

## 4.5 DIRECTIONAL COUPLERS

A *directional coupler* is a four-port waveguide junction as shown in Fig. 4-5-1. It consists of a primary waveguide 1-2 and a secondary waveguide 3-4. When all ports are terminated in their characteristic impedances, there is free transmission of power, without reflection, between port 1 and port 2, and there is no transmission of power between port 1 and port 3 or between port 2 and port 4 because no coupling exists between these two pairs of ports. The degree of coupling between port 1 and port 4 and between port 2 and port 3 depends on the structure of the coupler.

The characteristics of a directional coupler can be expressed in terms of its coupling factor and its directivity. Assuming that the wave is propagating from port 1 to port 2 in the primary line, the coupling factor and the directivity are defined,

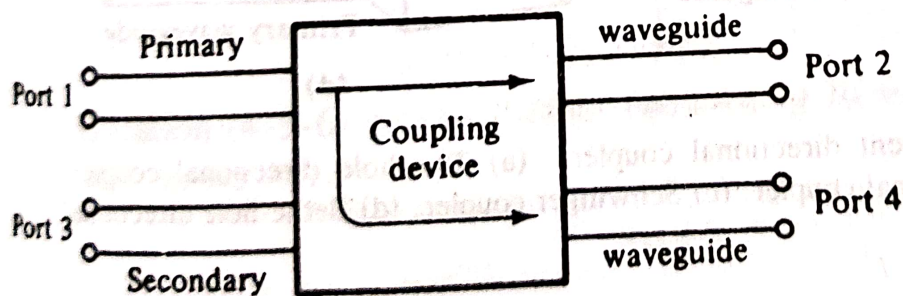


Figure 4-5-1 Directional coupler.

respectively, by

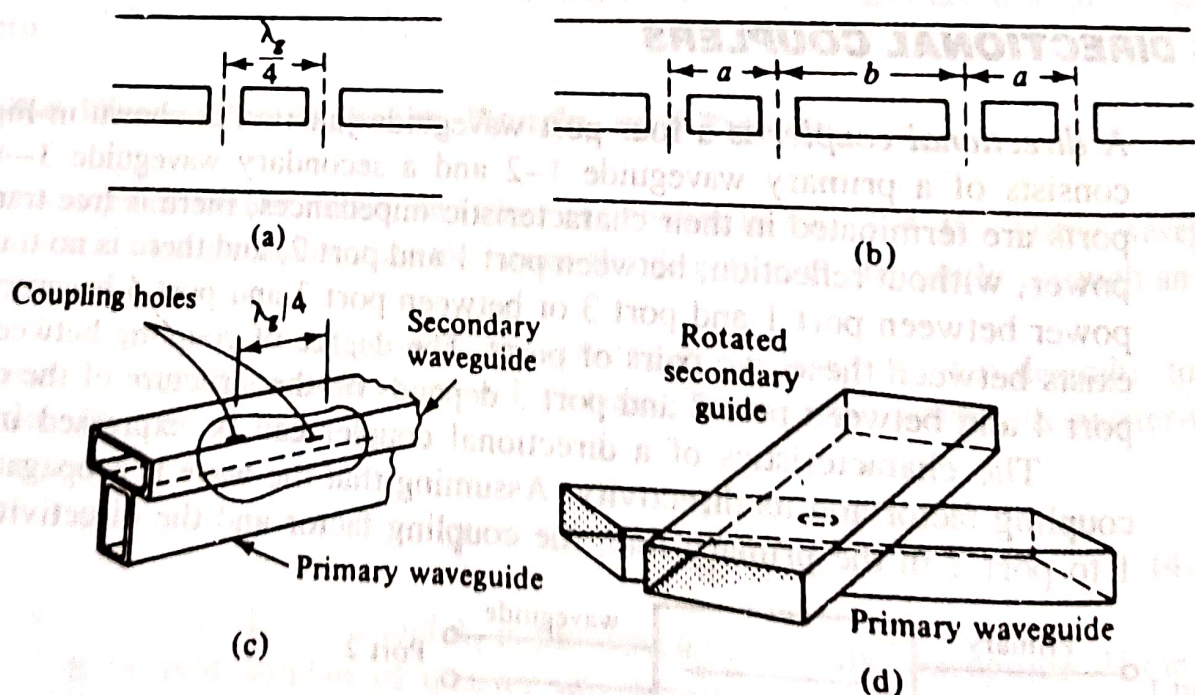
$$\text{Coupling factor (dB)} = 10 \log_{10} \frac{P_1}{P_4} \quad (4-5-1)$$

$$\text{Directivity (dB)} = 10 \log_{10} \frac{P_4}{P_3} \quad (4-5-2)$$

where  $P_1$  = power input to port 1  
 $P_3$  = power output from port 3  
 $P_4$  = power output from port 4

It should be noted that port 2, port 3, and port 4 are terminated in their characteristic impedances. The coupling factor is a measure of the ratio of power levels in the primary and secondary lines. Hence if the coupling factor is known, a fraction of power measured at port 4 may be used to determine the power input at port 1. This significance is desirable for microwave power measurements because no disturbance, which may be caused by the power measurements, occurs in the primary line. The directivity is a measure of how well the forward traveling wave in the primary waveguide couples only to a specific port of the secondary waveguide. An ideal directional coupler should have infinite directivity. In other words, the power at port 3 must be zero because port 2 and port 4 are perfectly matched. Actually, well-designed directional couplers have a directivity of only 30 to 35 dB.

Several types of directional couplers exist, such as a two-hole directional coupler, four-hole directional coupler, reverse-coupling directional coupler (Schwinger coupler), and Bethe-hole directional coupler (refer to Fig. 4-5-2). Only the very commonly used two-hole directional coupler is described here.



**Figure 4-5-2** Different directional couplers. (a) Two-hole directional coupler. (b) Four-hole directional coupler. (c) Schwinger coupler. (d) Bethe-hole directional coupler.



A two-hole directional coupler with traveling waves propagating in it is illustrated in Fig. 4-5-3. The spacing between the centers of two holes must be

$$L = (2n + 1) \frac{\lambda_g}{4} \quad (4-5-3)$$

where  $n$  is any positive integer.

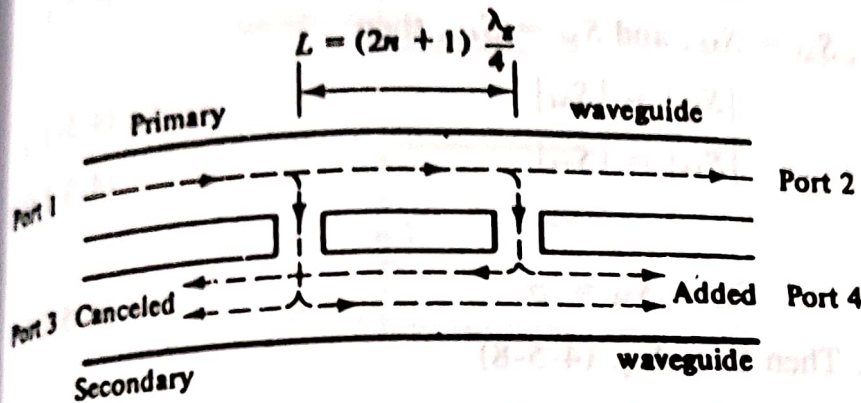


Figure 4-5-3 Two-hole directional coupler.

A fraction of the wave energy entered into port 1 passes through the holes and is radiated into the secondary guide as the holes act as slot antennas. The forward waves in the secondary guide are in the same phase, regardless of the hole space, and are added at port 4. The backward waves in the secondary guide (waves are progressing from right to left) are out of phase by  $(2L/\lambda_g)2\pi$  rad and are canceled at port 3.

## 4-5-2 S Matrix of a Directional Coupler

In a directional coupler all four ports are completely matched. Thus the diagonal elements of the S matrix are zeros and

$$S_{11} = S_{22} = S_{33} = S_{44} = 0 \quad (4-5-4)$$

As noted, there is no coupling between port 1 and port 3 and between port 2 and port 4. Thus

$$S_{13} = S_{31} = S_{24} = S_{42} = 0 \quad (4-5-5)$$

Consequently, the S matrix of a directional coupler becomes

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix} \quad (4-5-6)$$

Equation (4-5-6) can be further reduced by means of the zero property of the S matrix, so we have

$$S_{12}S_{14}^* + S_{32}S_{34}^* = 0 \quad (4-5-7)$$

$$S_{21} S_{23}^* + S_{41} S_{43}^* = 0 \quad (4-5-8)$$

Also from the unity property of the S matrix, we can write

$$S_{12} S_{12}^* + S_{14} S_{14}^* = 1 \quad (4-5-9)$$

Equations (4-5-7) and (4-5-8) can also be written

$$|S_{12}| |S_{14}| = |S_{32}| |S_{34}| \quad (4-5-10)$$

$$|S_{21}| |S_{23}| = |S_{41}| |S_{43}| \quad (4-5-11)$$

Since  $S_{12} = S_{21}$ ,  $S_{14} = S_{41}$ ,  $S_{23} = S_{32}$ , and  $S_{34} = S_{43}$ , then

$$|S_{12}| = |S_{34}| \quad (4-5-12)$$

$$|S_{14}| = |S_{23}| \quad (4-5-13)$$

Let

$$S_{12} = S_{34} = p \quad (4-5-14)$$

where  $p$  is positive and real. Then from Eq. (4-5-8)

$$p(S_{23}^* + S_{41}) = 0 \quad (4-5-15)$$

Let

$$S_{23} = S_{41} = jq \quad (4-5-16)$$

where  $q$  is positive and real. Then from Eq. (4-5-9)

$$p^2 + q^2 = 1 \quad (4-5-17)$$

The S matrix of a directional coupler is reduced to

$$S = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix} \quad (4-5-18)$$

**Example 4-5-1: Directional Coupler**



### 4-5-3 Hybrid Couplers

Hybrid couplers are interdigitated microstrip couplers consisting of four parallel strip lines with alternate lines tied together. A single ground plane, a single dielectric, and a single layer of metallization are used. This type of coupler, called a Lange hybrid coupler [3], has four ports, as shown in Fig. 4-5-6.

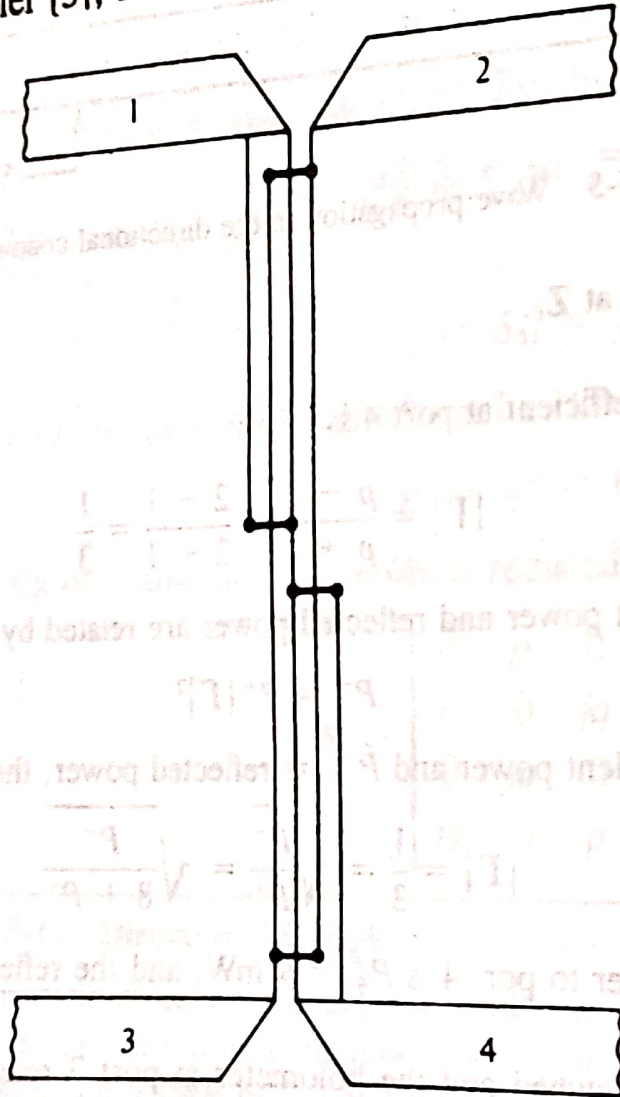


Figure 4-5-6 Lange hybrid coupler.

A signal wave incident in port 1 couples equal power into ports 2 and 4, but none into port 3. There are two basic types of Lange couplers:  $180^\circ$  hybrids and  $90^\circ$  (quadrature) hybrids. The latter are also called 3-dB directional couplers.

Hybrid couplers are frequently used as components in microwave systems or subsystems such as attenuators, balanced amplifiers, balanced mixers, modulators, discriminators, and phase shifters. The hybrid has a directivity of over 27 dB, a return loss of over 25 dB, an insertion loss of less than 0.13 dB, and an imbalance of less than 0.25 dB over a 40% bandwidth.

In modern microwave circuit design, Lange hybrid couplers are commonly

used in balanced amplifier circuitry for high-power and broad-bandwidth applications, as shown in Fig. 4-5-7.

Single-stage or cascaded double-stage GaAs MESFET chips are connected in parallel to two 3-dB and 90-degree Lange hybrid couplers. Their basic relationship can be expressed by the following three equations:

$$S_{11} = \frac{1}{2}(S_{11a} - S_{11b}) \quad (4-5-19)$$

$$S_{22} = \frac{1}{2}(S_{22a} - S_{22b}) \quad (4-5-20)$$

and

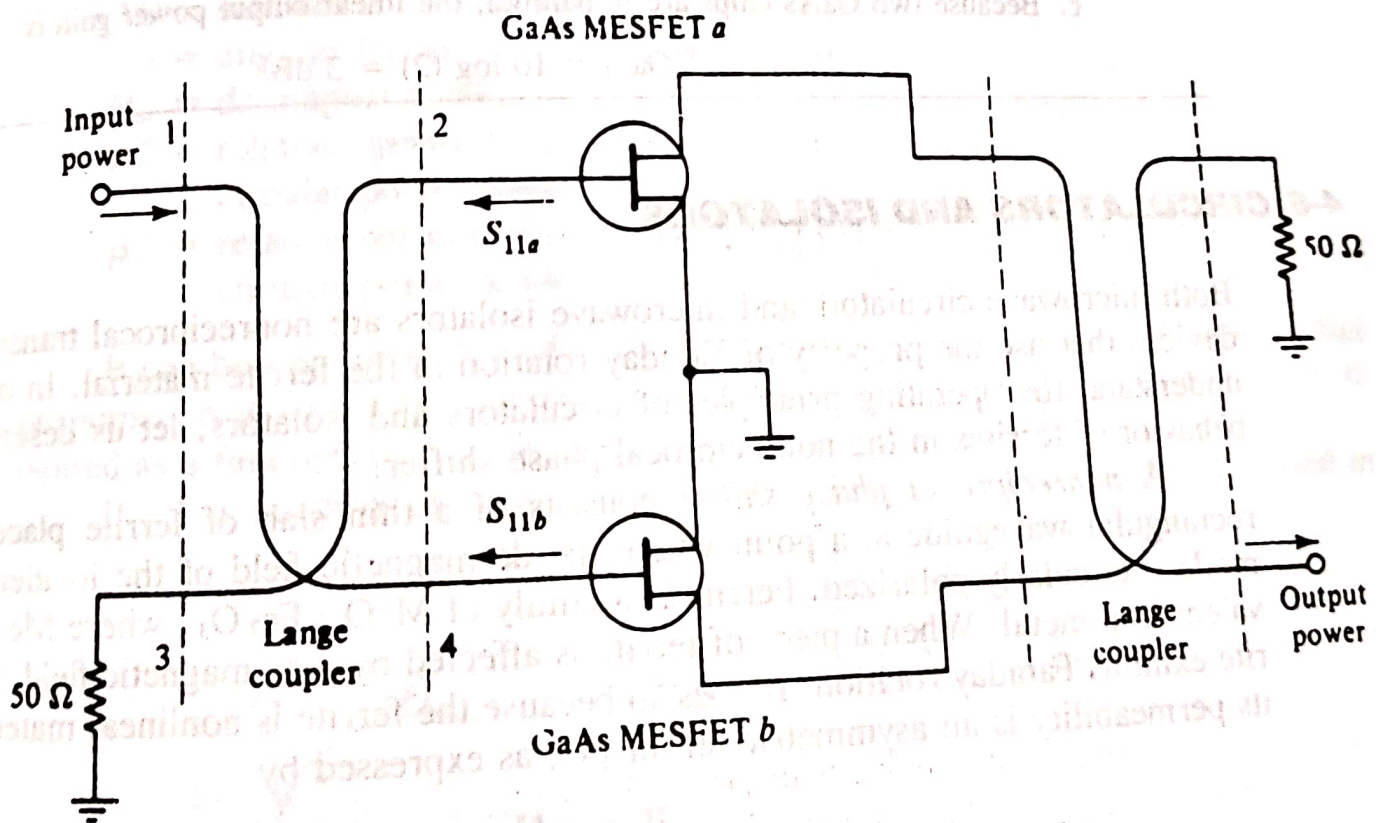
$$\text{Gain} = |S_{21}|^2 = \frac{1}{4}|S_{21a} + S_{21b}|^2 \quad (4-5-21)$$

where  $a$  and  $b$  indicate the two GaAs MESFET chips, and 1 and 2 refer to the input and output ports, respectively. The VSWRs of the balanced amplifier can be expressed as

$$\text{VSWR} = \frac{1 + |S_{11}|}{1 - |S_{11}|} \quad \text{for the input port} \quad (4-5-22)$$

and

$$\text{VSWR} = \frac{1 + |S_{22}|}{1 - |S_{22}|} \quad \text{for the output port} \quad (4-5-23)$$



**Figure 4-5-7** Balanced amplifier with Lange couplers.

Theoretically, if the two GaAs MESFET chips (or four chips in a double-stage amplifier circuit) are identical, the amplifier is balanced and its VSWR will be unity. Practically, however, characteristics of the two GaAs MESFET chips are not actually measured and they may not be the same. When their characteristics are different, the amplifier will not be balanced and manual tuning will be needed to balance it. Therefore, for mass production it is necessary to characterize the GaAs MESFET



chips in advance before placing them in the microwave integrated circuit in order to minimize the tuning work, reduce the production cost, and increase the hybrid reproductibility.