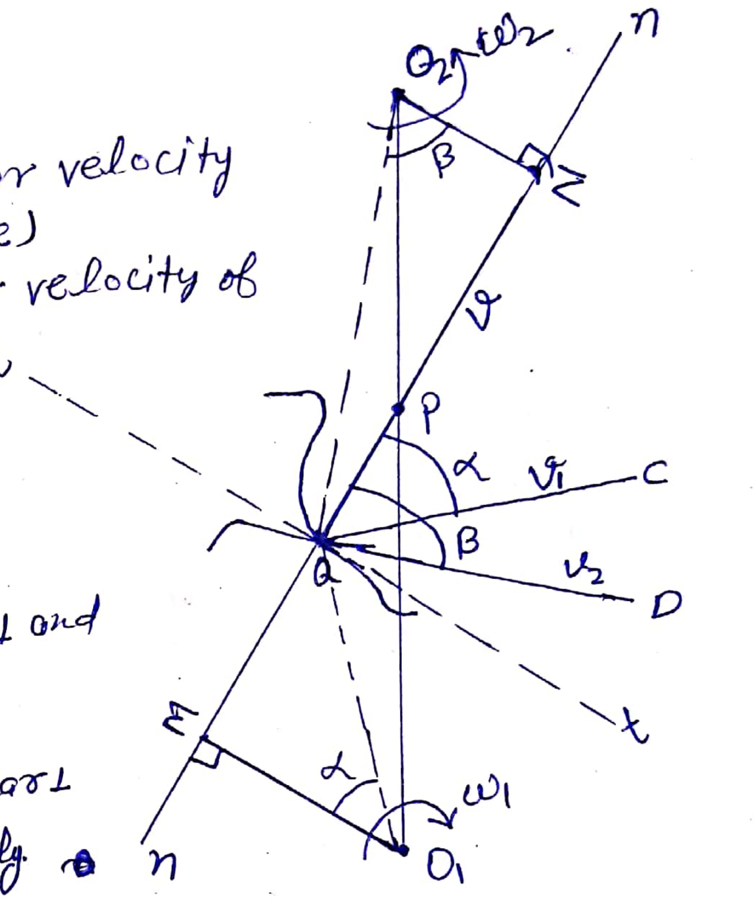
⇒ **Law of Gearing:-** Law of gearing states that common normal to the tooth profile at the point of contact should always pass through a fixed point, called as pitch point in order to obtain a constant velocity ratio.



Let
ω_1 – instentenious angular velocity of gear 1 (clock-wise)
ω_2 – instentenious angular velocity of gear 2 (Anti-clockwise)
tt → common tangent at contact point
nn – common normal
O_1 & O_2 are centre of gear 1 and gear 2, respectively
O_1Q and O_2Q are radii of gear 1 and gear 2, respectively.

Let us draw perpendicular from O_1 and O_2 to common normal i.e. O_1M & O_2N.
Angle made by O_1M w.r.t. O_1Q is α and angle made by O_2N w.r.t. O_2Q is β.

Let v_1 → is velocity along the direction QC ($QC \perp O_1Q$)
v_2 → is velocity along the direction QD ($QD \perp$ to O_2Q)

If the teeth are to remain in contact, then the component of these velocities along the common normal MN must be equal.

Therefore

$$v = v_1 \cos\alpha = v_2 \cos\beta \quad \text{--- (1)}$$

we know that linear velocity $v = r\omega$

$$v_1 = O_1Q \cdot \omega_1$$

$$v_2 = O_2Q \cdot \omega_2$$

Hence equation (1) becomes

$$v = O_1Q \cdot \omega_1 \cos\alpha = O_2Q \cdot \omega_2 \cdot \cos\beta \quad \text{--- (2)}$$

$$\triangle O_1MQ \Rightarrow \cos\alpha = \frac{O_1M}{O_1Q}$$

$$\triangle O_2NQ \Rightarrow \cos\beta = \frac{O_2N}{O_2Q}$$

Put in equa. (2)

$$v = O_1Q \cdot \omega_1 \cdot \frac{O_1M}{O_1Q} = O_2Q\,\omega_2 \cdot \frac{O_2N}{O_2Q}$$

$$v = \omega_1 \cdot O_1M = \omega_2 \cdot O_2N$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} \quad \text{--- (3)}$$

Now consider similar triangle $\triangle O_1MP$ + $\triangle O_2NP$

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} = \frac{R_2}{R_1} = \frac{D_2/2}{D_1/2} = \frac{PN}{PM} \quad \text{--- (4)}$$

$$\frac{O_2N}{O_1M} = \frac{D_2}{D_1} \quad \text{--- (5)}$$

from equa. (3) and (4)

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{D_2}{D_1}} \quad \text{--- (6)}$$

Hence velocity ratio is constant.

⇒ **Velocity of Sliding** :- If the curved surfaces of the two teeth of the gears 1 and 2 are to remain in contact, one can have a sliding motion relative to the other along the common tangent t-t.

Component of v_1 along t-t = $v_1 \sin\alpha$

component of v_2 along t-t = $v_2 \sin\beta$

Velocity of sliding = $v_1 \sin\alpha - v_2 \sin\beta$

$$= O_1Q \cdot \omega_1 \cdot \frac{QM}{O_1Q} - O_2Q \cdot \omega_2 \cdot \frac{QN}{O_2Q}$$

$$= \omega_1 QM - \omega_2 \cdot QN$$

$$= \omega_1 (PM - PQ) - \omega_2 (PN + PQ)$$

$$= \omega_1 \cdot PM - \omega_1 PQ - \omega_2 PN - \omega_2 PQ.$$

$$= -(\omega_1 + \omega_2) PQ + \underbrace{\omega_1 \cdot PM - \omega_2 PN}_{=0 \text{ (From eqn (4))}}$$

Velocity of sliding $= -(\omega_1 + \omega_2) PQ$

= sum of angular velocity × distance between pitch point and point of contact.

⇒ **Tooth profile or Forms of teeth :-** Two curve of any shape that fulfill the law of gearing can be used as the profiles of teeth. An arbitrary shape of one of mating teeth can be taken and applying the law of gearing the shape of the other can be determined. Such gear are said to have conjugate teeth. It will be very difficult to manufacture such gears and cost will be high. On wearing, it will be very difficult to replace them. Thus, there arises the need to standardize gear teeth.

Common form of teeth are:

1) Cycloidal profile teeth
2) Involute profile teeth

⇒ **1) Cycloidal Profile :-**

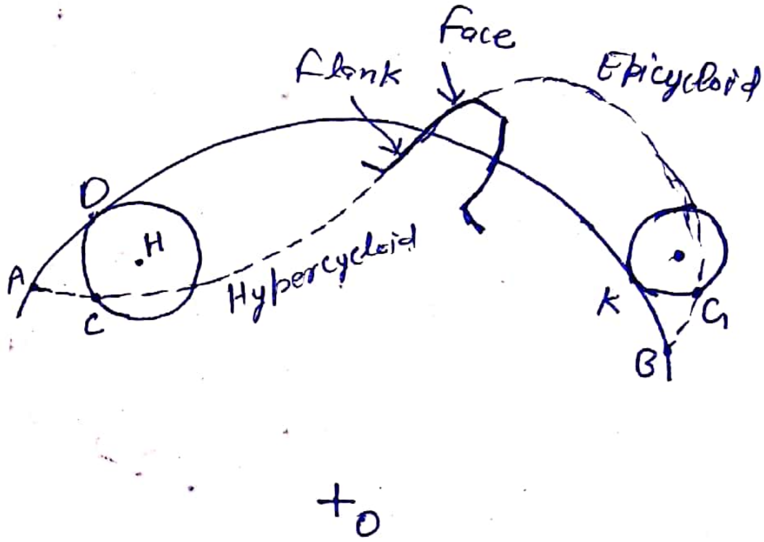
When a circle rolls on a straight line, a point on the circumference of the circle traces a path is known as cycloid curve. On a gear of a ~~getoid~~ finite diameter the face of a tooth is an Epicycloid whereas the flank has that of Hypercycloid. Therefore, the profile of cycloidal teeth is of double curvature.

The face of a cycloidal gear consist of a portion of an Epicycloid, whereas the flank ~~consist~~ consists of a portion of Hypocycloid. The Epicycloid is generated by a rolling circle, when it rolls outside ~~out~~ another circle called the pitch circle whereas the Hypocycloid is generated when the rolling circles rolls inside the pitch circle. The form of cycloidal gear tooth depends on the ratio R_a/R.

where R_a – is the rolling circle radius

R – Pitch circle radius

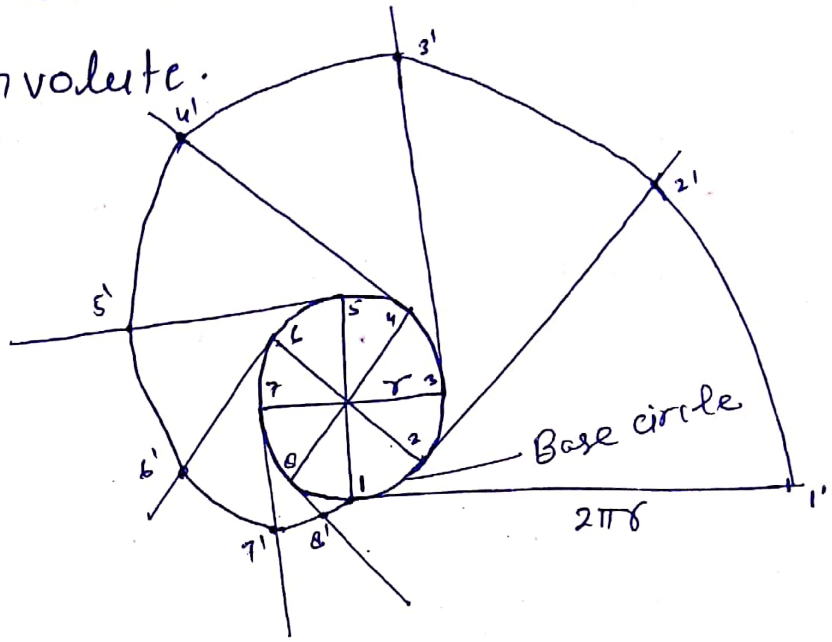
The formation of a cycloidal tooth has been shown in this figure.



★ To satisfy the law of gearing, the same rolling circles must generate the flank and face of two gears in contact.

⇒ 2) Involute Profile teeth:-

The involute of a circle is defind as the curve which is generated by the end points of a cord, which is kept taut while being unwound from the circle. The involute is a spiral beginning from the periphery of a circle called the base circle, which is the heart of the involute.



In case of involute gears:

1) Points of contact lie on the line of action which is the common tangent to the base circles.

2) The contact is made when the tip of a tooth of the driven wheel touches the flank of a tooth of the driving wheel and the contact is broken when the tip of driving wheel touches the flank of the driven wheel.

3) Initial contact occurs where the addendum circle of the driven wheel intersects the line of action. Final contact occurs at a point where the addendum circle of the driver intersects the line of action.

⇒ Interchangeable Gears :– The gears are interchangeable if they are standard ones. The gears can interchange easily when they have :

- Same module
- Same pressure angle
- Same addendum and dedendum
- Same thickness

⇒ System of Gear teeth :– Following four systems of a gear teeth are commonly used in practice :

1) $14\frac{1}{2}°$ Composite system
2) $14\frac{1}{2}°$ Full depth involute system
3) 20° Full depth involute system
4) 20° Stub involute system.

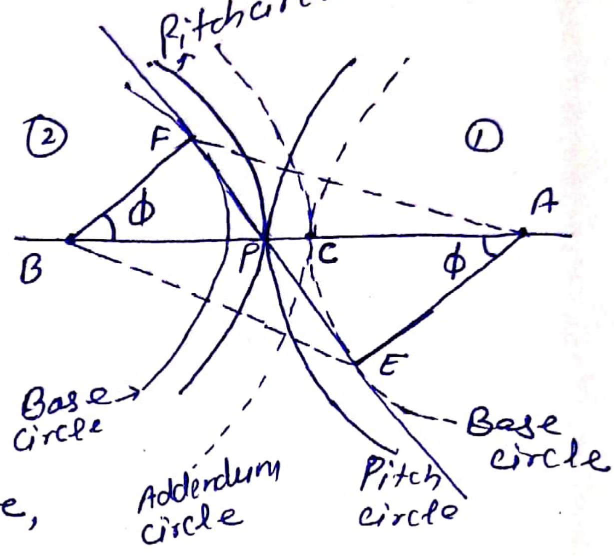
$14\frac{1}{2}°$ composite system is used for general purpose gear. Tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion. These are stronger but has no interchangeability.

$14\frac{1}{2}°$ full depth involute system tooth profile is developed for uses with gear hobs for spur & helical gears.

The increase in the pressure angle from $14\frac{1}{2}°$ to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base. 20° Stub involute system has a strong tooth to take heavy loads.

⇒ Interference in involute Gears :-

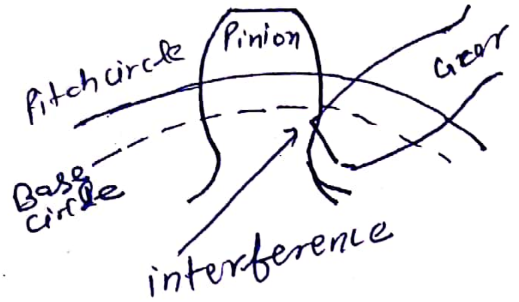
At any instant in a mating gears, the portions of tooth profiles which are in contact must be involutes so that the line of action does not deviate. If any of two surfaces is not an involute,



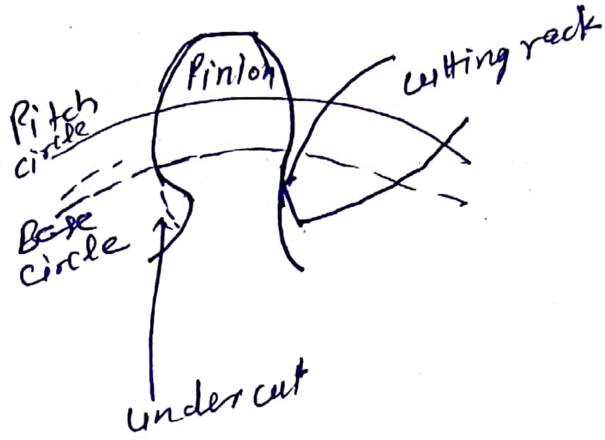
the two surfaces would not touch each other tangentially and transmission of power would not be proper. Mating of two non conjugate or non involute teeth is known as inteference because these two teeth do not slide properly and thus rough action and binding occurs.

The above figure shows two gears in mesh. We can see that addendum circle of gear 2 touches at point C with base circle of gear 1. Any further increase in the value of addendum circle radius BC will result in shifting the point of contact inside the base circle of the gear 1. Since an involute profile can exist only outside the base circle, therefore, any profile of teeth inside the base circle will be of a non-involute type. Therefore, inteference occurs in the mating of two gears.

⇒ Undercutting :–



(a)



(b)

As shown in the figure (a), a portion of pinion dedendum falls inside the base circle. The profile of the tooth inside the base circle is radial. If addendum of the mating gear is more than limiting value, it interferers with the dedendum of the pinion and the two gears are locked.

However, if a cutting rack having similar teeth is used to cut the teeth in the pinion, it will remove that portion of the pinion tooth which would have interfered with the gears as shown in fig. b. A gear having its material removed in this manner is said to be undercut and the precess, undercutting.