Lecture Notes

for

B.TECH. FIRST YEAR, IInd Semester

(Mechanical, EC, Electrical Engg.)

Subject Code: EE201

Subject: Basic Electrical Engineering

FACULTY OF ENGINEERING
UNIVERSITY OF LUCKNOW
LUCKNOW

D C Machine

By

Dr. Gaurav Gupta

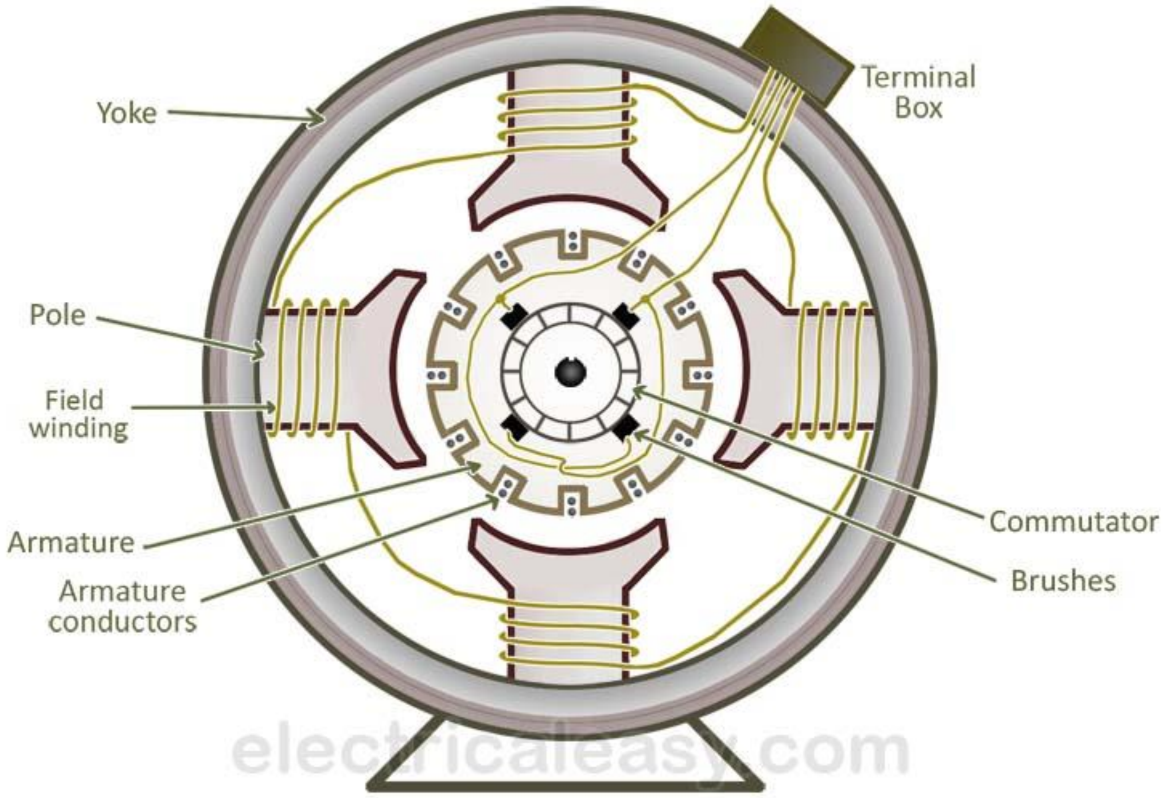
Direct current machines (DC machines)

- **DC Machines:** Rotating electrical machine.
- **Example: Generator and motor.**
- **DC generator:** Converts mechanical power to electrical power of DC nature.
- **DC motor:** Converts electrical power into mechanical power (converse of DC generator).
- Working principle: **Faraday's law of EM induction** (as already discussed during transformer lectures).

Construction

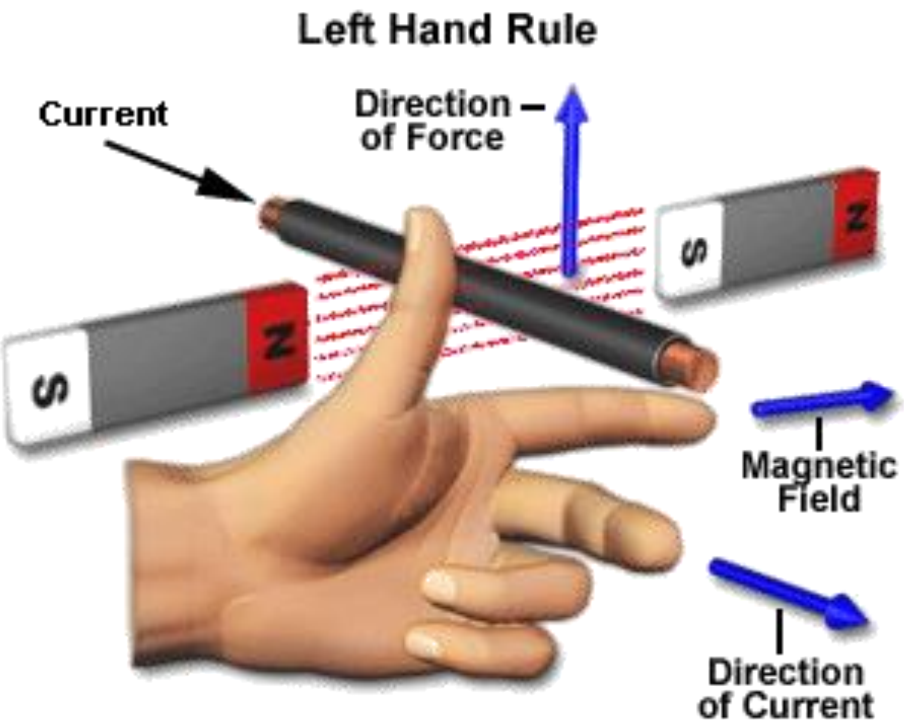
- **Armature (Rotatory part):** Comprising of a number of conductors suitably placed and connected so as to form a closed winding.
- **Field system (stationary part):** To produce magnetic field.
- **Airgap:** Used to separate stator and rotor.
- **Commutator:** Comprise of large number of commutator segments, properly insulated from each other.
- Commutator is used to convert an AC wave in the armature winding into a DC wave at the output terminals (in case of DC generator), and inverts the DC input wave into an AC wave in the armature winding (in case of DC motor).

Construction



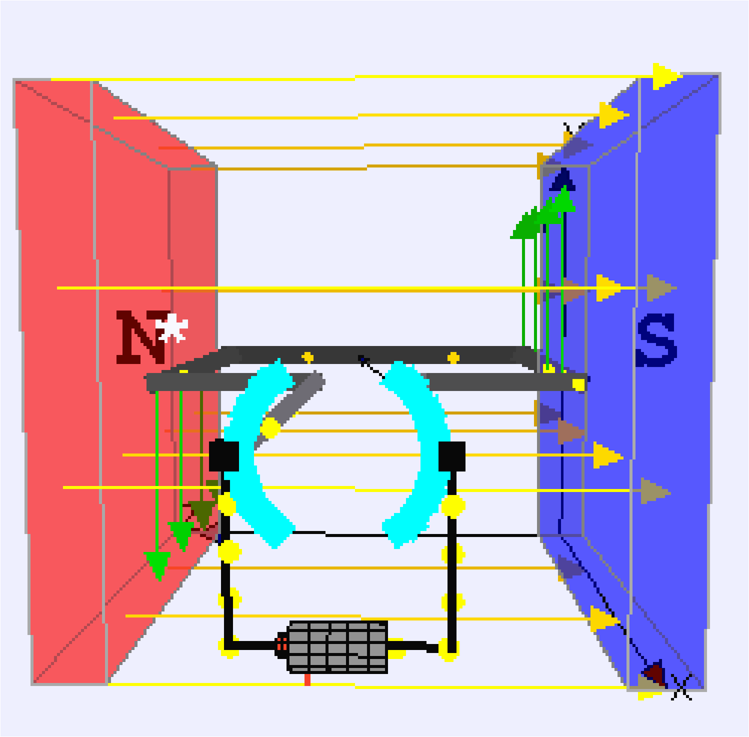
Armature core (rotor)

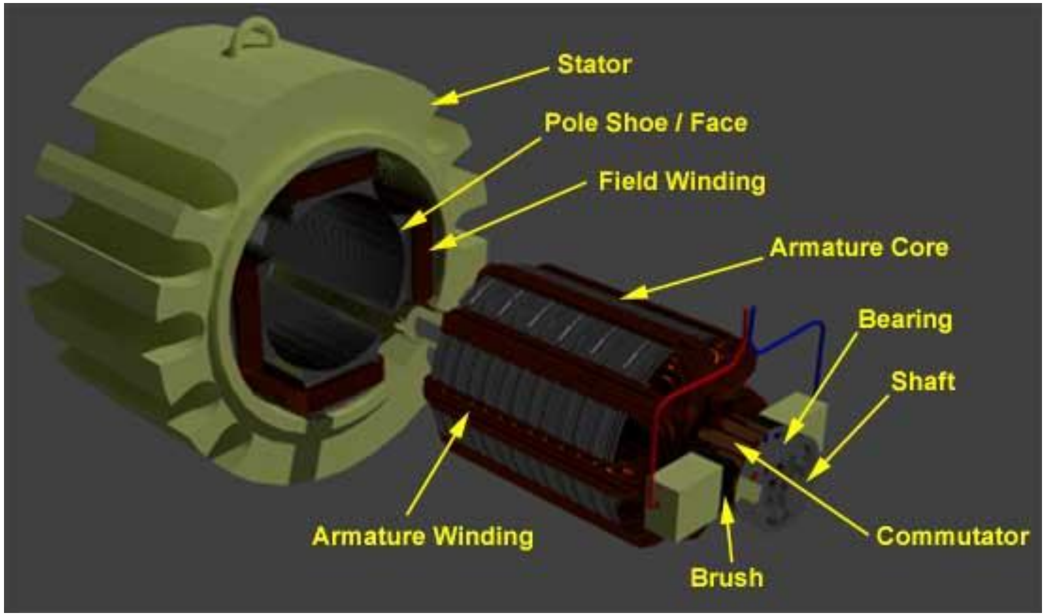
DC motor: Working principle



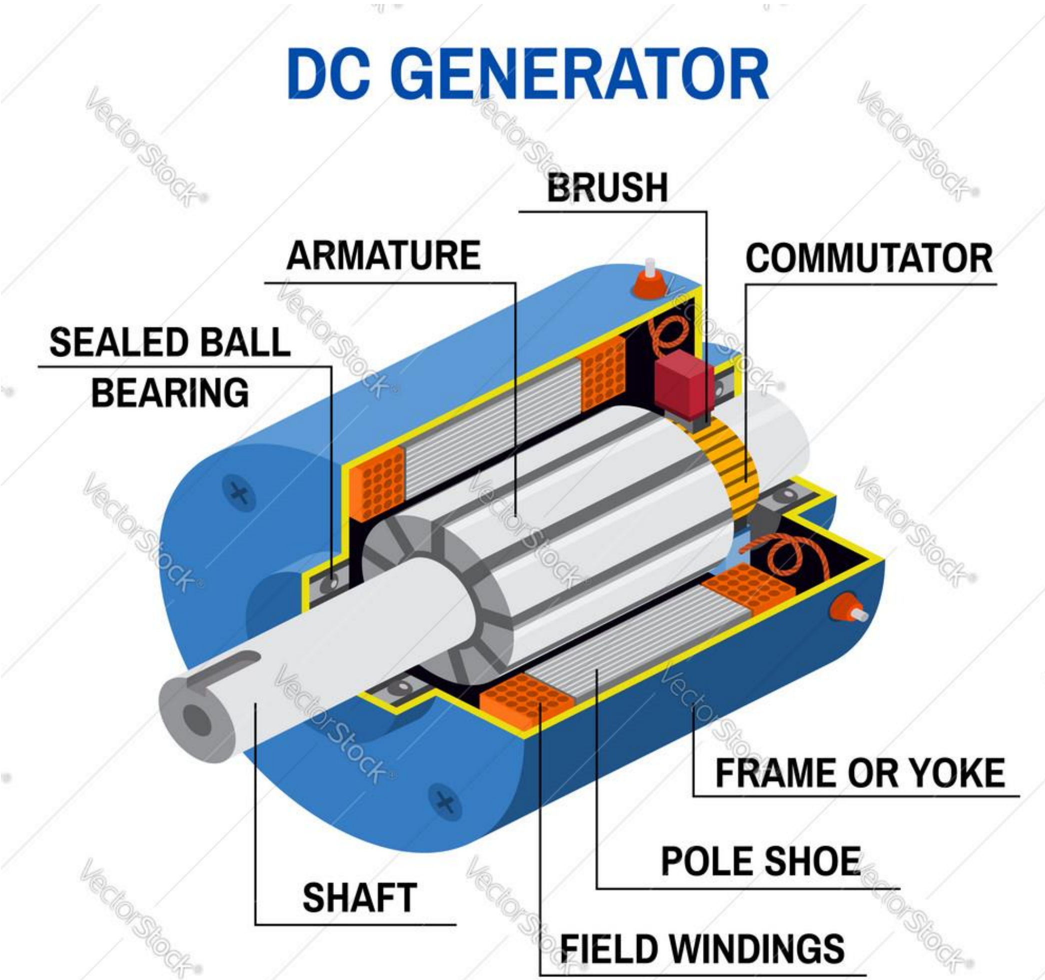
- When a current carrying conductor is placed in a magnetic field, it experiences a torque and has a tendency to move. This is known as motoring action.
- If the direction of current in the wire is reversed, the direction of rotation also reverses.
- When magnetic field and electric field interact they produce a mechanical force, and based on that the working principle of DC motor is established.

DC motor


N
S


Stator
Pole Shoe / Face
Field Winding
Armature Core
Bearing
Shaft
Armature Winding
Commutator
Brush

DC generator



General expression for resistance of DC armature winding

- If **Z- number of conductors**, each of length L.
- S- Cross-section area arranged in ‘a’ pairs of circuit.
- ‘p’- Pole pairs.
- ρ- Resistivity of the winding material.
- **Resistance of any conductor** is given as:

$$R = \frac{\rho L}{S}$$

$$R = \frac{\rho L}{S}$$

- ρ- Resistivity of the conducting material in ohm-cm.
- L- Total length of the conductor including overhang.
- S- Cross-section area of the conductor.
- **Armature winding consists of Z conductors arranged in "a" pairs of the circuits.**
- Thus, the **number of conductors in each parallel circuit= Z/a**
- **All the conductors in each parallel circuit are connected in series. As a result, the resistance of Z/a conductors connected in series** is given by:

$$R = \left(\frac{\rho L}{S}\right)\frac{Z}{a}\,\Omega$$

- There are "a" parallel circuits in the whole of the armature winding. Thus the resistance of armature winding is:

$$R = \left(\frac{\rho L Z}{S a}\right)\left(\frac{1}{a}\right) = \frac{1}{a^2}\left(\frac{\rho L Z}{S}\right)$$

General expression for the resistance of a DC armature winding

Numerical Problem: **Calculate the resistance of a 6-pole, lap connected armature winding using the following data:**

Number of slots: 150

Conductor per slot: 8

Mean length of one turn: 250 cm

Cross-section of each conductor: 10 mm × 2.5 mm

Value of resistivity (ρ): 2.1×10^{-6} Ohm-cm

Solution

- Total number of conductors of armature winding (Z)= 150 × 8= 1200.
- Number of turns = 1200/2= 600.

$$R = \frac{1}{a^2}\left(\frac{\rho L Z}{S}\right) = \frac{1}{(6)^2}\left(\frac{2.1\times10^{-6}\times250\times600}{0.25}\right)$$

EMF equation

- **Average EMF generated by the armature of a DC generator (voltage across the brushes of different polarity) = sum of the EMFs of all the conductors connected in series in one parallel path.**
- If **Z- total conductors on the armature** of a DC machine.
- Then the **number of conductors connected in series in one parallel path** will be Z/A.
- A- Number of parallel paths in the armature winding.
- Hence, the EMF generated by one parallel path is:

$$E_g = E_{average} \times \frac{Z}{A}$$

Where, $E_{avg.}$- Average EMF per conductor

Contd….

- **EMF generated by the armature = EMF generated by one of the parallel paths of the armature winding.**

$$E_g = E_{average} \times \frac{Z}{A}$$

- The **average EMF generated by one conductor= Flux cut/second** (As per the Faraday's law of EM induction).
- If Ø- Air gap flux/pole in Weber's.
- P-Total number of poles in the machine.
- Then, **total flux cut by one conductor in completing one revolution** of armature= PØ
- **Flux cut by one conductor per second** can be written as:

$$= P\phi \times \frac{N}{60}$$

- N- Speed of the machine in rpm

Contd…

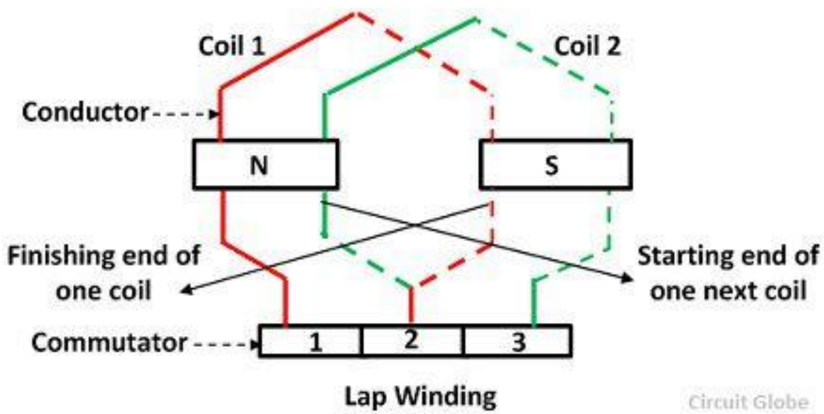
- Hence, **average EMF generated by one conductor** of the armature:

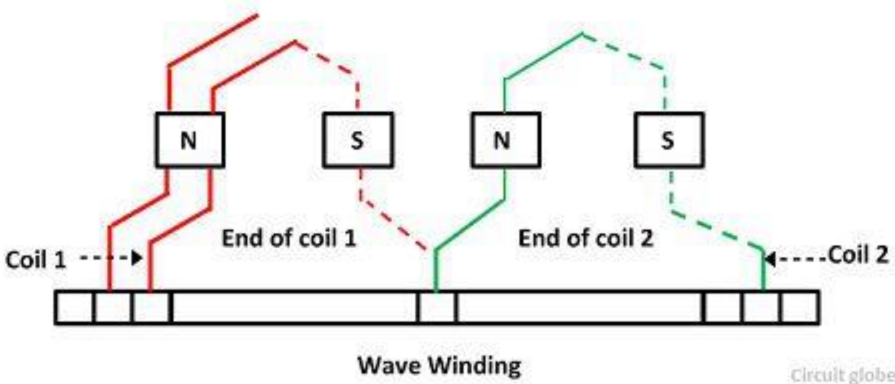
$$E_{avg.} = \frac{P\phi N}{60} V$$

- The **EMF generated by the armature of a DC machine** is:

$$E_g = \frac{P\phi N}{60} \times \frac{Z}{A} = \frac{P\phi NZ}{60A} V$$

Armature windings





- In **lap winding**, the two ends of a coil, designated "start'" and "finish", are connected to the adjacent commutator segments. **(side of successive coils overlap each coil, that's why called as lap winding).**
- In **wave winding**, "start" and "finish" ends are connected to the same commutator segment.

EMF for two different types of armature windings

- For a lap-wound armature,
- **Number of parallel paths in the armature winding = Number of poles in the machine.**
- Hence, EMF generated is:

$$E_g = \frac{P\phi NZ}{60P} = \frac{\phi NZ}{60} V$$

- For a wave-wound armature,
- **Number of parallel paths in the armature winding=2 (irrespective of number of poles).**
- Hence, EMF generated is:

$$E_g = \frac{P\phi NZ}{60 \times 2} = \frac{P\phi NZ}{120} V$$

Imp. Note **

- The expressions given for the induced EMF in previous slides hold equally for generators and motors.
- However, **in the case of a DC motor, the induced EMF is called as back EMF**, bcoz the induced EMF acts in a direction opposite to the applied voltage.
- Hence, **back EMF for a DC motor** is:

$$E_{back-emf} = \frac{P\phi NZ}{60A} V$$

Problems on EMF equation: Problem 1

- A 6-pole, lap wound armature has 840 conductors and flux per pole of 0.018 Wb. Calculate the EMF generated, when the machine is running at 600 rpm.
- Solution: For lap-type, A=P=6

$$E_g = \frac{P\phi NZ}{60A}V = \frac{6\times 0.018\times 600\times 840}{60\times 6}V$$

Problem 2

- A 6-pole, 2 circuit, wave-wound armature has 300 conductors and runs at 1000 rpm. The EMF generated on open-circuit is 400 V. Find the useful flux per cycle.
- Solution: For wave-wound, A=2

$$E_g = \frac{P\phi NZ}{60A} V$$

$$\phi = \frac{E_g \times 60A}{PNZ} = \frac{400 \times 60 \times 2}{6 \times 1000 \times 300}$$

Problem 3

- A lap-wound DC shunt generator having 80 slots with 10 conductors per slot generates at no load an EMF of 400 V, when running at 1000 rpm. At what speed , should it be rotated to generate a voltage of 220 V on open circuit?
- Solution: Total number of conductors on armature (Z)= Number of slots on armature × conductor per slot = 80 × 10 = 800

$$E_g = \frac{P\phi NZ}{60A} V$$

Contd…

- For lap-wound, A=P, therefore, EMF is

$$E_g = \frac{\phi NZ}{60} V$$

$$400 = \frac{\phi \times 1000 \times 800}{60}$$

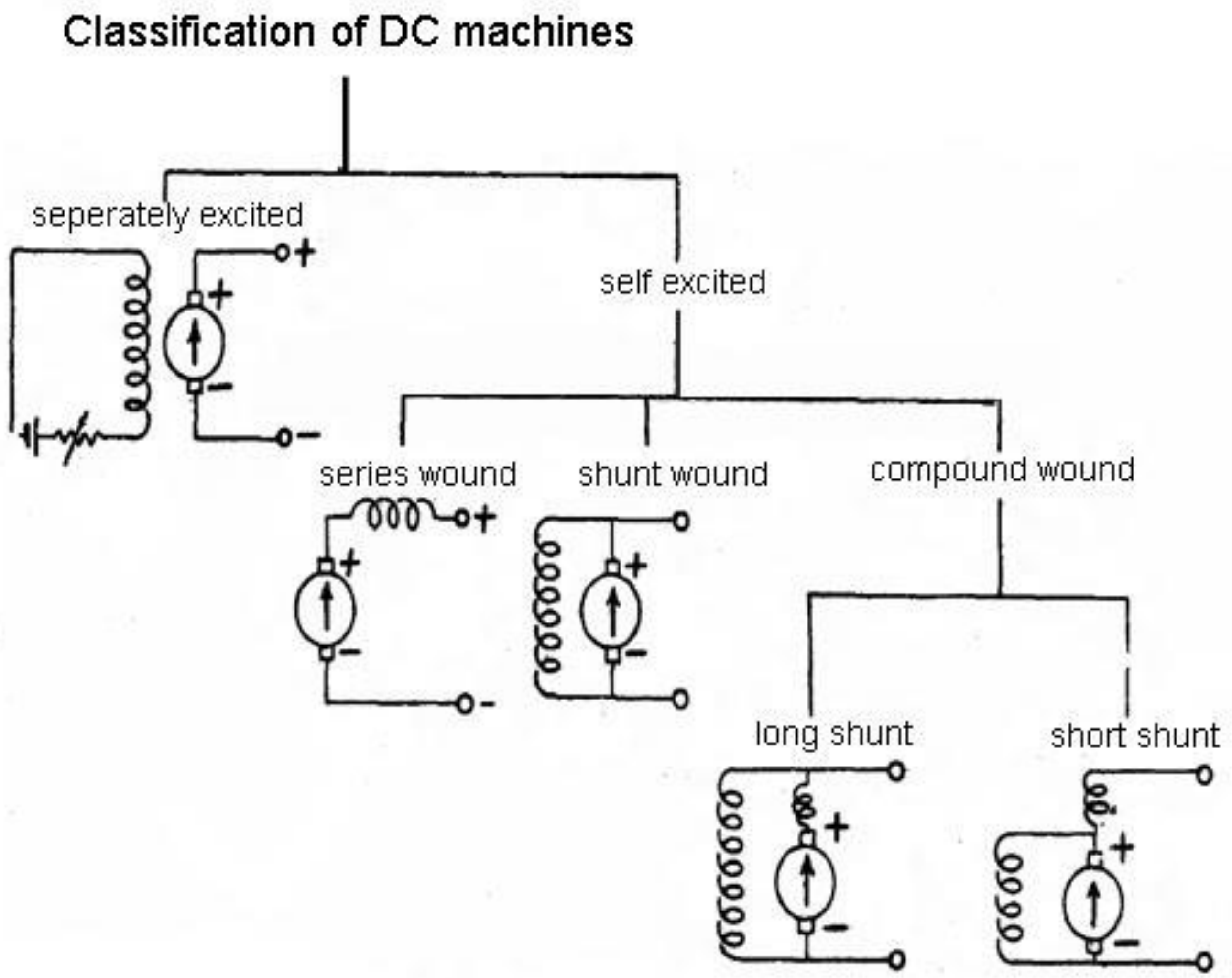
$$\phi = 0.03 Wb$$

- Desired value of generated voltage= 220 V, thus

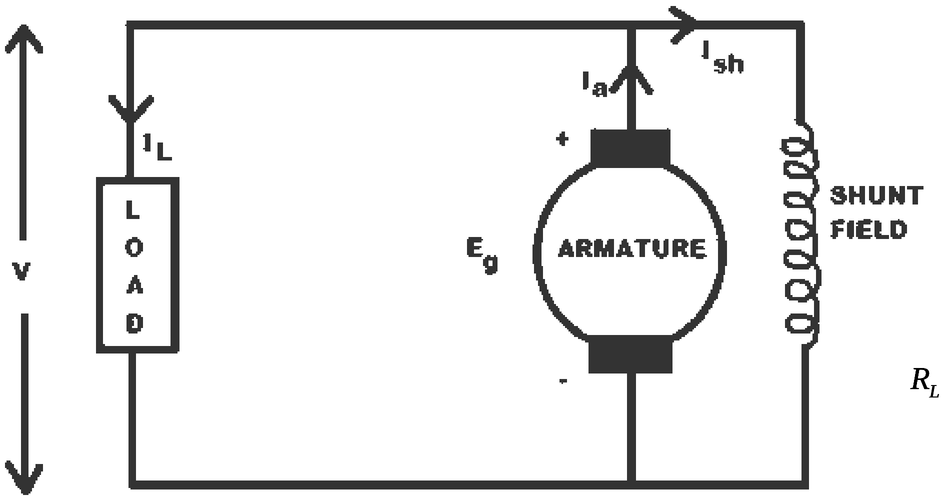
$$220 = \frac{0.03 \times N \times 800}{60}$$

$$N = 550 rpm$$

Classification of DC machines



DC shunt generator



- **Applying Kirchhoff's laws**

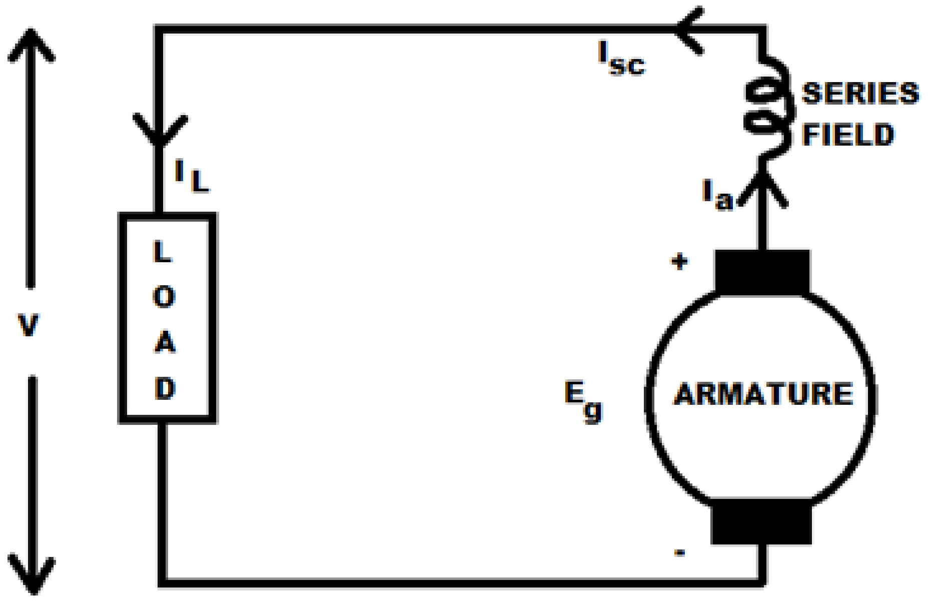
$$I_a = I_L + I_{sh}$$

- Also **EMF generated= Terminal voltage + voltage drop in the armature**

$$E_g = V + I_a R_a$$

- **Applying Ohm's law,**
- **Load current** $I_L = \frac{V}{R_L}$
- **Shunt field current** $I_{sh} = \frac{V}{R_{sh}}$

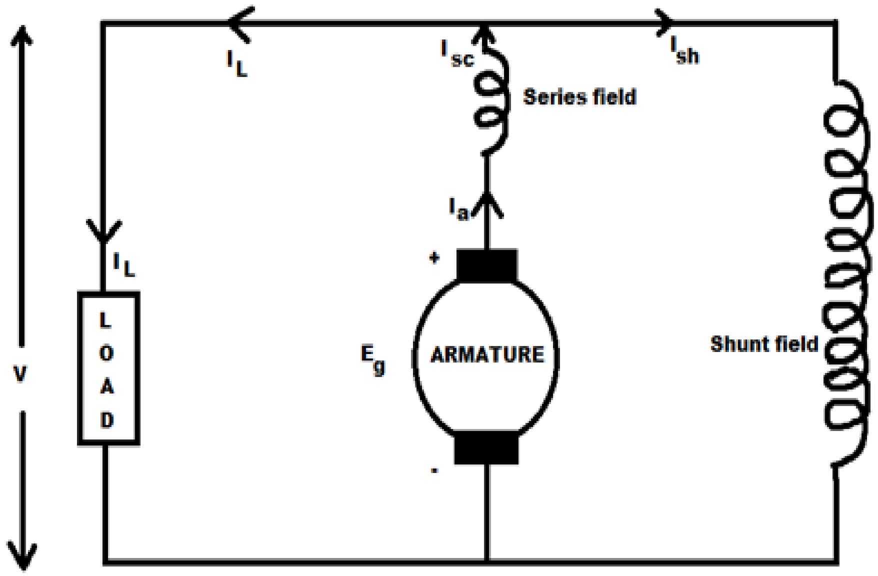
DC series generator



- **Excitation current= Armature current**
- EMF generated:

$$E_g = V + I_a(R_a + R_{sc})$$

DC compound generator



$$I_a = I_L + I_{sh}$$

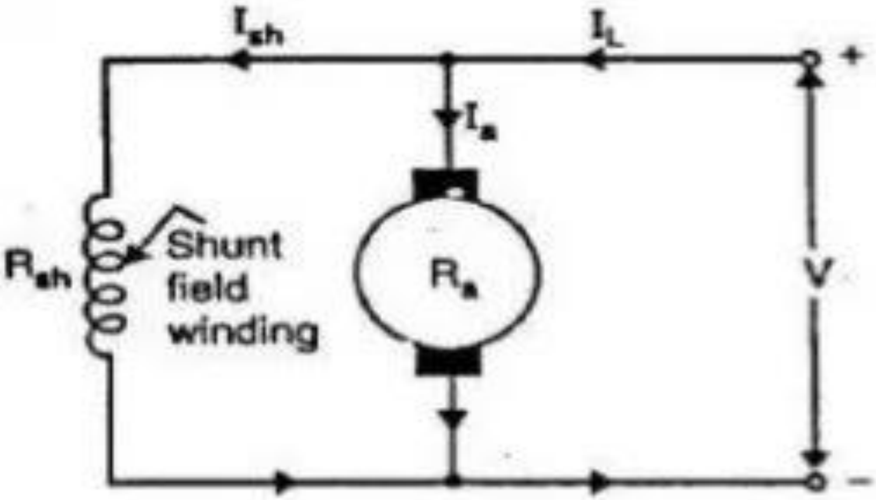
$$I_{sc} = I_L$$

$$E_g = V + I_a R_a + I_L R_{sc}$$

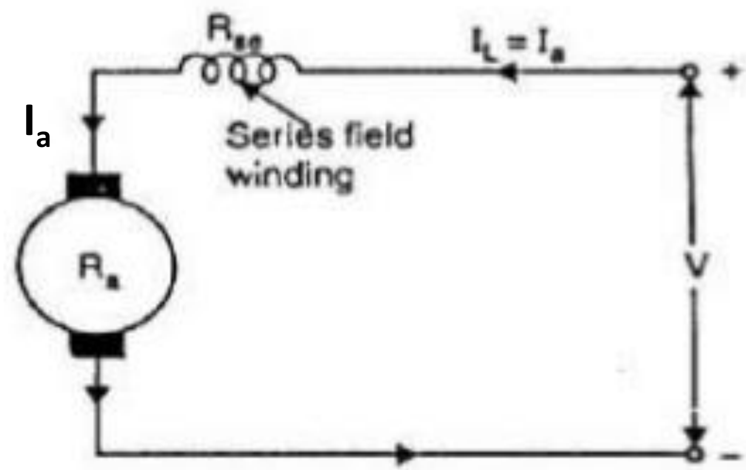
$$I_{sh} = \frac{V + I_L R_{sc}}{R_{sh}}$$

DC shunt and series motors

DC shunt motor



DC series motor



$$I_a = I_L - I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

$$I_L = I_{sc}$$

$$E_{back-emf} = V - I_a R_a$$

$$E_{back-emf} = V - I_a(R_a + R_{sc})$$

Problem 1

- The armature of a 4-pole, lap-wound shunt generator has 120 slots with 4 conductors per slot. The flux per pole is 0.05 Wb. The armature resistance is 0.05 Ohm, and the shunt field resistance is 50 Ohm. Find the speed of the machine when supplying 450 A at a terminal voltage of 250 V.

Solution: Terminal voltage, V= 250 V

- Load current, I_L= 450 A
- Shunt field resistance, R_{sh}= 50 Ohm
- Shunt field current, I_{sh}= V/R_{sh}=250/50= 5 A
- Armature current, I_a= I_L+I_{sh}= 450+5= 455 A
- Armature resistance, R_a= 0.05 Ohm
- Generated EMF, E_g= V + I_aR_a= 250+ 455 ×0.05= 272.75 V

$$E_g = \frac{P\phi NZ}{60A} V$$

$$272.75 = \frac{4 \times 0.05 \times N \times (120 \times 4)}{60 \times 4}$$

$$N = 682 rpm$$

Problem 2

- A 120 V DC shunt motor draws a current of 200 A. The armature resistance is 0.02 Ohm, and shunt field resistance of 30 Ohm.
- (i). Find the back EMF.
- (ii). If the lap-wound armature has 90 slots with 4 conductors per slot, at what speed will the motor run, when the flux per pole is 0.04 Wb?

Solution

(i)

$$I_{sh} = \frac{V}{R_{sh}} = \frac{120}{30} = 4A$$

$$I_a = I_L - I_{sh}$$
$$= 200 - 4 = 196A$$

$$E_{back-emf} = V - I_a R_a$$

$$E_{back-emf} = 120 - 196 \times 0.02$$

$$E_{back-emf} = 116.08V$$

- **(ii).** For lap-wound, A=P, therefore, EMF is

$$E_g = \frac{\phi NZ}{60} V$$

$$116.08 = \frac{0.04 \times N \times (90 \times 4)}{60}$$

Torque equation for a DC motor

- The **back-emf** of DC motor can be written as:

$$E_b = V - I_a R_a$$

- Multiplying this equation on both sides by I_a,

$$E_b I_a = V I_a - I_a^2 R_a$$

- Where, **VI_a- Total electrical power supplied to the armature of the DC motor (armature input).**
- **$I_a^2 R_a$- Power wasted in the DC armature (Armature copper loss).**

Contd….

- The **difference between the armature input and the armature copper loss =Mechanical power developed by the armature.**
- Hence, **mechanical power developed= $E_b I_a$ Watts.**
- If **T_a- Torque** in Newton meter developed by the armature running at N revolutions/minute, then
- **Mechanical power developed** $= \frac{2\pi N T_a}{60}$

$$E_b I_a = \frac{2\pi N T_a}{60}$$

$$T_a = \left(\frac{60}{2\pi}\right)\frac{E_b I_a}{N}$$

We already know, back-emf can be expressed as:

$$E_b = \frac{P\phi NZ}{60A}$$

After substituting the value of E_b

$$T_a = \left(\frac{60}{2\pi}\right)\left[\frac{P\phi NZ}{60A} \times \frac{I_a}{N}\right]$$

$$T_a = 0.159\left[\frac{P\phi I_a Z}{A}\right] Nm$$

Conclusions drawn from Torque equation

- For a particular DC motor, the **number of poles (P), number of conductors on armature (Z), and the number of parallel paths in the armature winding (A) are fixed.**
- Hence, $T_a \alpha \phi I_a$
- Thus, the **torque developed by the armature is proportional to the product of flux per pole and the armature current.**

Speed of the motor

- The back EMF of DC motor is:

$$E_b = \frac{P\phi NZ}{60A}$$

$$E_b = V - I_a R_a$$

- Combining the above equations,

$$\frac{P\phi NZ}{60A} = V - I_a R_a$$

Speed of the motor

$$N = (V - I_a R_a)\frac{60A}{PZ} \times \frac{1}{\phi}$$

Contd…

- For a given motor, **P, Z, and A are fixed**, hence,

$$N = K\left(\frac{V - I_a R_a}{\phi}\right)$$

$$N = K\left(\frac{E_b}{\phi}\right)$$

- Thus, the **speed of motor is directly proportional to the voltage applied to the armature (or back EMF).**
- **Inversely proportional to the flux/pole.**

Problem 1

- The lap-wound armature of a 6-pole, 6-circuit DC shunt motor takes 300 A at the speed of 400 revolutions per minute. The flux per pole is 75×10^{-3} Wb. The number of armature turns is 500. The torque lost windage, friction, and iron losses can be assumed as 2.5 percent. Compute:
- **Torque developed by the armature.**
- **Shaft torque.**
- **Shaft power in kW**

Solution

$$T_a = 0.159\left[\frac{P\phi I_a Z}{A}\right] Nm$$

- A=P=6
- Number of armature turns=500
- Thus, total number of conductors on the armature= 2× 500=1000

$$T_a = 0.159\left[\frac{6\times 75\times 10^{-3}\times 300\times 1000}{6}\right]$$

$$T_a = 3577.5 Nm$$

Computation of Shaft torque and shaft power

- Torque lost in windage, friction, and iron losses= 2.5 percent of T_a= 0.025 × 3577.5

T_a= 89.44 Nm

- Thus, **shaft torque, T_{sh}= 3577.5-89.44=3488.06 Nm**
- **Shaft power,**

$$P = \frac{2\pi N T_{sh}}{60 \times 1000} kW$$

$$P = \frac{2\pi \times 400 \times 3488.06}{60 \times 1000} kW$$

Problem 2

- A 200V DC shunt motor takes a total current of 100 A and runs at 750 rpm. The resistance of the armature winding and of shunt field winding is 0.1 and 40 Ω, respectively.
- Compute (i) the torque developed by the armature, and (ii) copper losses.
- If the friction and iron losses amount to 1500 W, also calculate (iii) shaft power, (iv) shaft torque, and (v) efficiency.

Solution part (i)

- Voltage applied across the motor, V= 200 V
- Resistance of the shunt field winding= 40 Ω
- Shunt field current, I_{sh}= 200/40=5 A
- Total current drawn by motor= 100 A
- Thus, armature current, I_a= 100-5= 95 A
- Armature resistance, R_a= 0.1 Ω
- Back EMF, E_b= V-$I_a R_a$= 200-95×0.1=190.5V
- Mechanical power developed= E_b I_a=190.5 × 95= 18097.5 W

$$E_b I_a = \frac{2\pi N T_a}{60}$$

$$T_a = \left(\frac{60}{2\pi}\right)\frac{E_b I_a}{N}$$

Contd….

$$T_a = \left(\frac{60}{2\pi}\right)\frac{E_b I_a}{N}$$

$$T_a = \left(\frac{60}{2\pi}\right)\frac{18097.5}{750} = 230.3Nm$$

Solution part (ii)

- The back EMF for a DC motor

$$E_b = V - I_a R_a \qquad E_b I_a = V I_a - I_a^2 R_a$$

- The armature copper loss, $I_a^2 R_a = V I_a - E_b I_a$

$$I_a^2 R_a = 200 \times 95 - 18097.5 = 902.5 W$$

- Field copper losses $= I_{sh}^2 R_{sh} = (5)^2 \times 40 = 1000 W$

Contd….

- Total Cu losses= 902.5+1000=1902.5 W
- **Solution of Part (iii):**
- Friction and iron losses= 1500 W
- Total Cu loss= 1902.5 W
- Input to the motor= 200 × 100= 20000 W
- Output= 20000-(1500+1902.5)=16597.5 W
- Or **shaft power= 16.6 W**

Solution parts (iv) and (v)

$$P = \frac{2\pi N T_{sh}}{60 \times 1000} kW$$