De Broglie wave

Photoelectric Effect and Compton Effect showed that a wave could behave like a particle and could make collision with other particle like a billiard ball collision. Davisson and Germer Experiment established that a moving particle could behave like a wave.

$$E=h\nu=hc/\lambda=pc$$

$$h/\lambda=p$$

$$\lambda=h/p$$

Ψ is complex

$$\Psi = A + i\,B$$

$$\Psi^* = A - i\,B$$

$$|\Psi|^2 = A^2 + B^2$$

The probability density $|\psi|^2$ is given by the product $\psi^*\psi$

De Broglie Wave Speed

What is the speed of de Broglie wave?

Since a de Broglie wave is associated with a moving body this wave should travel at the same speed as that of the body.

We may quote the usual formula as following:

$$\lambda = \frac{h}{mv}$$

$$E = hv = mc^2$$

$$v = \frac{mc^2}{h}$$

$$w = v\lambda = \frac{mc^2}{h}\frac{h}{mv} = \frac{c^2}{v}$$

So, the de Broglie wave has speed greater than the moving body !

Is it OK?

How to represent a wave associated with a moving particle?

Can the following equation of a plain progressive wave represent de Broglie wave associated with a moving particle?

$$y = A \sin(\omega t - kx)$$

A way of representing a wave group is in terms of a series of individual waves differing slightly in wavelength, whose interference with one another results in the variation of amplitude that defines the group shape.

$$y_1 = A\cos(\omega t - kx)$$

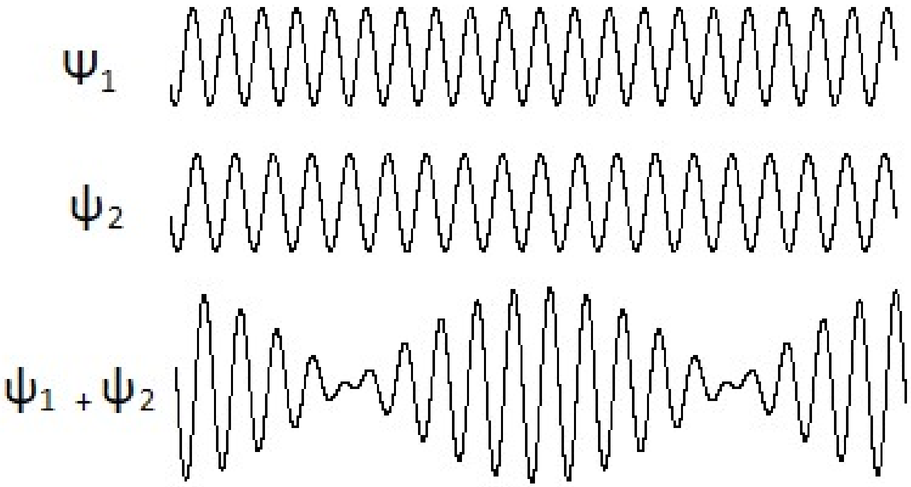
$$y_2 = A\cos[(\omega+d\omega)t - (k+dk)x]$$

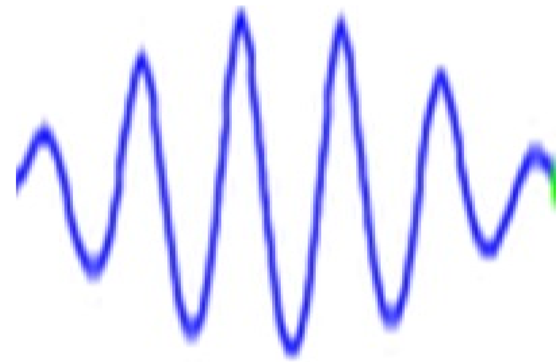
$$y = 2A\cos(\omega t - kx)\cos(d\omega/2 - dk/2)$$

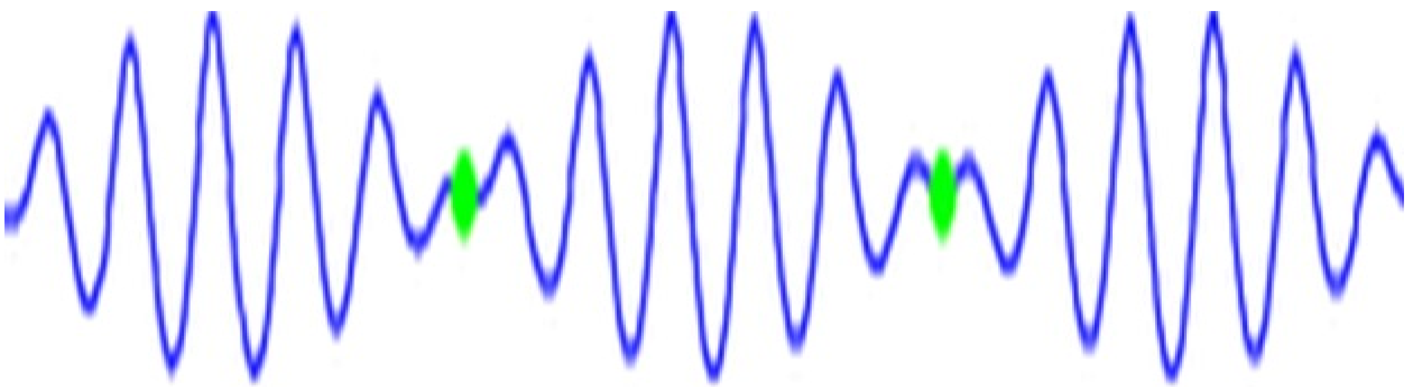
Wave velocity $w = \omega/k$; Group velocity $u = d\omega/dk$

$$w = \nu\lambda \qquad \lambda = \frac{h}{mv} \qquad E = h\nu = mc^2 \qquad \nu = \frac{mc^2}{h}$$

$$w = \nu\lambda = \frac{mc^2}{h}\frac{h}{mv} = \frac{c^2}{v}$$


ψ_1
ψ_2
$\psi_1 + \psi_2$





$$\omega = 2\pi\nu = \frac{2\pi mc^2}{h} = \frac{2\pi mc^2}{h\sqrt{1-\frac{v^2}{c^2}}}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h} = \frac{2\pi m_o v}{h\sqrt{1-\frac{v^2}{c^2}}}$$

$$w = \frac{\omega}{k} = \frac{c^2}{v}$$

$$u = \frac{d\omega}{dk} = \frac{d\omega}{dv} \bigg/ \frac{dk}{dv} = v$$

Properties of Wave Function Ψ

- Ψ contains all the information about the particle.
- Ψ summed over all space must be 1.

$$\int_{+\infty}^{-\infty} |\Psi|^2 \, dx = 1$$

- Must be a solution of the Schrodinger equation.
- Must be a continuous function of x.
- The slope of the function in x must be continuous. Specifically $\frac{\partial \Psi}{\partial x}$ must be continuous.
- These constraints are applied to the boundary conditions on the solutions, and in the process help determine the energy eigenvalues.

Mathematical Expression of the wave associated with a particle

$$\Psi = e^{-i\omega(t-x/v)}$$

$$\Psi = e^{-2\pi i\,(\nu t-x/\lambda)}$$

$$E=h\nu=2\pi\hbar\nu$$

$$\lambda = h/p = 2\pi\hbar/p$$

$$\Psi = e^{-\,i/\hbar\,(Et-px)}$$