So,

$$(\text{mul})\, o/p = \{s(t) + n(t)\} \cos 2\pi f_c t$$

$$= \{A_c m(t) \cos 2\pi f_c t + n_c(t) \cos 2\pi f_c t + n_q(t) \sin 2\pi f_c t\} \cdot \cos 2\pi f_c t$$

And,

$$(\text{LPF})\, o/p = \underbrace{\frac{A_c m(t)}{2}}_{\text{Signal}} + \underbrace{\frac{n_c(t)}{2}}_{\text{Noise}}$$

So,

$$S_0 = \text{Power}\left[\frac{A_c m(t)}{2}\right]$$

$$S_0 = \frac{A_c^2 m^2(t)}{4}$$

{∵ $A_c m(t)$ is either A·C or D·C is not specified. So, Squaring gives Power}

$$S_0 = \frac{A_c^2 P}{4}$$

And,

$$N_0 = \text{Power}\left[\frac{n_c(t)}{2}\right]$$

$$N_0 = \frac{1}{4} \cdot 2N_0 W$$

So,

$$\boxed{S_0/N_0 = \frac{A_c^2 P}{8 N_0 W}}$$

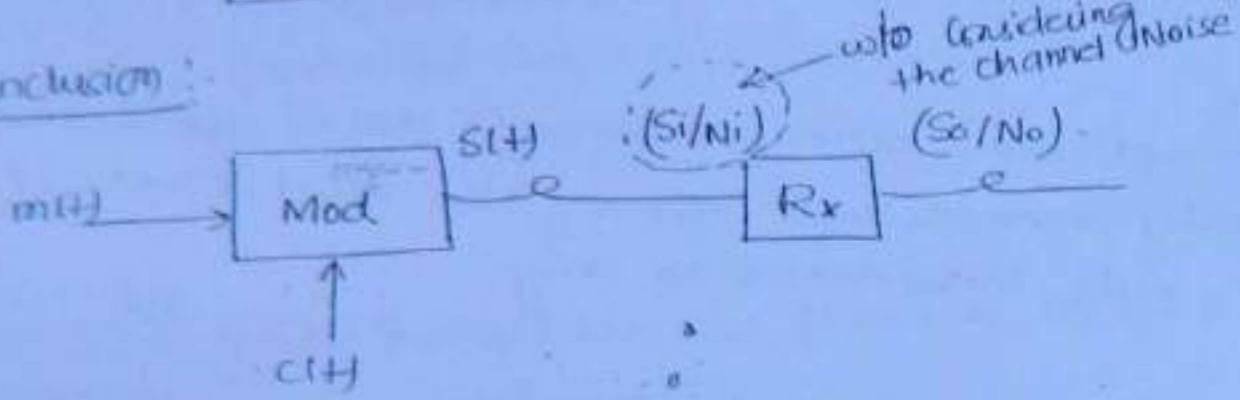
Now,

$$F.O.M = \frac{(S_o/N_o)}{(S_i/N_i)}$$

8.12

$$\boxed{F.O.M = 1 \Rightarrow (S_o/N_o) = (S_i/N_i)}$$

Conclusion:



* (Si/Ni) is calculated by considering the effect of Noise on the msg signal. But (Si/Ni) is to be calculated at the i/p of Rx by considering the effect of Noise on the modulated signal. But (So/No) = (Si/Ni); hence it may be concluded that the demodulator is eliminating the effect of channel Noise.

* Synchronous detector is working efficiently in nullyfying the white Noise affecting DSB signal in the channel.

* F.O.M of SSB Rx:

General exp of SSB is given as:

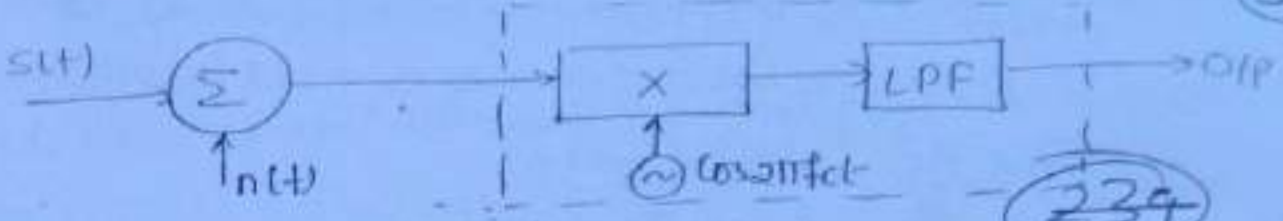
$$s(t) = \frac{A_c m(t)}{2}\cos 2\pi f_c t \mp \frac{A_c \hat{m}(t)}{2}\sin 2\pi f_c t$$

$$S_i = \frac{S_{DSB}}{2} \; \{\text{half of DSB power}\}.$$

$$S_i = \frac{A_c^2 P}{4}$$

N_i: Noise power affecting msg signal

$$\boxed{N_i = N_0 \omega \text{ watts}} \Rightarrow \boxed{S_i/N_i = A_c^2 P / 4 N_0 \omega}$$



~~234~~

$$(\text{mul})\, o/p = \{s(t) + n(t)\} \cos 2\pi f_c t$$

$$= \left\{ \frac{A_c m(t)}{2} \cos 2\pi f_c t \mp \frac{A_c \hat{m}(t)}{2} \sin 2\pi f_c t + n_c(t) \cos 2\pi f_c t + n_q(t) \sin 2\pi f_c t \right\} \cos 2\pi f_c t$$

$$(\text{LPF})\, o/p = \underbrace{\frac{A_c m(t)}{4}}_{\text{Signal}} + \underbrace{\frac{n_c(t)}{2}}_{\text{Noise}}$$

Now,

$$S_0 = \text{Power}\left\{\frac{A_c m(t)}{4}\right\}$$

$$S_0 = \frac{A_c^2 m^2(t)}{16}$$

$$\boxed{S_0 = \frac{A_c^2 P}{16}}$$

$$N_0 = \text{Power}\left\{\frac{N_c(t)}{2}\right\}$$

$$= \frac{1}{4} n_c^2(t)$$

$$N_0 = \frac{1}{4} \times N_0 \omega$$

$$\boxed{N_0 = N_0 \omega / 4}$$

So,

$$\boxed{\left(\frac{S_0}{N_0}\right) = \frac{A_c^2 P}{4 N_0 \omega}}$$

$$\therefore \boxed{\left(\frac{S_0}{N_0}\right) = (S_i/N_i)}$$

So, $\boxed{\text{F.O.M} = 1}$

* F.O.M OF AM Rx:

General exp of AM signal is given as

(314)

$$S_{AM}(t) = A_c\{1+K_a m(t)\}\cos 2\pi f_c t$$

$$= A_c\cos 2\pi f_c t + A_c K_a m(t)\cos 2\pi f_c t$$

So,

$$S_i = \frac{A_c^2}{2} + \frac{A_c^2 K_a^2 m^2(t)}{2}$$

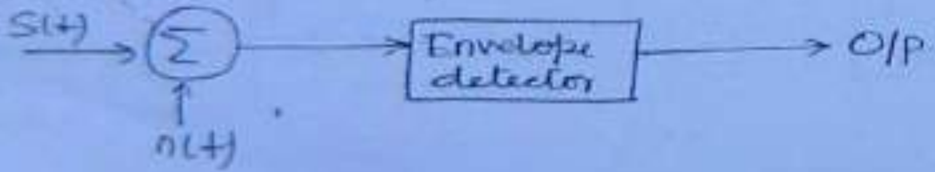
$$S_i = \frac{A_c^2}{2} + \frac{A_c^2 K_a^2 P}{2}$$

$$\boxed{S_i = \frac{A_c^2}{2}(1+K_a^2 P)}$$

Ni = Noise power affecting msg

$$\boxed{N_i = N_0W \text{ watts}}$$

So, $$\boxed{(S_i/N_i) = \frac{A_c^2}{2N_0W}(1+K_a^2 P)}$$



Now, $(ED)_{o/p} = \{s(t)+n(t)\}$.

$$= \{A_c\cos 2\pi f_c t + A_cK_a m(t)\cos 2\pi f_c t + n_c(t)\cos 2\pi f_c t + n_q(t)\sin 2\pi f_c t\}.$$

NOW, as

$$A\cos 2\pi f_0 t + B\sin 2\pi f_0 t \xrightarrow{E/D\ O/P} \sqrt{A^2+B^2}$$

$$(ED)_{o/p} = \{\underbrace{A_c + A_cK_a m(t) + n_c(t)}_{A}\}\cos 2\pi f_c t + \{\underbrace{n_q(t)}_{B}\}\sin 2\pi f_c t$$

So, $(ED)O/P = \sqrt{\{A_c + A_c K_a m(t) + n_c(t)\}^2 + \{n_s(t)\}^2}$

(3.15)

As, we know that the effect of quadrature component is 0

And, the amplr blocks D.C components also

So, $\boxed{(ED)O/P = \underbrace{A_c K_a m(t)}_{\text{Signal}} + \underbrace{n_c(t)}_{\text{Noise}}}$

Now,

$$S_0 = \text{Power}\{A_c K_a m(t)\} = A_c^2 K_a^2 m'(t) = A_c^2 K_a^2 P$$

$$N_0 = \text{Power}\{n_c(t)\} = 2N_0W$$

So,

$$\boxed{\frac{S_0}{N_0} = \frac{A_c^2 K_a^2 P}{2N_0W}}$$

Then,

$$F.O.M = \frac{(S_0/N_0)}{(S_i/N_i)}$$

$$= \frac{\cancel{A_c^2} K_a^2 P \times \cancel{2N_0W}}{\cancel{2N_0W} \times \cancel{A_c^2}(1 + K_a^2 P)}$$

$$\boxed{F.O.M = \frac{K_a^2 P}{(1 + K_a^2 P)}}$$

let,

$m(t) = A_m \cos 2\pi f_m t$

then $\text{Power}\{m(t)\} = P = A_m^2/2$

putting in above we get :-

$$F.O.M = \frac{\frac{K_a^2 A_m^2}{2}}{\left(1 + K_a^2 \frac{A_m^2}{2}\right)}$$

$$F.O.M = \frac{(K_a A_m)^2}{\{2+(K_a A_m)^2\}}$$

8/6a

$$\boxed{F.O.M = \frac{\mu^2}{2+\mu^2}}$$

* Also, $\boxed{\eta = \frac{\mu^2}{2+\mu^2} = FOM}$

Now,

$\mu = 0.5 \Rightarrow \eta = 0.11$

$\mu = 0.707 \Rightarrow \eta = 0.2$

$\mu = 1 \Rightarrow \eta = 1/3 = 0.33$

Conclusion:

FOM ↑ as μ ↑

$$\boxed{(FOM)_{max} = 1/3 \text{ for } \mu = 1}$$

so,

$$\boxed{\left(\frac{S_o}{N_o}\right) = \frac{1}{3}\left(\frac{S_i}{N_i}\right)}$$

Note:

1. The performance of Envelope detector against chann
Noise is Poor.

No Imp

* F.O.M OF FM Reciever:

The F.O.M of FM Recievers is given by:-

$$\boxed{F.O.M = \frac{3K_f^2 P}{\omega^2}}$$

where,

K_f = freq^n sensitivity of FM mod

P = Power of m(t)

ω = msg B.W

let $m(t) = A_m \cos 2\pi f_m t$

$$P = \frac{A_m^2}{2} \quad \& \quad W = f_m$$

~~245~~ (317)

$$\text{So, } F.O.M = \frac{3K_f^2 \cdot A_m^2/2}{f_m^2}$$

$$F.O.M = \frac{3}{2}\left\{\frac{K_f A_m}{f_m}\right\}^2$$

$$= \frac{3}{2}\left\{\frac{\Delta f}{f_m}\right\}^2$$

$$\boxed{F.O.M = \frac{3}{2}\beta^2}$$

***For NBFM:**

For NBFM ; $\beta \leq 1$ (small)

$\beta_{max} = 1$

So $\boxed{F.O.M = 3/2 = 1.5}$

*let $\beta = 0.5 = 1/2$

$$\boxed{F.O.M = \frac{3}{2} \times \frac{1}{4} = 3/8 = 0.3}$$

Conclusion: FOM of NBFM is small.

*** For WBFM:**

For WBFM ; $\beta > 1$ (high) ~~{$\beta_{min} = 1$}~~

$$FOM = \frac{3}{2} \cdot \beta^2$$

As $\beta \uparrow$ FOM ↑

but $\beta \uparrow \rightarrow \uparrow B.W = 2(\beta\uparrow + 1) f_m$

Conclusion: So, Generally, the value of $\boxed{\beta \text{ is Restricted to } 10}$

$\boxed{\beta = 10 \quad , FOM = 150}$

WBFM is preferred over NBFM because of its high FOM.

Q1 For an FM, given

$$(S/N)_{o/p} = 30dB \; ; \; (S/N)_{i/p} = 20dB.$$

2M

Find the value of β.

Soln: $(S/N)_o = 30\,dB \Rightarrow 10\log_{10}(S/N)_o = 30$

$$(S/N)_{o/p} = 10^3 = 1000$$

$$(S/N)_i = 20dB \Rightarrow 10\log_{10}(S/N)_i = 20$$

$$(S/N)_i = 100$$

So,

$$\frac{(S/N)_o}{(S/N)_i} = 10 = \frac{3}{2}\beta^2$$

$$\beta = \sqrt{\frac{20}{3}}$$

$$\boxed{\beta = 2.58}$$

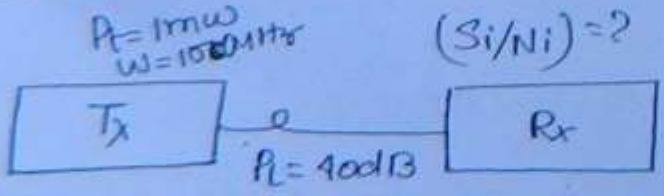
Q2 A Video Signal of having BW of 10MHz. power of 1mw is transmitted through a channel. Power loss in the channel is given by 40dB. Noise PSD is given by 10^{-20} watts/Hz. Find S/N at the i/p of the Reciever.

Soln: Given, B.W = 10MHz.

Power = 1mw.

Power loss in channel = 40dB.

Noise PSD ; No = 10^{-20} watts/Hz



As we know that,

$$N_i = N_0(W)$$

$$= 10^{-20} \times 100 \times 10^{6}$$

$$N_i = 10^{-12} \text{ watt}$$

If there is no path loss $\Rightarrow P_t = S_i$

But Path loss = 40dB (P_L).

$$\text{So, } S_i = (P_t - P_L)$$

$$(S_i)_{dB} = (P_t)_{dB} - (P_L)_{dB}$$

$$10\log_{10}(S_i) = 10\log_{10}(P_t) - 10\log_{10}(P_L)$$

$$\Rightarrow \cancel{10\log_{10}}(S_i) = \cancel{10\log}(P_t/P_L)$$

$$S_i = (P_t/P_L).$$

$$S_i = \frac{1\times10^{-3}}{(10^{4})}$$

$$S_i = 10^{-7} \text{ watts.}$$

then,
$$(S_i/N_i) = \frac{(10^{-7})}{(10^{-12})}$$

$$(S_i/N_i) = 10^{5}.$$

$$\boxed{\left(\frac{S_i}{N_i}\right)_{dB} = 50\text{dB}}$$

Q3 Audio Signal Band limited to 15KHz is transmitted through a channel after modulation. Power loss in the channel is given by 50dB. 2 sided Noise PSD is 10^{-10} watts/Hz. Find transmitted power required to get $(S/N)_{o/p}$ of 40 dB if the modulation scheme used is:-

a) DSB.

b) AM with $\mu = 1$.

c) FM with $\beta = 5$.

Solⁿ Given,

$B.W, W = 15KHz$

$P_l = 50dB$

$\frac{N_0}{2} = 10^{-10}$ watts/Hz

$(S/N)_{O/P} = 40dB$

~~246~~ 320

a) For DSB

$(S/N)_{O/P} = (S/N)_{I/P}$

So, $(S_i/N_i) = \left(\frac{S_i}{N_0 W}\right) = 40dB$

$$\frac{S_i}{(2\times10^{-10}\times15\times10^3)} = 10000$$

$S_i = 0.03$.

Now, $S_i = \frac{P_t}{P_L} = 0.03$

So $P_t = (0.03\times P_L)$

$= 0.03\times1\times10^5$

$\boxed{P_t = 3KW}$ Ans

b) For AM :-

$$F.O.M = \frac{\mu^2}{2+\mu^2} = 1/3$$

So, $\left(\frac{S}{N}\right)_{O/P} = \frac{1}{3}(S_i/N_i)$

So, $\left(\frac{S_i}{N_i}\right) = 3\times$ ~~40dB~~ 10^4

$S_i = 3\times10^4\times2\times10^{-10}\times15\times10^3$

$\frac{P_t}{P_L} = 0.09$

$P_t = 0.09 \times 10^{5}$

$\boxed{P_t = 9KW}$ Ans

) For FM:

35P

$FOM = \frac{3}{2} \beta^2$

for $\beta = 5$

$FOM = 37.5$

So, $(S_0/N_0) = 37.5 \times (S_i/N_i)$

$(S_i/N_i) = \frac{10^4}{37.5}$

$S_i = \frac{10^4}{37.5} \times 2 \times 10^{-10} \times 15 \times 10^3$

$\frac{P_t}{P_L} = \frac{1}{1250}$

$\boxed{P_t = 80W}$ Ans

24-11-20

*NOISE IN DIGITAL COMMUNICATION:

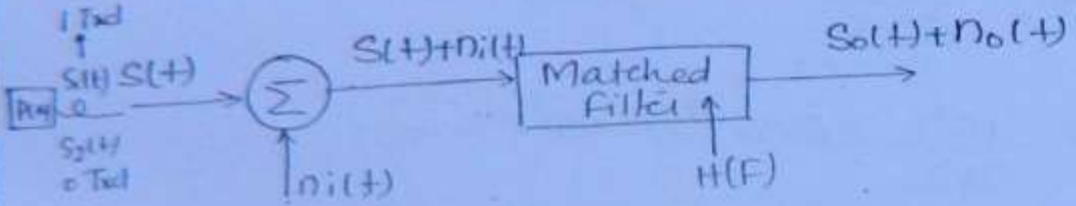
*MATCHED FILTER:

*Importance is there of Matched Filter is to reduce the Prob. of Error

Note:

*Matched Filter is used in digital Rx before threshold comparator

*It increases S/N↑ ratio so that Pe↓ will be decreased.



$$(SNR)_{O/P} = \frac{|S_0(t)|^2}{N_0} \; ; \; N_0 = \text{O/P Noise power}$$

*Matched Filter increases the SNR, so we have to calcula the characteristic of MF ie H(F)

*Let,

$n_i(t)$ = white Noise possessing Gaussian density func with '0' mean, and having 2 sided PSD of $N_0/2$ ω/Hz

Now,

$$S_0(t) = S(t) * h(t)$$

where,

$S(t)$ = Input Signal Power

$h(t)$ = Impulse Response of MF.

Taking FT we get:-

$$S_0(F) = S(f) \cdot H(f).$$

$$\& \; S_0(t) = \text{IFT}\{S_0(F)\}.$$

$$= \int_{-\infty}^{\infty} S_0(f) \cdot e^{j2\pi ft} df$$

$$\boxed{s_o(t) = \int_{-\infty}^{\infty} S(f)H(f)e^{j2\pi ft}\,df}$$

Now, N_o: O/P ~~of~~ Noise power

$$(\text{Noise PSD})_{O/P} = (\text{Noise PSD})_{I/P} \cdot |H(f)|^2$$

$$S_o(f) = \frac{N_o}{2} \cdot |H(f)|^2$$

So. O/P Noise power $\{N_o\}$ = Area$[S_o(f)]$.

$$N_o = \int_{-\infty}^{\infty} S_o(f)\,df$$

$$N_o = \int_{-\infty}^{\infty} \frac{N_o}{2} |H(f)|^2\,df$$

× (SNR) at a specific time instant of $t = T$ is given by:-

Then $$\boxed{(SNR)_{O/P} = \frac{|s_o(T)|^2}{N_o}}$$

So, $$\left(\frac{S}{N}\right)_o = \frac{\left|\int_{-\infty}^{\infty} S(f)H(f)e^{j2\pi fT}\,df\right|^2}{N_o}$$

× (S/N) depends upon H(f). Hence for H(f) value the $(S/N)_o$ reaches max^m has to be calculated

Now, according to schwartz's inequality

$$\left|\int_{-\infty}^{\infty} S(f)H(f)\right) \quad \left|\int_{-\infty}^{\infty} x_1(f)\,x_2(f)\,df\right|^2 \le \int_{-\infty}^{\infty}|x_1(f)|^2 df \cdot \int_{-\infty}^{\infty}|x_2(f)|^2 df$$

So, applying above to $(S/N)_o$ we get:-

$$\cancel{\left|\int_{-\infty}^{\infty} \underbrace{S(f)}_{x_1}\,\underbrace{H(f)e^{j2\pi fT}}_{x_2}\,df\right|^2} \le \int_{-\infty}^{\infty} |\cancel{S(f)}|^2 df$$

applying [illegible] in [illegible] inequality

$$\left|\int_{-\infty}^{\infty} \underbrace{H(f)}_{x_1} \underbrace{S(f)\, e^{j2\pi f T}}_{x_2} df\right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |S(f)\, e^{j2\pi f T}|^2 df$$

$$\left|\int_{-\infty}^{\infty} H(f)\, S(f) e^{j2\pi f T} df\right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |S(f)|^2 df$$

$$\left\{ \because |e^{j2\pi f T}|^2 = 1 \right\}$$

Provided, $\boxed{H(f) = S^*(f)\, e^{-j2\pi f T}}$

the equality Relation holds good.

Now,

$$\left(\frac{S}{N}\right)_0 = \frac{\left|\int_{-\infty}^{\infty} S(f)\, H(f)\, e^{j2\pi f T} df\right|^2}{\int_{-\infty}^{\infty} \frac{N_o}{2} |H(f)|^2 df}$$

$$\left(\frac{S}{N}\right)_0 \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_o}{2} \int_{-\infty}^{\infty} |H(f)|^2 df}$$

provided. $H(f) = S^*(f)\, e^{-j2\pi f T}$, the equality Relation holds good.

So,

$$\left(\frac{S}{N}\right)_{0\,max} = \frac{\cancel{\int_{-\infty}^{\infty} |H(f)|^2 df} \cdot \int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_o}{2} \cancel{\int_{-\infty}^{\infty} |H(f)|^2 df}}$$

$$\left(\frac{S}{N}\right)_{0\,max} = \int_{-\infty}^{\infty} \overset{\text{ESD of } s(t)}{|S(f)|^2} df \Big/ (N_o/2)$$

$$\boxed{\left(\frac{S}{N}\right)_{0\,max} = E/(N_0/2)}$$

where, E = Energy of s(t)

So,

$$\boxed{\left(\frac{S}{N}\right)_{max} = (2E/N_0)}$$

$$\left(\frac{S}{N}\right) = E/(N_0/2)$$

S → mapped to Energy of Input signal

N → mapped to Input PSD of Input $n_i(t)$

Note:

For max (S/N) is corresponding to the Ratio of Input Signal Energy and Input Noise PSD.

Imp ✱✱

✱ Impulse Response of Matched filter.

As, $h(t) = IFT[H(f)]$

$$= \int_{-\infty}^{\infty} H(f) e^{+j2\pi ft} df$$

$$= \int_{-\infty}^{\infty} S^*(f) e^{-j2\pi fT} \cdot e^{+j2\pi ft} df$$

Now, if s(t) is Real, then $S^*(f) = S(-f)$.

So,

$$h(t) = \int_{-\infty}^{\infty} S(-f) e^{-j2\pi fT} \cdot e^{j2\pi ft} df$$

let $-f \to f$

$$h(t) = \int_{\infty}^{-\infty} S(f) e^{j2\pi fT} \cdot e^{-j2\pi ft} (-df)$$

$$\because -\int_a^b = \int_b^a$$

So,

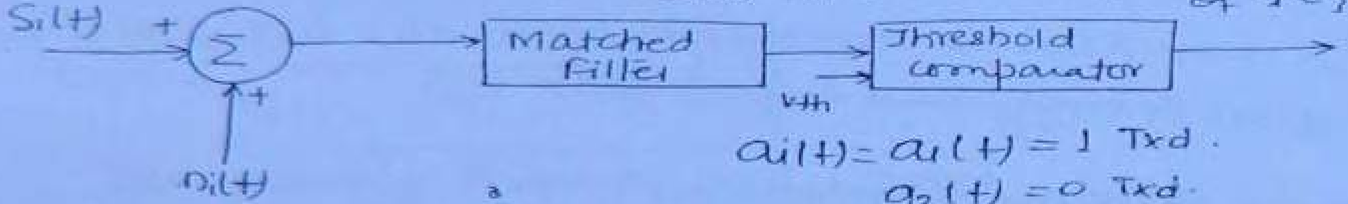
$$h(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi fT} \cdot e^{-j2\pi ft} df$$

$$h(t) = \int_{-\infty} S(f)\, e^{j2\pi f(T-t)}\, df$$

So, $\boxed{h(t) = s(T-t)}$ ***

* Probability Error of digital Signalling schemes:

$z(t) = a_i(t) + n_o(t)$ { $z \rightarrow$ input voltage expelled as input of T.C }



$a_i(t) = a_1(t) = 1$ Txd.

$a_2(t) = 0$ Txd.

$S_i(t) = s_1(t) \longrightarrow$ binary '1' was Txd.

$s_0(t) \longrightarrow$ binary '0' was Txd

* Assume $n_i(t)$ corresponds to white Noise of having 2 sided PSD $N_0/2$, and possessing Gaussian density func^n with 0 mean.

Case 1:- Assume no signal component was transmitted by the Tx. $(a_i(t) = 0)$.

So, $z(t) = n_o(t)$

$z = n_o$.

So, $E[z] = E[n_o]$.

$E[z] = 0$ { $\because$ mean of n_i is 0, hence mean of n_o is also 0. }.

So, $f(z) = \dfrac{1}{\sqrt{2\pi\sigma^2}}\, e^{-(z-a)^2/2\sigma^2}$ { $\because$ Gaussian density func^n }.

$\sigma^2 \leftarrow$ variance of z = AC power.

Now, $z = n_o$

variance[z] = variance[n_o]

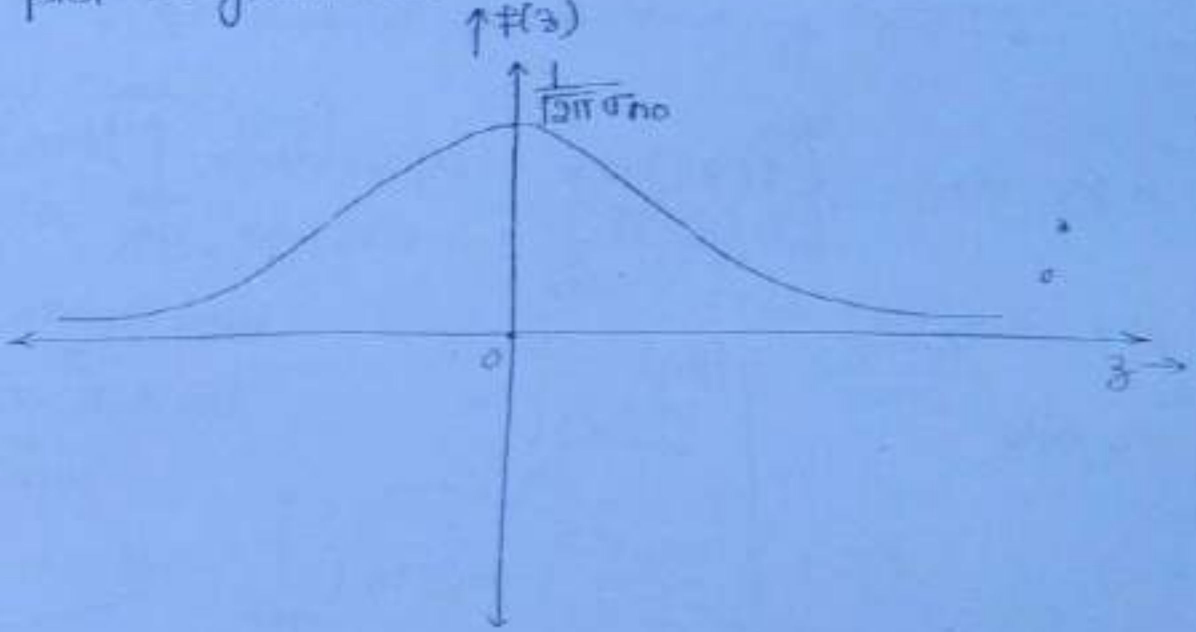
A-C power(z) = AC power(n_o).

So, let $\sigma = \sigma_{n0}$ = variance of noise

$$f(z) = \frac{1}{\sqrt{2\pi\sigma_{n0}^2}} e^{-\frac{(z-a)^2}{2\sigma_{n0}^2}}$$

So, $$f(z) = \frac{1}{\sqrt{2\pi\sigma_{n0}^2}} \cdot e^{-z^2/2\sigma_{n0}^2}$$ { ∵ mean = 0, a = 0 }

The plot is given as:-



* As the Noise is white Noise, so the strength of Noise is very small. So,

P(no small) = high.

p(no large) = low.

{ ∵ $P(X \le x) = \int_{-\infty}^{x} F_X(a) \cdot dx$.

Case 2:

Assume binary 1 was transmitted

So, $z = a_1(t) + n_0(t)$

$z = a_1 + n_0$

$E[z] = E[a_1 + n_0] = E[a_1] + E[\overset{0}{\cancel{n_0}}]$.

$E[z] = E[a_1] = a_1$

$E[z] = a_1$

The density function is given by:- ③②⑧

$$f(z/1) = \frac{1}{\sqrt{2\pi\sigma_{no}^2}} \cdot e^{-\frac{(z-a_1)^2}{2\sigma_{no}^2}}$$

{∵ σ a.c power
and a_1 is dc term
so a.c power of $a_1 = 0$
a.c power of $no = \sigma^2_{no}$}

Now,

Pe_1 = Prob of Rx 0 when 1 was Txd. = $P\{z < V_{th}\}$

Pe_0 = — " — 1 — " 0 — " — $= P\{z > V_{th}\}$

So,

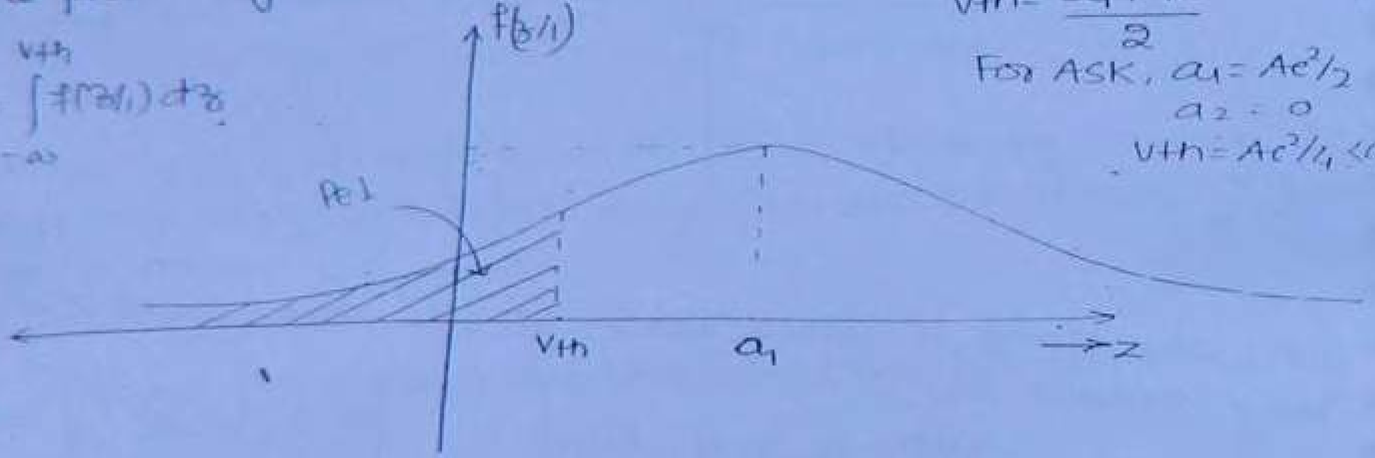
$$P\{z < V_{th}\} = \int_{-\infty}^{V_{th}} f(z/1)\,dz \;;\; P\{z > V_{th}\} = \int_{V_{th}}^{\infty} f(z/0)\,dz.$$

*The plot is given as:-

$$V_{th} = \frac{a_1 + a_2}{2}$$

For ASK, $a_1 = Ac^2/2$
$a_2 = 0$
$V_{th} = Ac^2/4 < a_1$

$$Pe_1 = \int_{-\infty}^{V_{th}} f(z/1)\,dz$$



Case 3: when Binary '0' was transmitted.

$Z = a_2 + no.$

$E[Z] = E[a_2] + 0.$

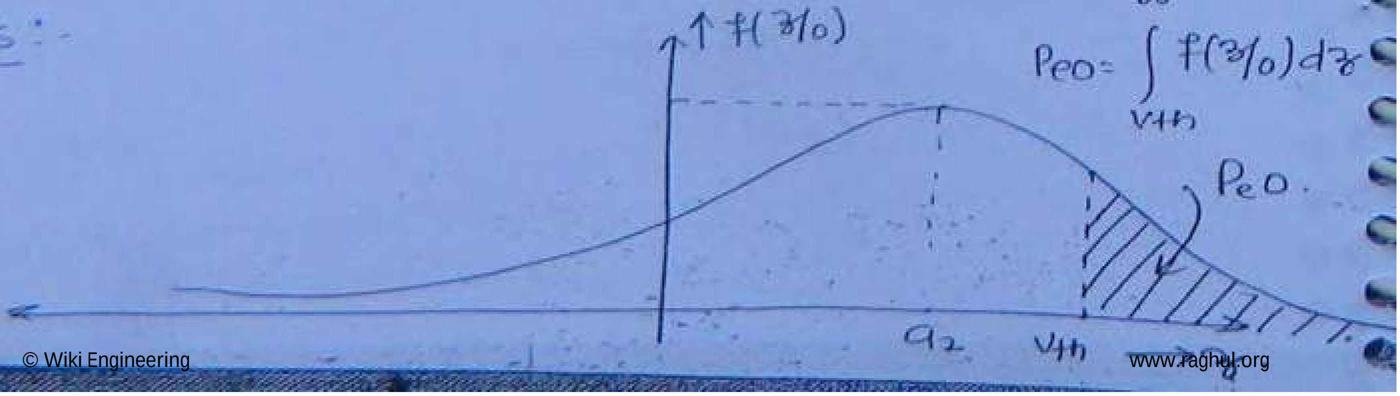
$E[z] = a_2$

So,
$$f(z/0) = \frac{1}{\sqrt{2\pi\sigma_{no}^2}} e^{-\frac{(z-a_2)^2}{2\sigma_{no}^2}}$$

$V_{th} = Ac^2/4 > a_2 = 0.$

*Plot is:-

$$Pe_0 = \int_{V_{th}}^{\infty} f(z/0)\,dz$$



Note:

1 when Binary 1 was transmitted, no error occurs if z>Vth

then $\boxed{P_{e1} = P(z<V_{th})}$ (329)

2. when Binary '0' was transmitted, no error occurs if z<Vth

$$\boxed{P_{e0} = P(z>V_{th}).}$$

* Assume the channel was Binary symmetric channel so that, $\boxed{P_{e1} = P_{e0}}$

So, $P_{e0} = P(z>V_{th}). = \int_{V_{th}}^{\infty} f(z/0)\,dz.$

$$= \int_{V_{th}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_{n0}^2}} \cdot e^{-\frac{(z-a_2)^2}{2\sigma_{n0}^2}} \cdot dz$$

$$P_{e0} = \int_{V_{th}}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_{n0}} \cdot e^{-\frac{(z-a_2)^2}{2\sigma_{n0}^2}} \cdot dz$$

$$P_{e0} = \frac{1}{\sqrt{2\pi}\,\sigma_{n0}^2} \int_{+V_{th}}^{\infty} e^{-\frac{(z-a_2)^2}{2\sigma_{n0}^2}} \cdot dz$$

Now as error func^n is given as:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-y^2/2}\,dy.$$

Now, let, $\frac{z-a_2}{\sigma_{n0}} = y$

$z - a_2 = \sigma_{n0}\, y.$

$dz = \sigma_{n0} \cdot dy$

$z = 0 \Rightarrow y = \infty.$

$z = V_{th} \Rightarrow y = \left(\frac{V_{th} - a_2}{\sigma_{n0}}\right)$

$= \frac{\frac{a_1+a_2}{2} - a_2}{\sigma_{n0}}$

$= \frac{(a_1-a_2)/2}{\sigma_{n0}}$

$\{y = (a_1-a_2)/2\sigma_{n0}\}$

So $P_e = P_{e_1} = P_e = \frac{1}{\sqrt{2\pi}\,\sigma_{no}} \int_{\frac{a_1-a_2}{2\sigma_{no}}} e^{-y^2/2}\, \sigma_{no}\, dy$

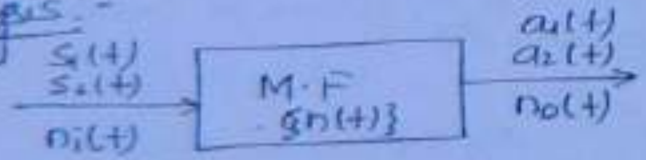
$$P_e = \frac{1}{\sqrt{2\pi}} \int_{\frac{a_1-a_2}{2\sigma_{no}}}^{\infty} e^{-y^2/2}\, dy$$

$$P_e = Q\left\{\frac{a_1-a_2}{2\sigma_{no}}\right\} \quad \left[\because Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-y^2/2}\, dy\right].$$

× The Plot of $Q(x)$ is given as:-

$$\text{Now, } P_e = Q\left\{\sqrt{\frac{(a_1-a_2)^2}{4\sigma_{no}^2}}\right\}$$

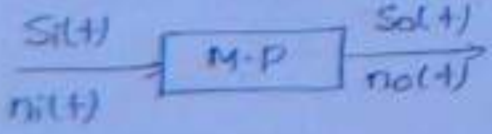
× Analysis:-



$$\text{Now } \frac{(a_1-a_2)^2}{\sigma_{no}^2} = \frac{\text{diff signal Power}}{\text{total Noise power}}$$

$$\because \sigma^2 = \underset{\text{(total power)}}{m_2} - \overset{\to 0}{\cancel{m_1}^2} \text{ (mean)}$$

×



$$\left(\frac{S}{N}\right)_o = \frac{|S_o(t)|^2}{N_o} = E/(N_o/2)$$

when, $h(t) = S(T-t)$.

Now, $\frac{(a_1-a_2)^2}{\sigma_{no}^2}$ ↘ $E_d/(N_0/2)$; Ed = diff Signal Energy
$h(t) = s(T-t)$ ie energy $\{S_1(t)-S_2(t)\}$

Now, $s(t) = S_1(t) - S_2(t)$ (331)

So, $$P_{emin} = Q\left[\sqrt{\frac{1}{4}\cdot\frac{E_d}{N_0/2}}\right]$$

$$\boxed{P_{emin} = Q\left[\sqrt{\frac{E_d}{2N_0}}\right]}$$

* From M.F opⁿ:-

$\frac{a_1-a_2}{\sigma_{no}^2}$ is maximised to $(E_d/N_0/2)$ so that P_e will be minimised

× P_e of ON-OFF SIGNALLING SCHEMES:

$1 \rightarrow S_1(t) = A_c$

$0 \rightarrow S_2(t) = 0V$

So, $$P_e = Q\left[\sqrt{\frac{E_d}{2N_0}}\right]$$

$$E_d = \text{Energy}\left[S_1(t) - S_2(t)\right]$$

$$= \int_0^{T_b} \{S_1(t) - S_2(t)\}^2 dt$$

$$= \int_0^{T_b} (A_c - 0)^2 dt$$

$$\boxed{E_d = A_c^2 T_b}$$

So, $$\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{2N_0}}\right]} = P_e = Q[x_1]$$

* Pe of NRZ signalling scheme

$1 \rightarrow S_1(t) = A_c$

$0 \rightarrow S_2(t) = -A_c$

$$E_d = \int_0^{T_b} \{S_1(t) - S_2(t)\}^2 dt$$

$$= \int_0^{T_b} (2A_c)^2 dt$$

$$\boxed{E_d = 4A_c^2 T_b}$$

So, $P_e = Q\left[\sqrt{\frac{4A_c^2 T_b}{2N_0}}\right]$

$$\boxed{P_e = Q\left[\sqrt{\frac{2A_c^2 T_b}{N_0}}\right]} \Rightarrow P_e = Q[x_2]$$

* As $\boxed{x \uparrow \rightarrow Q[x] \downarrow}$

Conclusion:

As $x_2 > x_1$

So, $\boxed{P_e(NRZ) < P_e(ON\text{-}OFF)}$

* Pe OF ASK:

$1 \rightarrow S_1(t) = A_c \cos 2\pi f_c t$

$0 \rightarrow S_2(t) = 0$

So, $E_d = \int_0^{T_b} \{S_1(t) - S_2(t)\}^2 dt$

$$= \int_0^{T_b} \{A_c \cos 2\pi f_c t\}^2 dt$$

$$= \int_0^{T_b} \frac{A_c^2}{2} dt + \frac{A_c^2}{2} \int_0^{T_b} \cancelto{0}{\cos 4\pi f_c t}\, dt$$ {∵ fc is integer multiple of complete cycles}

$$E_d = \frac{A_c^2 T_b}{2}$$

So, $P_e = Q\left[\sqrt{\frac{E_d}{2N_0}}\right]$ (333)

$$\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{4N_0}}\right]}$$

***P_e OF PSK:**

$1 \rightarrow s_1(t) = A_c \cos 2\pi f_c t$

$0 \rightarrow s_2(t) = -A_c \cos 2\pi f_c t$

$$\text{So, } E_d = \int_0^{T_b} \{s_1(t) - s_2(t)\}^2 dt$$

$$= \int_0^{T_b} \{2A_c \cos 2\pi f_c t\}^2 dt$$

$$= \cancel{2A_c} \int_0^{T_b} 4A_c^2 \cos^2 2\pi f_c t \, dt$$

$$= \frac{4A_c^2}{2} T_b + \frac{4A_c^2}{2} \int_0^{T_b} \cancelto{0}{\cos 4\pi f_c t} \, dt$$

$(E_d = 2A_c^2 T_b)$

So, $P_e = Q\left[\sqrt{\frac{E_d}{2N_0}}\right]$

$$\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{N_0}}\right]}$$

***P_e OF FSK:**

$1 \rightarrow s_1(t) = A_c \cos 2\pi f_1 t$ $(f_1 > f_2)$.

$0 \rightarrow s_2(t) = A_c \cos 2\pi f_2 t$

$$E_d = \int_0^{T_b} \{s_1(t) - s_2(t)\}^2 dt$$

$$E_d = \int_0^{T_b} (A_c \cos 2\pi f_1 t - A_c \cos 2\pi f_2 t)^2 dt$$

$$= \int_0^{T_b} A_c^2 \cos^2 2\pi f_1 t \, dt + \int_0^{T_b} A_c^2 \cos^2 2\pi f_2 t \, dt$$

$$- 2 \int_0^{T_b} A_c^2 \cos 2\pi f_1 t \cos 2\pi f_2 t \, dt$$

$$= \frac{A_c^2}{2} \int_0^{T_b} dt + \frac{A_c^2}{2} \int_0^{T_b} \underbrace{\cos 4\pi f_1 t}_{\to 0} \, dt + \frac{A_c^2}{2} \int_0^{T_b} dt + \frac{A_c^2}{2} \int_0^{T_b} \underbrace{\cos 4\pi f_2 t}_{\to 0} \, dt$$

$$- 2 \frac{A_c^2}{2} \int_0^{T_b} \left\{ \underbrace{\cos 2\pi (f_1 + f_2) t}_{\to 0} + \cos 2\pi (f_1 - f_2) t \right\} dt$$

$$E_d = \frac{A_c^2 T_b}{2}$$

So, $P_e = Q\left[\sqrt{\frac{E_d}{2N_0}}\right]$

$$P_e = Q\left[\sqrt{T_b A_c^2 / 4N_0}\right]$$

$$\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{2N_0}}\right]}$$

~~$\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{4N_0}}\right]}$~~

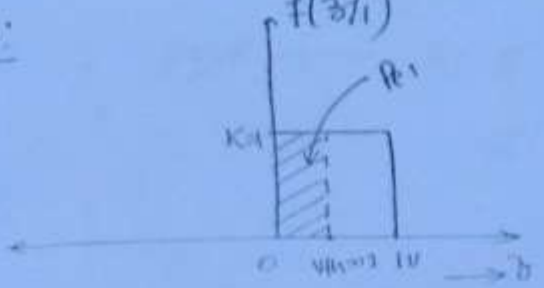
Conclusion:

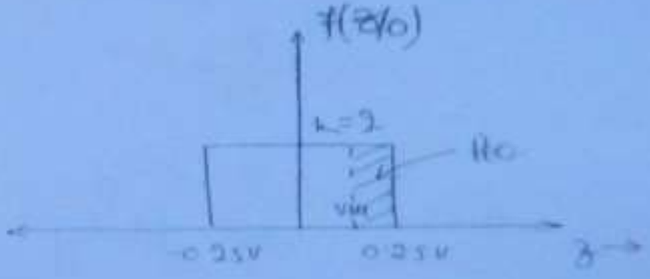
Signalling scheme	Pe
ASK	$Q\left[\sqrt{\frac{A_c^2 T_b}{4N_0}}\right]$
FPSK	$Q\left[\sqrt{\frac{A_c^2 T_b}{2N_0}}\right]$
PSK	$Q\left[\sqrt{\frac{A_c^2 T_b}{N_0}}\right]$

So, $\boxed{P_e(PSK) < P_e(FSK) < P_e(ASK)}$ 335

Q1 A Binary Tx is transmitting 2 possible binary symbols specified by 0 & 1. when Binary 1 was transmitted, the ~~[illegible]~~ signal voltage at the I/P of Threshold Comparator will be in b/w 0V & 1V with equal probability. when Binary 0 was transmitted signal voltage lies b/w −0.25V & 0.25V with equal probability. Threshold voltage is given by 0.2V. find avg. Pe?

Sol^n:





Now, $P_{e\,avg} = \frac{P_{e1} + P_{e0}}{2}$

∴ channel is not BSC:

$$P_{e1} = P(z < V_{th}) = \int_{-\infty}^{V_{th}} f(z/1)\,dz$$

$$= \int_{0}^{0.2} 1 \cdot dz = 0.2$$

Now,

$$P_{e0} = P(z > V_{th}) = \int_{V_{th}}^{\infty} f(z/0)\,dz$$

$$= \int_{0.2}^{0.25} 2\,dz = 0.1$$

So, $P_{e\,avg} = \frac{P_{e0} + P_{e1}}{2} = \frac{0.1 + 0.2}{2}$

$$\boxed{P_{e\,avg} = 0.15}$$

x Complementary Error Function, erfc(x) :-

Mathematically given as:

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-y^2}\, dy$$

&

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-z^2/2}\, dz$$; z = dummy variable.

let $\frac{z}{\sqrt{2}} = y$

$z = \sqrt{2}\, y$

$dz = \sqrt{2}\, dy$

$z = x \Rightarrow y = x/\sqrt{2}$

$z = \infty \Rightarrow y = \infty$

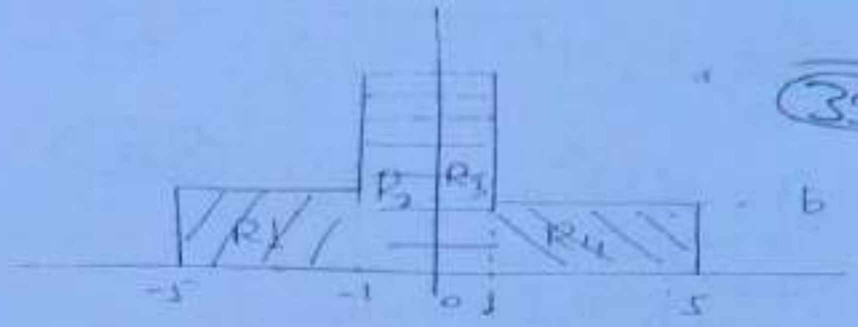
So,
$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x/\sqrt{2}}^{\infty} e^{-y^2} \cdot \sqrt{2}\, dy$$

$$Q(x) = \frac{1 \times 1 \times 2}{2\sqrt{\pi}} \int_{x/\sqrt{2}}^{\infty} e^{-y^2}\, dy$$

$$\boxed{Q(x) = \frac{1}{2} \operatorname{erfc}\left(x/\sqrt{2}\right)}$$

And,
$$\boxed{\operatorname{erfc}[x] = \frac{e^{-x^2}}{x\sqrt{\pi}}}$$

337



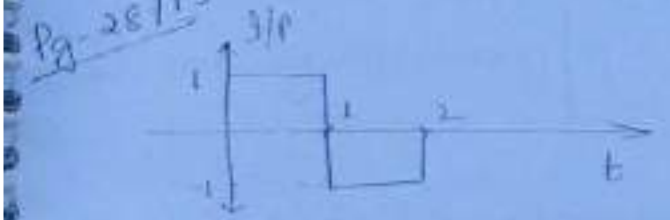
Prob(R1) = 1/4 = Area(R1)

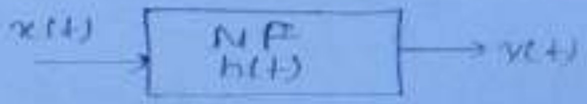
$\Rightarrow 4b = 1/4 \rightarrow \boxed{b = 1/16}$

Area(R3) = Prob(R2) = 1/4

$\boxed{a = 1/4}$

Pg-28/13





y(t) = x(t) * h(t)

h(t) = s(T - t)

when T is not given then take T = ~~a~~ total duration of given signal = 2

T=2

So h(t) = S(2 - t)

So, y(t) = x(t) * S(2 - t)

S(t) ⟶ S(2 - t)

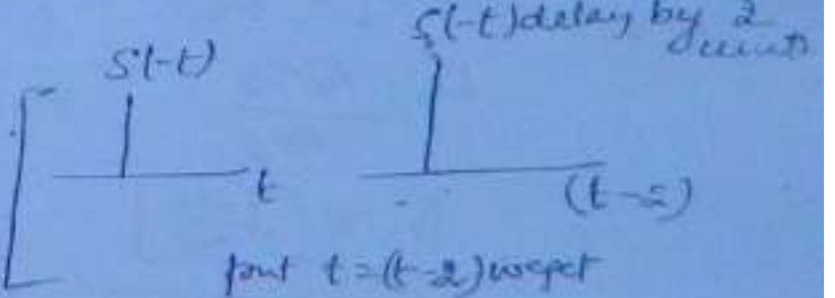
time Reversal

↓

S(-t)

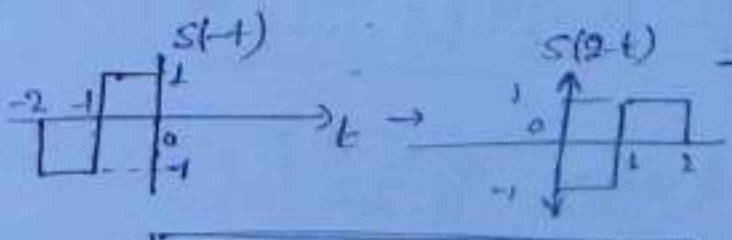
↓ delay by 2 unit

S(-(t-2)) = S(-t+2)



put t = (t-2) wrspt

S(-t+2)



$\boxed{\text{So, } y(t) = x(t) * S(2-t)}$

Reg

31/2 Given. $P_t = 1kW$ | Tx | — C — | Rx | $N = 10^{-4}$

1st model P_e

$P_L = 60dB$. $P_L = 60dB$

$$S_i = \frac{P_t}{P_L} = \frac{1 \times 10^3}{10^6} = 0.1mW$$ 338

$N = 10^{-4}$ watt.

$$P_e \text{ of ASK} = Q\left[\sqrt{\frac{A_c^2 T_b}{4N_0}}\right]$$

∴ No data is given regarding above.

And o/p of MF is matched to output energy & PSD of noi

$$S/N = E/(N_0/2)$$

Now,

$$\text{Energy/bit}, E_b = \frac{A_c^2 T_b}{2} \quad \& \quad P_e = Q\left[\sqrt{E_b/2N_0}\right].$$

So, $E_b \to S$

$\frac{N_0}{2} \to N$

So. $\boxed{P_e = Q\left[\sqrt{S/2 \cdot 2N}\right]}$; Before matched filter operation

② For PSK

$$P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{N_0}}\right]$$

$$E_b = \frac{A_c^2 T_b}{2}$$

$$P_e = Q\left[\sqrt{\frac{2E_b}{N_0}}\right]$$

$$= Q\left[\sqrt{\frac{2S}{2N}}\right] = \boxed{Q\left[\sqrt{S/N}\right] : P_e}$$

P_e 2nd model

③ $R_b = 2.5 \times 10^6$ bits/sec

$\frac{N_0}{2} = 10^{-20}$ watt/Hz

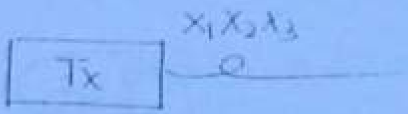
$A_c = 1\mu$volt

$$P_e \text{ of FSK} = Q\left[\sqrt{\frac{A_c^2 T_b}{2N_0}}\right]$$

$$= Q\left[\sqrt{\frac{1 \times 10^{-12} \times}{2.5 \times 10^6 \times 2 \times 2 \times 10^{-20}}}\right]$$

As $Q[x] = \frac{1}{2} \operatorname{erfc}\left[x/\sqrt{2}\right]$

× SOURCE CODING THEOREM



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$x_1 = 001 \rightarrow$ Code length $= 3$

$x_2 = 0010 \rightarrow$ code length $= 4$

$x_3 = 0011 \rightarrow$ Code length $= 4$

× how efficiently is the code working is calculated by the coding efficiency.

× Avg. code length $= L =$ bits/symbol.
Mathematically given as:-

$$L = \sum_i n_i P(x_i).$$

× So, coding efficiency, $\boxed{\eta = \frac{L_{min}}{L}}$

× According to the Source coding theorem $\Rightarrow \boxed{L \geq H}$

$$\boxed{\% \eta \uparrow \quad H / L \downarrow}$$

× If avg. code length is small, then it is called to be as that the coding efficiency is high.

$$\boxed{H = -\sum_i P(x_i) \log P\{x_i\}}$$

Q:

[illegible]
possible symbol with prob. of 0.3, 0.25, 0.2, 0.12, 0.08, 0.05
Find coding efficiency

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Solⁿ:

Step1 → Arrange all prob. in decreasing order

0.3
0.25
0.2
0.12
0.08
0.05

Step2 → divide the whole prob in such that the 2 sets have equiprobable probabilities (i.e almost close prob)

0.55	0.3	0
	0.25	0
0.45	0.2	1
	0.12	1
	0.08	1
	0.05	1

Note: Assign 0 to above set and 1 to the lower set or Vice-versa, but only one has to be followed.

Step3: Again divide the 2 set with the same process as in 2nd step

	0.3	0	0			→ $n_1 = 2$
	0.25	0	1			→ $n_2 = 2$
0.2	0.2	1	0			→ $n_3 = 2$
0.25	0.12	1	1	0		→ $n_4 = 3$
	0.08	1	1	1	0	→ $n_5 = 4$
	0.05	1	1	1	1	→ $n_6 = 4$

Step4: Repeat 3

Step 5:

$$L = \sum_{i=1}^{6} n_i \, p(x_i)$$

$$L = (2\times0.3 + 2\times0.25 + 2\times0.2 + 3\times0.12 + 4\times0.08 + 4\times0.05)$$

$$\boxed{L = 2.38 \text{ bits/symbol}}$$

(341)

Step 6:

$$H = -\sum_{i=1}^{6} p(x_i)\log\{p(x_i)\}$$

$$\boxed{H = 2.36 \text{ bits/symbol}}$$

So,

$$\% \eta = H/L$$

$$\% \eta = \frac{2.36}{2.38} \times 100$$

$$\boxed{\% \eta = 99.9\%}$$ Ans

Q2. Construct HOFFMAN coding for the above problem?

Soln: Step 1: Arrange in decreasing order

0.3
0.25
0.2
0.12
0.08
0.05

Step 2: Sum the last 2 prob

ie 0.08+0.05 = 0.13

0.3
0.25
0.2
0.12
0.08] — 0.13
0.05]

Step 3: by taking step2 in consideration again arrange prob in decreasing order

0.3	0.3
0.25	0.25
0.2	0.2
0.12	→ 0.13] — 0.25
0.08]	→ 0.12]
0.05]	

step 4 Repeat step 3

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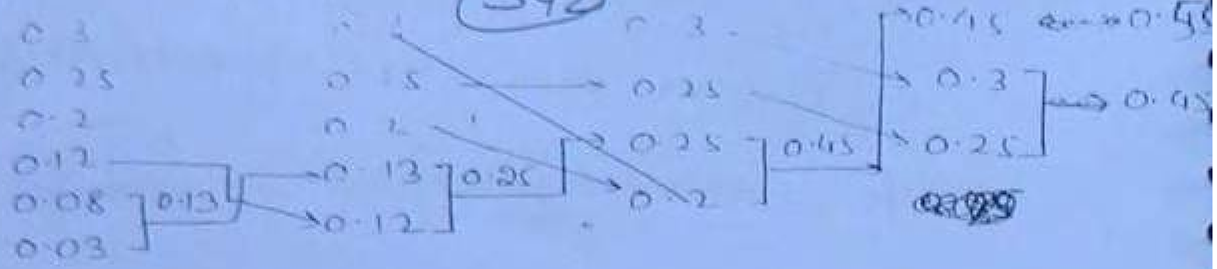
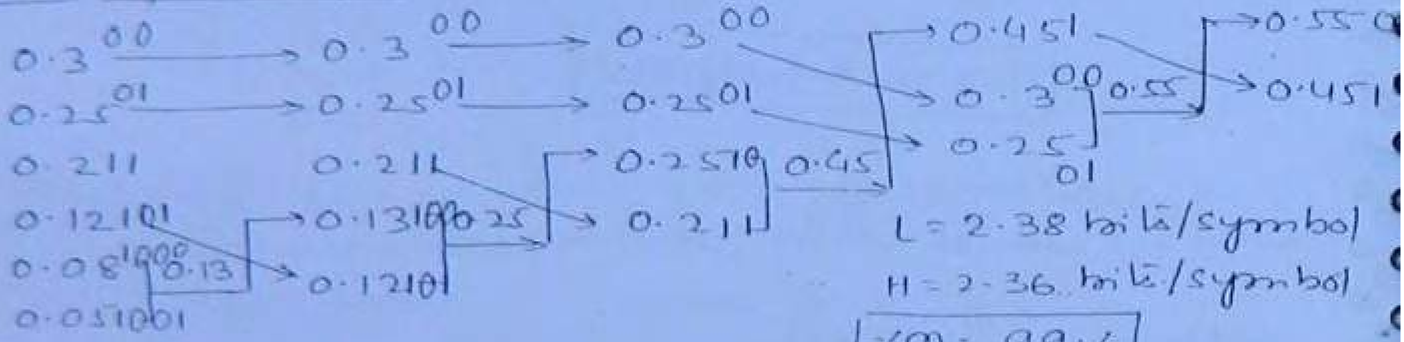


Step 4 Repeat step 3



L = 2.38 bits/symbol

H = 2.36 bits/symbol

$\boxed{\% \eta = 99\%}$

Step 5 · Associate '0' to all prob corresponding to 0.55 &
1 to prob corresponding to 0.45

Step 6: Move Backward and repeat above.

p34/1

1. i) $H = -\sum_{i=1}^{8} P(x_i) \log \{P(x_i)\}$.

$= 1.96$ bits/symbol.

ii) Prob. of occur of '0' $= (3 \times 1/2 + 2 \times 1/4 + 2 \times 1/8 + \cdots) \times 1/3$

Prob. of occuring of '1' $= 0.2$

iii) $\eta = \frac{H}{H_{max}} = \frac{1.96}{\log_2 8}$ | ∵ no coding technique was given, so $L = H_{max}$

$\eta = \frac{1.96}{3}$

$\eta = 65.3\%$

iv) by Shanon formu. [illegible]

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v) $\eta = 100\%$

Q2. $x_1 \updownarrow$ 0.5 0 $\quad n_1 = 1$

$x_2 \updownarrow$ 0.4 1 0 $\quad n_2 = 2$

x_3 0.1 1 1 $\quad n_3 = 2$

So, $L = 1 \times 0.5 + 2 \times 0.4 + 2 \times 0.1$

$= 0.5 + 0.8 + 0.1$

$= 1.4$ bits/symbol

$\eta = 90.7\%$

× 2nd order extension code:

	Pe
$x_1 x_1 \longrightarrow$	0.25
$x_1 x_2 \longrightarrow$	0.2
$x_1 x_3 \longrightarrow$	0.05
$x_2 x_1 \longrightarrow$	0.2
$x_2 x_2 \longrightarrow$	0.16
$x_2 x_3 \longrightarrow$	0.04
$x_3 x_1 \longrightarrow$	0.05
$x_3 x_2 \longrightarrow$	0.04
$x_3 x_3 \longrightarrow$	0.01

$\longrightarrow$ S.F Coding.

Q. Given $z = x + y$, where x and y are Random variables having density function in the form of Rectangular pulse. Density funcn of z will be:

a) Rectangular pulse
~~b)~~ Triangular pulse
c) Gaussian Pulse
d) None

$\because f(z) = f(x) * f(y)$

$f(z) =$ 

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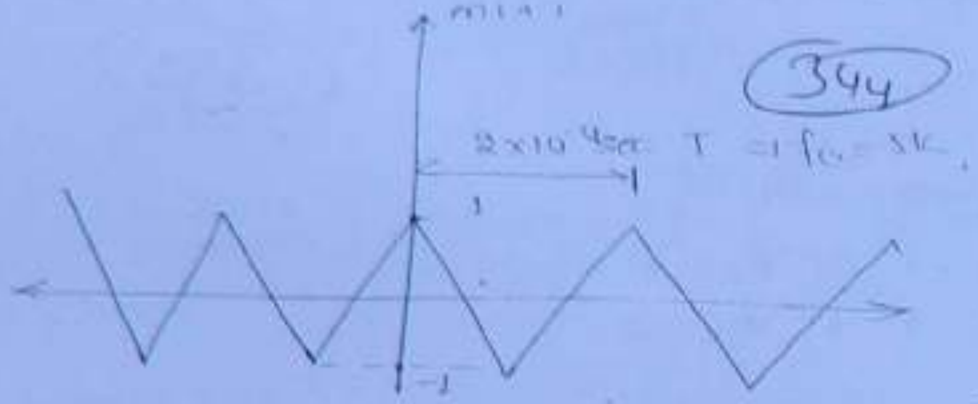


$$m(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn2\pi f_0 t}$$

upto 3rd Harmonic freq^n

$m(t) = 3f_0, 2f_0, 1f_0 \ldots$ } ... multi tone modulation }

$f_{max} = 15KHz \{3f_0\}$

Now,

$K_f = 2\pi\times10^5$; $K_p = 5\pi$

The units are not mentioned & π terms are involved hence,

$\omega_i = \omega_c + K_f m(t)$

but all the analysis was done for

$f_i = f_c + K_f m(t)$

So, $\frac{\omega_i}{2\pi} = \frac{\omega_c}{2\pi} + \frac{K_f m(t)}{2\pi}$

$\therefore K_f = \frac{2\pi\times10^5}{2\pi}$; $K_p = 5\pi/2\pi$

$= 1\times10^5$ $\quad K_p = 5/2$

$B_{FM} = 2(\Delta f + f_m)$

$= 2\{\frac{K_f A_m}{[illegible]} + f_m\}$

$= 2\{1\times10^5 \times 1 + 15\times10^3\}$

$= 230K$

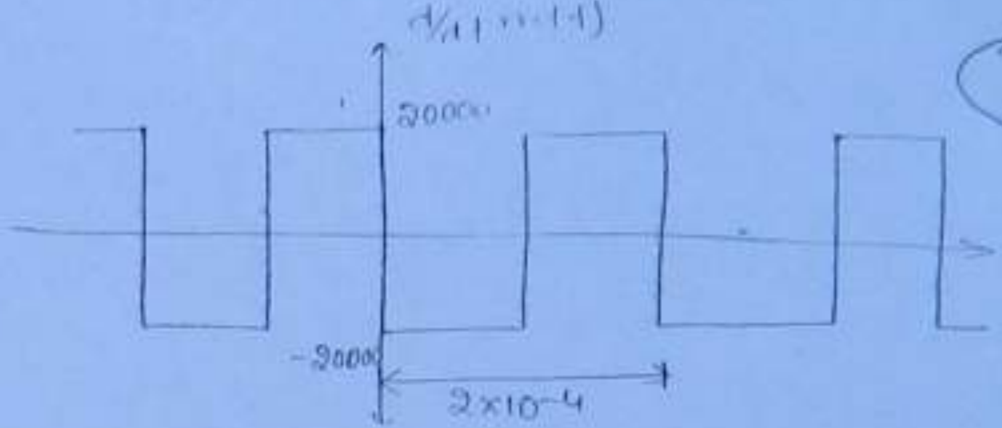
For PM, if the msg. is not sinusoidal, the formula of B.W & Power changes.

So,

PM of $m(t)$ = FM of $\frac{d}{dt}m(t)$.

B.W of PM of $m(t)$ = B.W of FM of $\frac{d}{dt}m(t)$

Now, $\frac{d}{dt} m(t) = \text{slope of } m(t) = \pm\frac{2}{10^{-4}} = \pm 20{,}000$



So, $B.W = 2\{\Delta f + f_{max}\}$

$= 2\{K_p A_m' + f_{max}\}$

If BW of PM is to be calculated by $\frac{d}{dt} m(t)$, then K_f replaced by K_p

$B.W = 2\{K_p A_m' + f_{max}\}$

$= 2\{\frac{5}{\pi} \times \overset{100000}{20000} + 15000\}$

$= 2 \times 55k$

$\boxed{B.W = 130k}$

) $B.W\uparrow = 2(\Delta f\overset{2}{\uparrow} + f_m)$.

$B.W\uparrow = 2(\Delta \phi\overset{2}{\uparrow} + 1) f_m$

Conclusion [illegible]

PAM:

Generation → AND gate

demod → LPF (Integrator)

PWM:

Generation → Monostable M.V

demod → LPF (Integrator).

PPM:

Generation $\longrightarrow$ PWM $\longrightarrow \boxed{\frac{d}{dt}} \longrightarrow \boxed{\text{clipper}} \longrightarrow$ PPM

demod $\longrightarrow$ PPM $\longrightarrow \boxed{\text{Bistable M.V}} \longrightarrow$ PWM.

* Granular Noise power: $\frac{\Delta^2}{3}$; Δ = Step Size

* $\boxed{\text{\% of modulation for FM} = \frac{\Delta f}{\Delta f_{max}} \quad ; \Delta f_{max} = 75K \text{ standard.}}$

* For PAM → Roll off factor (α)

$$\boxed{B.W = \frac{Rb}{2}(1+\alpha)}$$