# UNIT-2 (Remaining Parts)

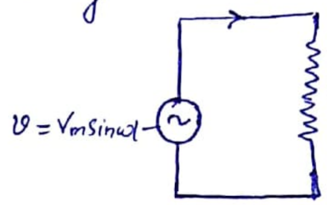
## Steady State Analysis of Single phase AC Circuit

### 2.1 Circuit with pure resistance only

A pure resistance is that in which there is ohamic voltage drop only. consider a circuit having a pure resistance R as shown in fig. 1 below.

Let the instantaneous value of the alternating voltage applied be

$$v = V_m \sin \omega t$$



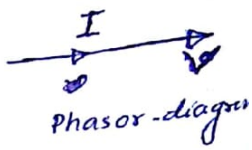




Fig.1

The instantaneous value of current

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

* Phase difference between current and voltage waveform is zero
* instantaneous power $P(t) = v(t)\, i(t)$

$$P(t) = V_m \sin \omega t \; I_m \sin \omega t$$

$$\boxed{P(t) = \frac{V_m I_m}{2}\left[1 - \cos 2\omega t\right]}$$

### 2.2 Circuit with Pure inductance only

A pure inductive circuit possesses only inductance and no resistance or capacitance as shown in fig. 2

Let the applied voltage $v = V_m \sin \omega t$ —— ①

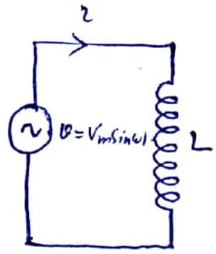



Fig. 2

when an alternating voltage is applied to it, a back emf $\left(-L\frac{di}{dt}\right)$ of self inductance is induced in it. As there is no ohmic resistance drop, the applied voltage has to oppose the self induced emf only. So the applied voltage is equal to and opposite to back emf at all instants.

instantaneous value of self induced emf is $v'$.

$$v' = -L\frac{di}{dt} = -v$$

$$\boxed{v' = -v}$$

$$-L\frac{di}{dt} = -V_m \sin\omega t$$

$$di = \frac{V_m}{L}\sin\omega t \cdot dt$$

integrating both side

$$\int di = \frac{V_m}{L}\int \sin\omega t \cdot dt$$

$$i = \frac{V_m}{\omega L}(-\cos\omega t)$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = \frac{V_m}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)} \quad \text{— (2)}$$

$\boxed{X_L = \omega L}$ ohm

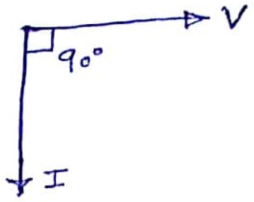



Fig. 3(a) Phasor diagram

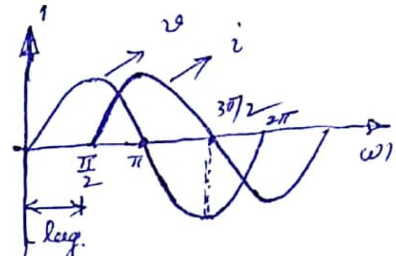



Fig. 3(b) wave form

Observing eq (1) and eq (2) we find that current lags the applied voltage by 90° or $\frac{\pi}{2}$ radian.

$\boxed{j = 1\angle 90°}$

Circuit impedance $Z = \frac{V_m}{I_m} = \frac{V_m \angle 0°}{\frac{V_m}{\omega L}\angle -\frac{\pi}{2}} = \omega L \angle \frac{\pi}{2} = j\omega L$

The quantity $\omega L$ is called inductive reactance and is usually devoted by symbol $X_L$ and units is ohms

Average Power :-

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} v \cdot i \, d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t \cdot I_m \sin\left(\omega t - \frac{\pi}{2}\right) d(\omega t)$$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} - V_m I_m \sin\omega t \cos\omega t \cdot d(\omega t)$$

$$= -\frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{\sin 2\omega t}{2} \cdot d\omega t)$$

$$= 0$$

Note. (1) The average power consumption in an inductive circuit is zero, and is periodic with twice the supply frequency.



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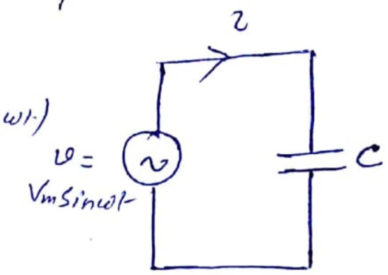
## 2.3 Circuit with Pure Capacitance only:

Applied voltage $v = V_m \sin \omega t$ —— (1)

Current $i(t) = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin\omega t)$




$i(t) = \underbrace{V_m \omega C}_{\downarrow} \cos\omega t$

$i(t) = I_m \cos\omega t$

Fig. 4 (a)

$i(t) = I_m \sin(\omega t + 90^\circ)$ —— (2)

from eq (1) and eq (2) it is observed that current lead the applied voltage by 90°.

Here $I_m = \omega C V_m = \frac{V_m}{1/\omega C}$ , $\boxed{X_c = \frac{1}{\omega C}}$

The quantity $1/\omega C$ is called inductive capacitance and is usually denoted by $X_c$.

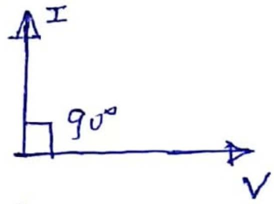



Fig. 4 (b) Phasor diagram

### (a) Circuit impedance :-

impedance $Z = \frac{V}{I} = \frac{V_m \angle 0^\circ}{I_m \angle 90^\circ}$

$Z = \frac{V_m}{I_m} \angle -90^\circ$

$Z = X_c \angle -90^\circ$ [ Since $\frac{V_m}{I_m} = X_c$ ]

$Z = -jX_c = -\frac{j}{\omega C}\ \Omega$

$$\boxed{Z = -\frac{j}{\omega C}\ \Omega}$$

### (b) Average Power :-

instantaneous power $P = v \cdot i = V_m \sin\omega t \cdot I_m \sin(\omega t + \frac{\pi}{2}) = V_m I_m \sin\omega t \cos$

$P = \frac{V_m I_m}{2} \sin 2\omega t$

$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \, d(\omega t) = 0$

This shows that power consumed in purely capacitive circui is zero.

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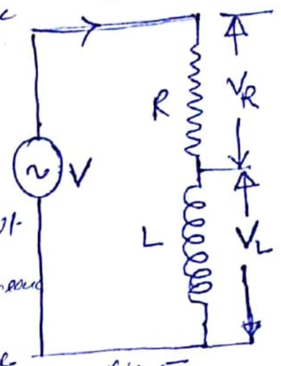
# AC Through Series RL Circuit

$V_R \rightarrow$ voltage drop across resistance

$V_L \rightarrow$ voltage across reactance

$V_R = IR \rightarrow$ in phase with Current Vector $I$

$V_L = IX_L = I\omega L \rightarrow 90^\circ$ ahead of Current vector $I$




$\rightarrow v = V_m \sin\omega t$ — instantaneous value of voltage

fig. 5

$V$ = rms or effective value of applied voltage

$I$ = rms or effective value of current

(a) Phasor diagram:-

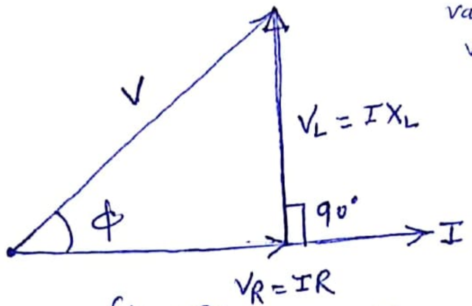



fig. 5 (a) voltage Triangle

(b) Circuit impedance -

Applying KVL in fig. 5

$$V = V_R\angle 0^\circ + V_L\angle 90^\circ$$

$$IZ = I\cdot R + jIX_L$$

$$\boxed{Z = R + jX_L}$$

Magnitude of circuit impedance

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$Z = |Z|\angle\phi$$

(c) Circuit Current

$$I = \frac{V}{Z} = \frac{V\angle 0^\circ}{|Z|\angle\phi}$$

$$I = \frac{|V|}{|Z|}\angle -\phi$$

instantaneous value of current — $\boxed{i = I_m \sin(\omega t - \phi)}$

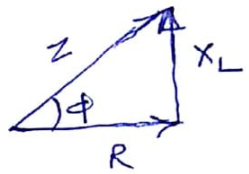
where $I_m = \frac{|V|}{|Z|}$

(d) Phase difference b/w Applied voltage and Current:-

In R-L Circuit current lags the applied voltage by angle $\phi$.

$\phi = \tan^{-1}\frac{X_L}{R}$

(e) Impedance Triangle -




$$\cos\phi = \frac{R}{Z}$$

$\cos\phi$ is called the power factor of the circuit.

The power factor is lagging in an inductive circuit.

Wasid Ast. Prof. Dr. Gaurav Gupta

## 2.5: AC Through Series RC Circuit

$v = V_m \sin\omega t \rightarrow$ instantaneous applied voltage

$V =$ rms or effective value of applied voltage

$V_R = IR \rightarrow$ in phase with current $I$

$V_C = I \cdot \frac{1}{\omega C} = I X_C$, $90^\circ$ lagging with current $I$

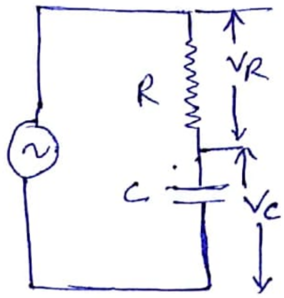



fig. 6

(a) Phasor diagram -

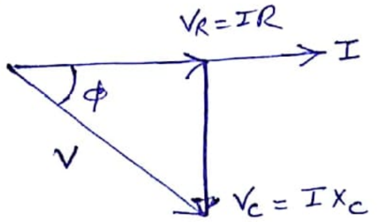



fig. 6 (a) Voltage triangle

$$V = \sqrt{V_R^2 + V_C^2}$$

Circuit current $I$ leads the applied voltage by an angle $\phi$.

(b) Circuit Impedance :-

By KVL -

$$V = V_R \angle 0^\circ + V_C \angle -90^\circ$$

$$IZ = IR + \frac{I_C}{j\omega C} \qquad \left( \frac{1}{j} = 1\angle -90^\circ \right)$$

$$\boxed{Z = R - jX_C}$$

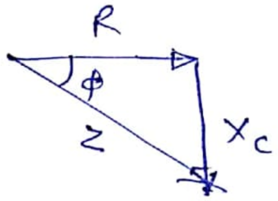



fig. 6 (b) impedance Triangle

$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\phi = \tan^{-1}\left[\frac{\text{Imaginary Part}}{\text{real part}}\right] = \tan^{-1}\left(\frac{-X_C}{R_C}\right) = -\tan^{-1}\left(\frac{X_C}{R}\right)$$

(c) Power factor :-

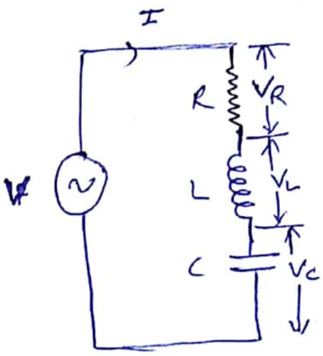
$$\text{Power factor } \cos\phi = \frac{V_R}{V} = \frac{R}{Z} \quad (\text{Leading})$$

(d) instantaneous value of Current -

$$I = I_m \sin(\omega t + \phi)$$

## 2.6 AC Through Series RLC Circuit

~~By KVL~~




(a) **Circuit Impedance:-**

By KVL

$$V = V_R \angle 0 + V_L \angle 90^\circ + V_C \angle -90^\circ$$

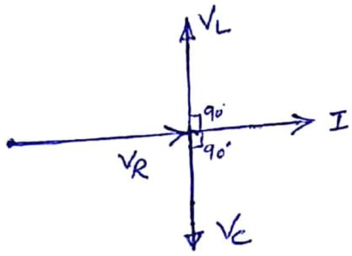
$$IZ = IR + jIX_L - jIX_C$$

$$\boxed{Z = R + j(X_L - X_C)}$$

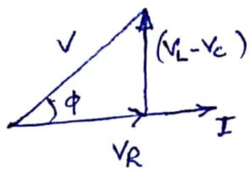
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

(b) Phasor diagram:-



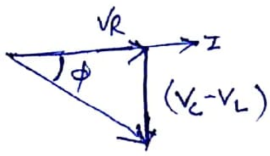

→ Case-1 when $V_L > V_C$




$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$\phi$ → lagging

→ Case-2 - when $V_C > V_L$




$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$\phi$ → leading

→ Case-3, when $V_L = V_C$


$$\boxed{V = V_R}$$

$$\phi = 0^\circ$$

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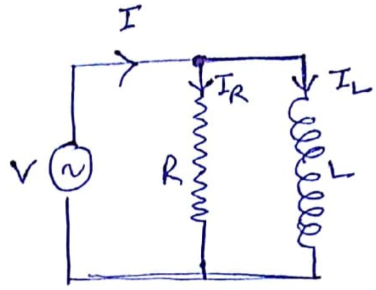
## 2.7 R-L Parallel Circuit:-

By KCL -

$$\vec{I} = \vec{I_R} + \vec{I_L}$$

$$\cancel{I = V\phi}$$




Note:- In parallel ckt voltage is taken as reference.

$$\bar{I} = I_R \angle 0^\circ + I_L \angle -90^\circ$$

$$\frac{V}{Z} = \frac{V}{R} - j\frac{V}{X_L}$$

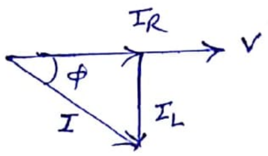
$$\frac{1}{Z} = \frac{1}{R} - j\frac{1}{X_L}$$

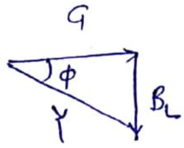
$$\boxed{Y = G - jB_L}$$

$Y \rightarrow$ admittance

$B_L \rightarrow$ Inductive susceptance $= \frac{1}{X_L}$

(a) Phasor diagram




(a)

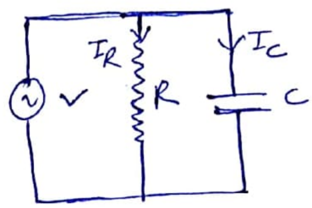
$$I = \sqrt{I_R^2 + I_L^2} \qquad Y = \sqrt{G^2 + B_L^2}$$

$$\phi = \tan^{-1}\left(\frac{-I_L}{I_R}\right) \qquad \phi = \tan^{-1}\left(\frac{-B_L}{G}\right)$$

## 2.8 R-C- Parallel Circuit

(a) By KCL

$$I = I_R \angle 0^\circ + I_C \angle +90^\circ$$

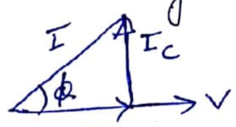



$$\frac{V}{Z} = \frac{V}{R} + j\frac{V}{X_C}$$

$$\frac{1}{Z} = \frac{1}{R} + j\frac{1}{X_C}$$

$$\boxed{Y = G + jB_C}$$

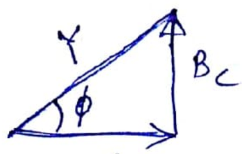
(b) Phasor diagram




$Y$ = admittance $= \frac{1}{Z}$

$G$ = conductance $= \frac{1}{R}$

$B_C$ = Capacitive susceptance $= \frac{1}{X_C}$



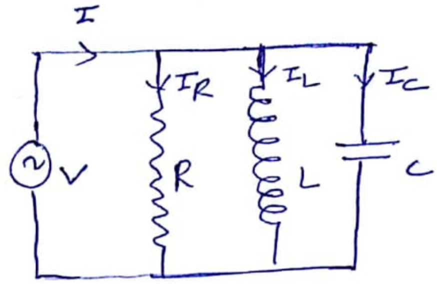
## 2.9. Parallel RLC Circuit:

By KCL.

$$I = I_R \angle 0^\circ + I_L \angle -90^\circ + I_C \angle +90^\circ$$

$$\frac{V}{Z} = \frac{V}{R} - j\frac{V}{X_L} + j\frac{V}{X_C}$$

$$\boxed{Y = G + j(B_C - B_L)}$$

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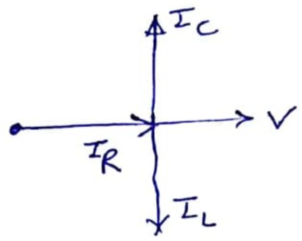
$$G = \frac{1}{R}$$

$$B_C = \frac{1}{X_C}$$

$$B_L = \frac{1}{X_L}$$

**Phasor diagram:-**

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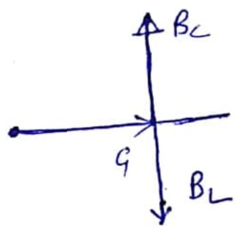


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