M. Sc. (Physics) Semester IV

PHYC-402

NUCLEAR PHYSICS –2

Collective model, Rotational and Vibrational Hamiltonian, Vibrational and rotational spectra of different nuclei

VINOD KUMAR

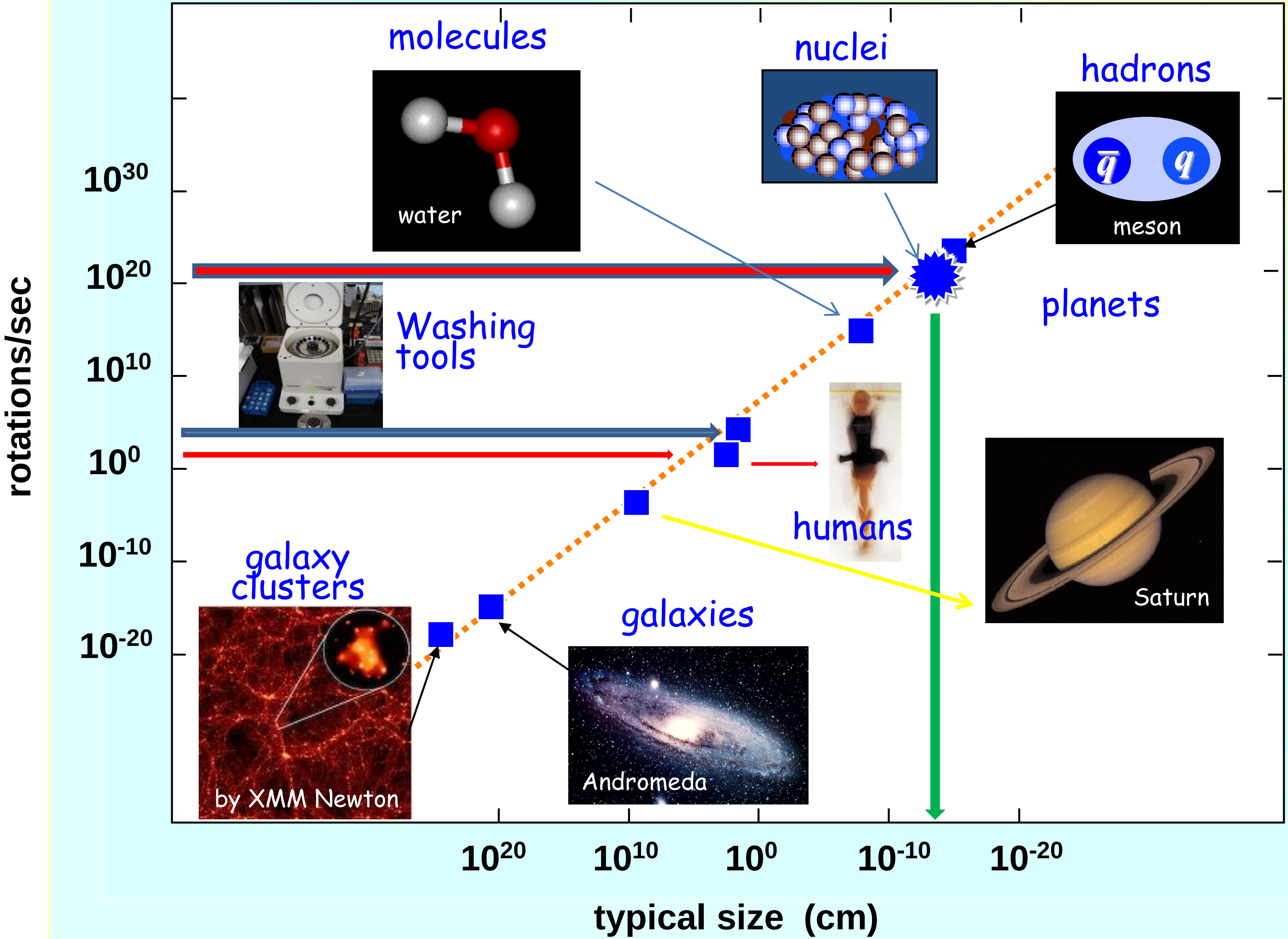
Physics Department

University of Lucknow

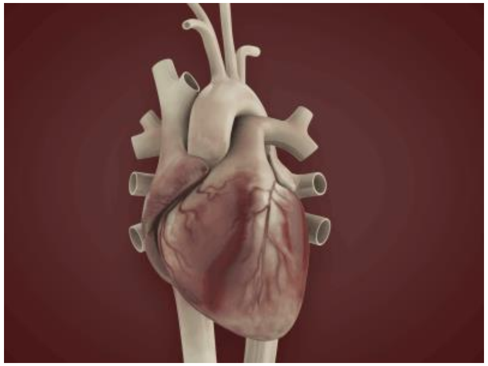
- Every single thing in the Universe is either vibrating or spinning and both at a particular frequency. Nothing rests.

- Whenever mass get energy or has energy, it starts to either vibrate or rotate to sustain stability. In other we can say vibration and rotation are the keys of stability.

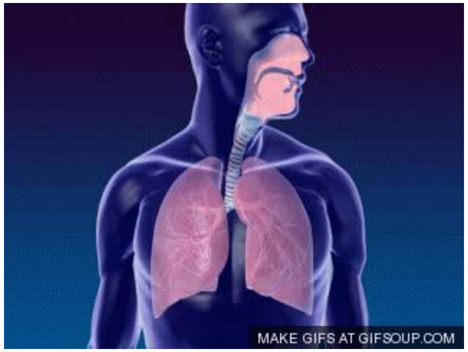
rotations in the universe



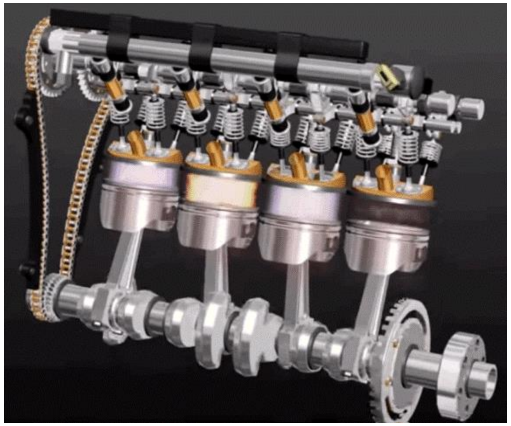
Vibrations in the universe



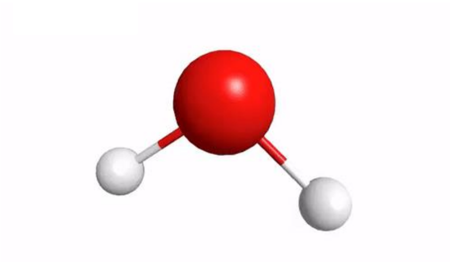
Human Heart



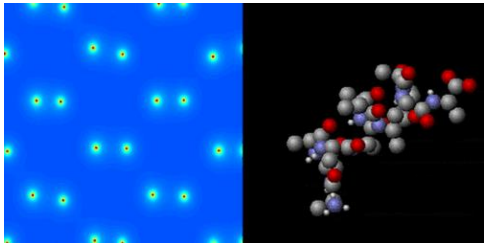
Milky Way



Engine



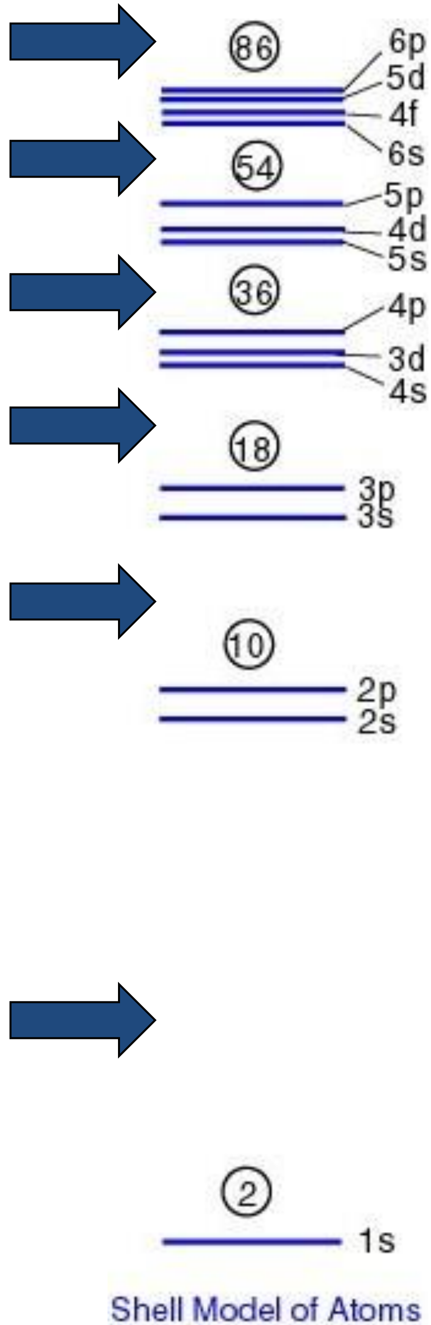
Vibration of a H2O molecule



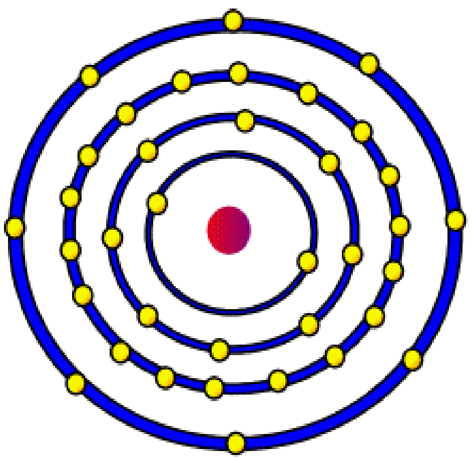
Vibrations of an Alpha Protein

Atom:
electron shells

Nucleus:
proton/neutron shells

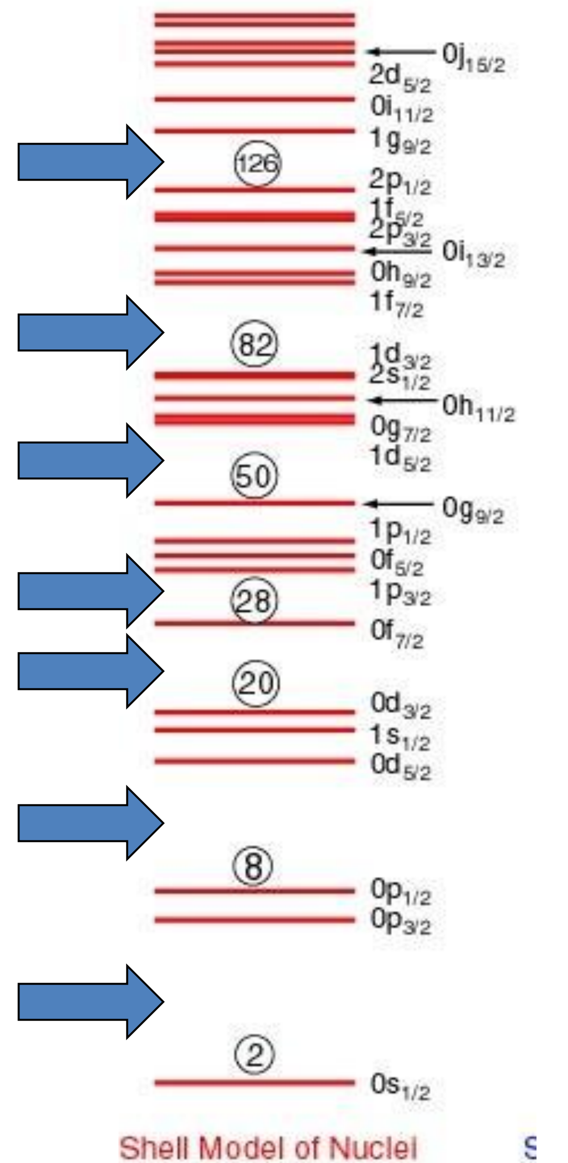


noble gases
(closed shells)

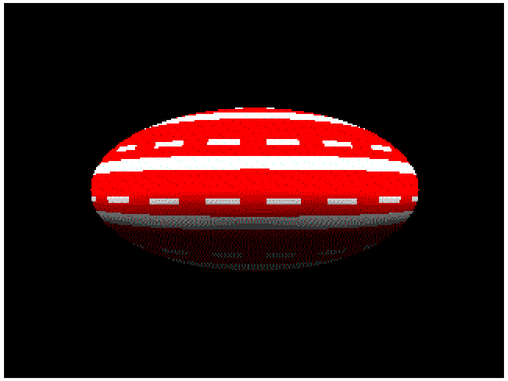


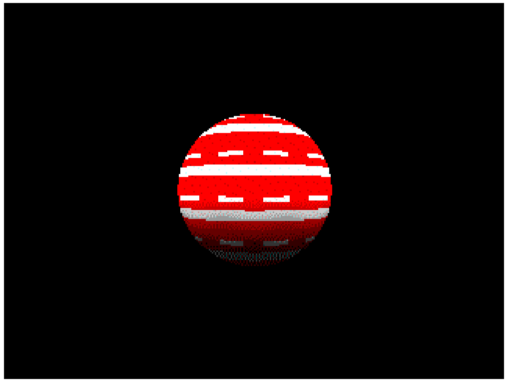
Krypton Atom

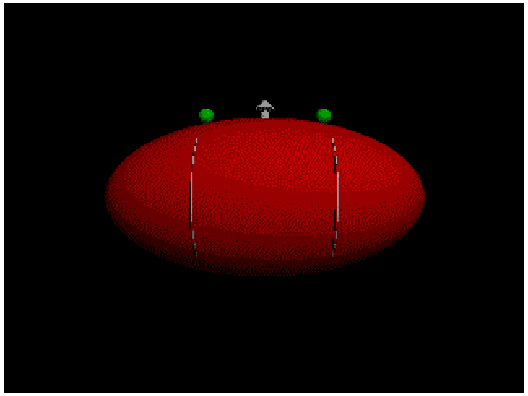
magic nuclei
(closed shells)

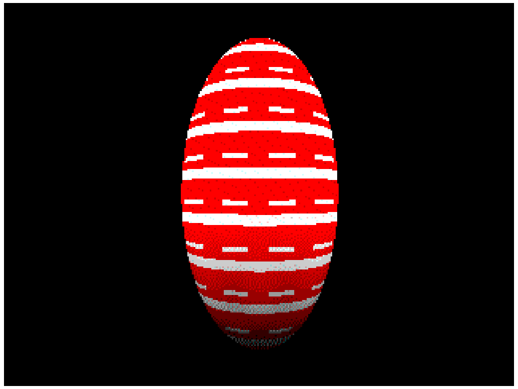


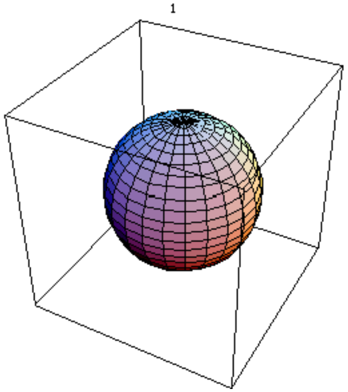
Rotation in Nuclei & Vibration

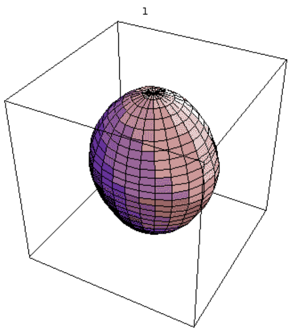




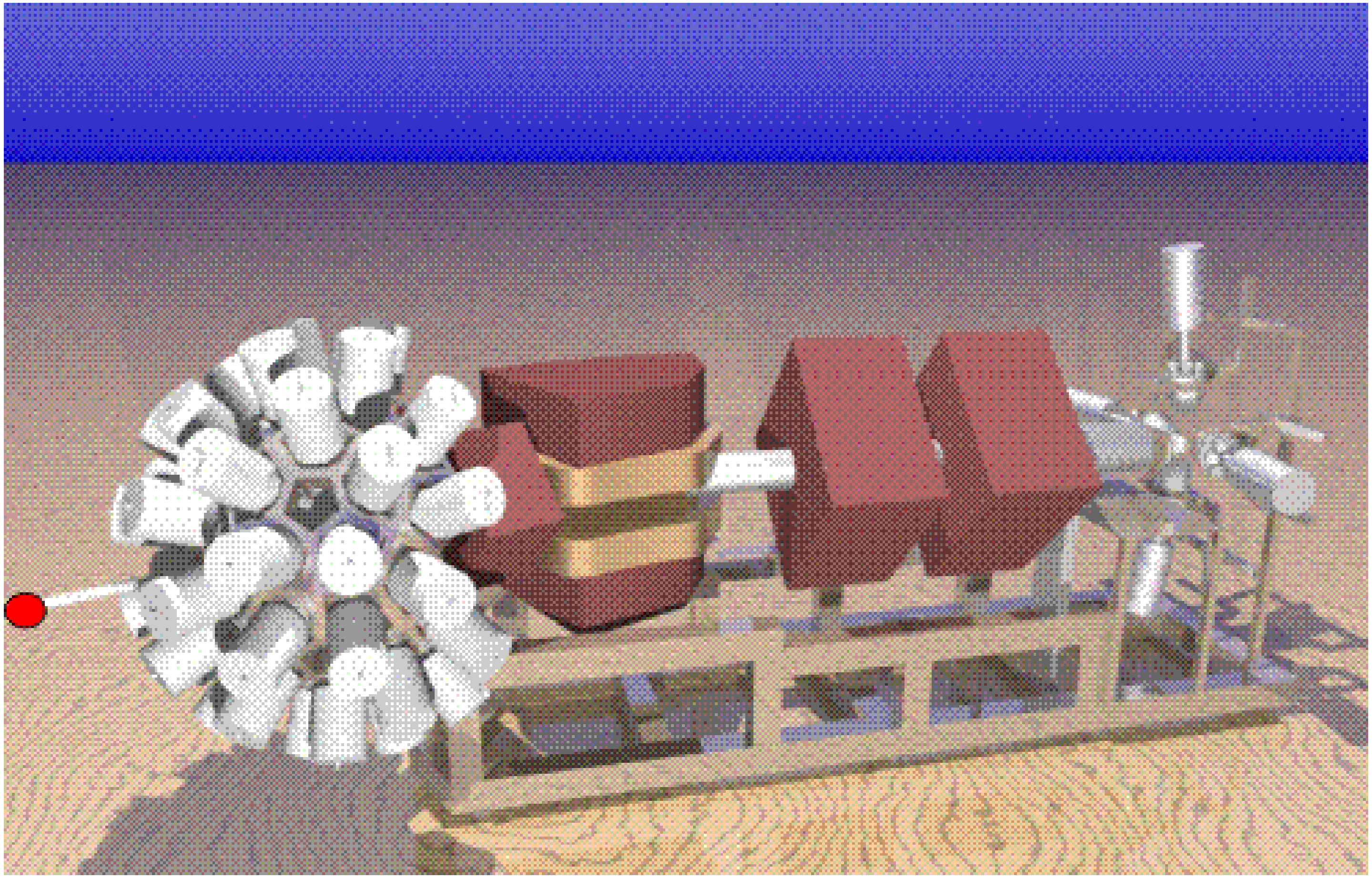




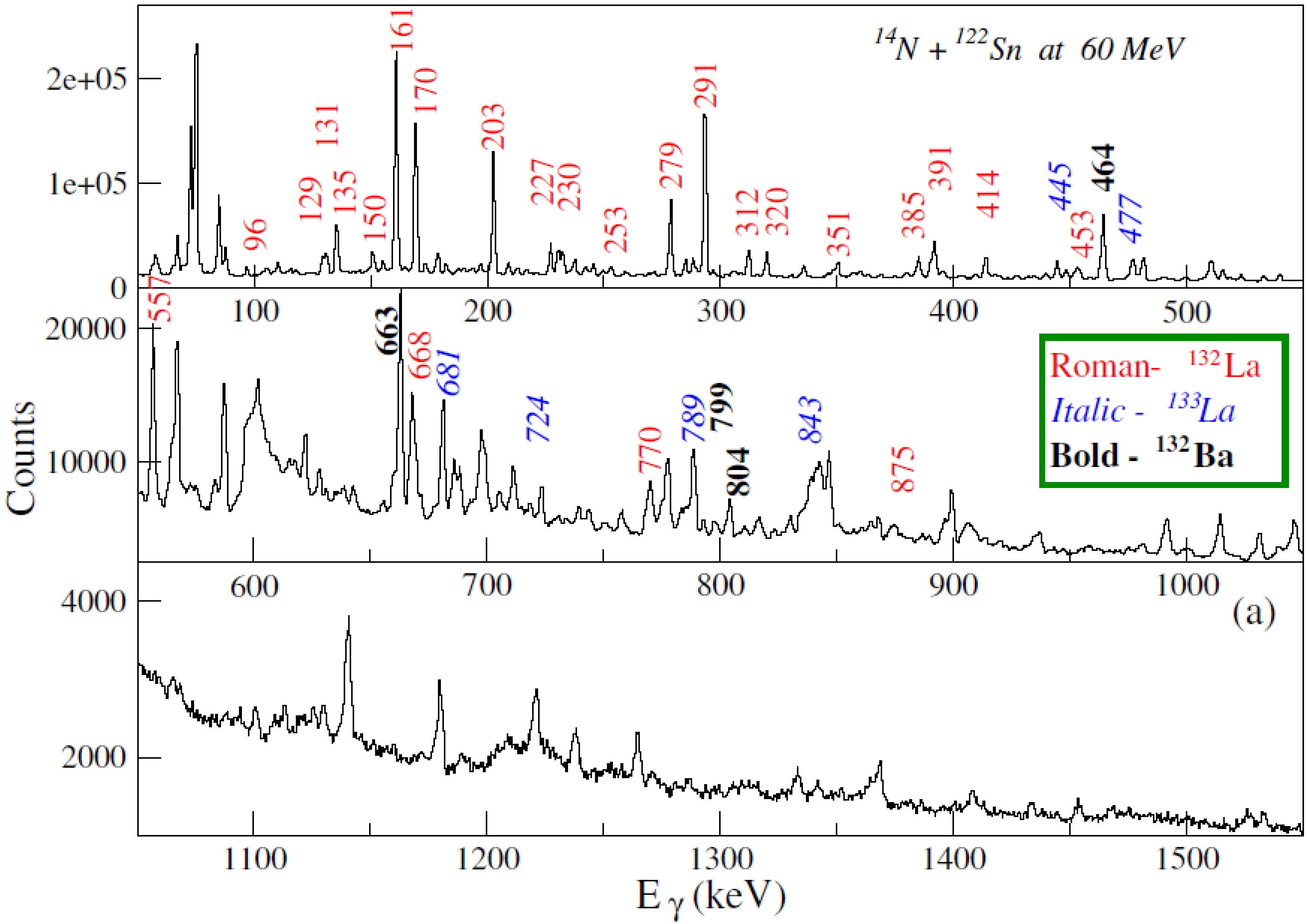


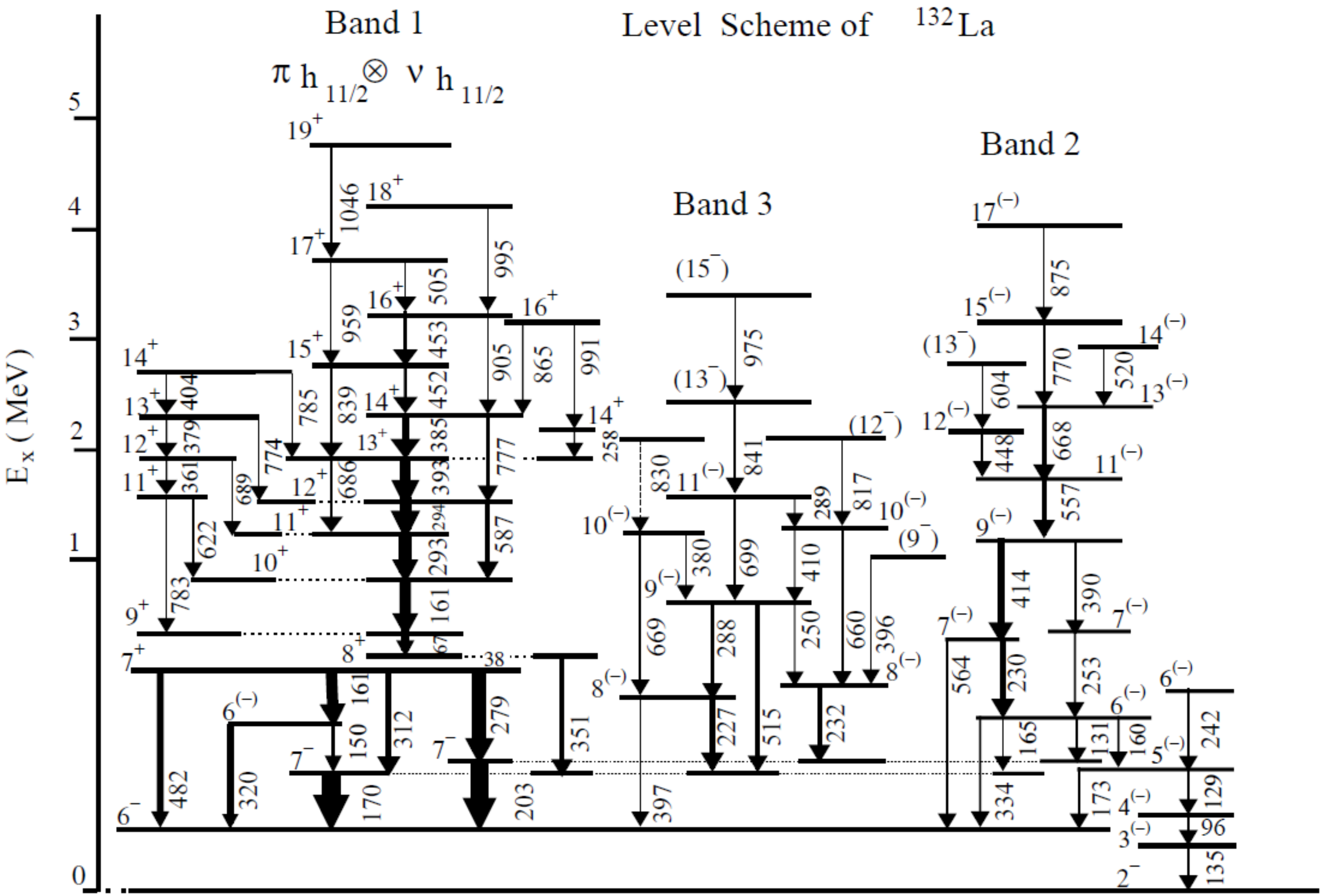


Experimental setup for γ-ray spectroscopy



γ-ray spectrum

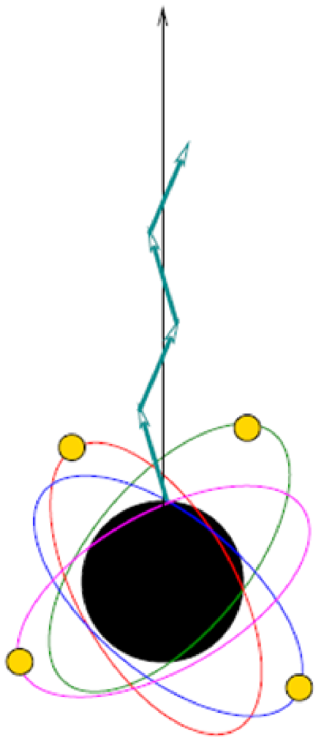


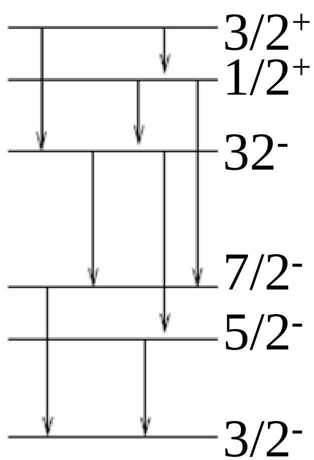

Level Scheme of 132La
Band 1
π h 11/2 ⊗ ν h 11/2
Band 2
Band 3
Ex (MeV)

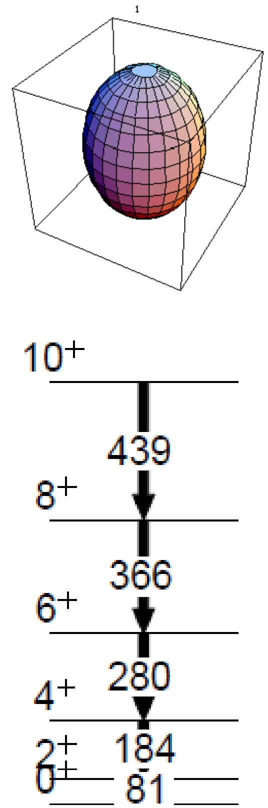
Excited States in Nuclei, Generation of Angular momentum

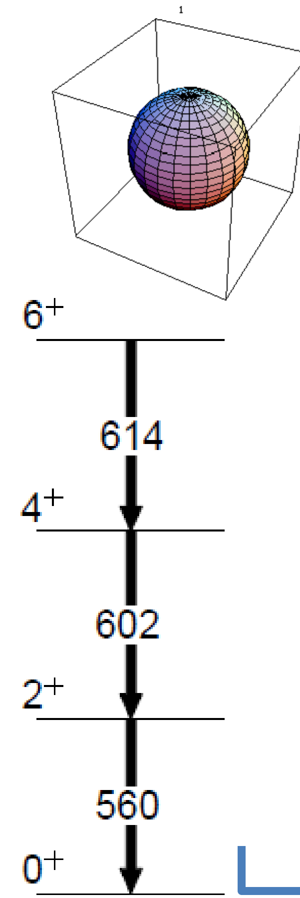
(Noncollective (out of phase) motions of nucleons) (collective (in phase) motions of nucleons)

Single Particle | Rotation | Vibration









$$E_{rot}(I) = \frac{\hbar^2}{2\zeta} I(I+1)$$

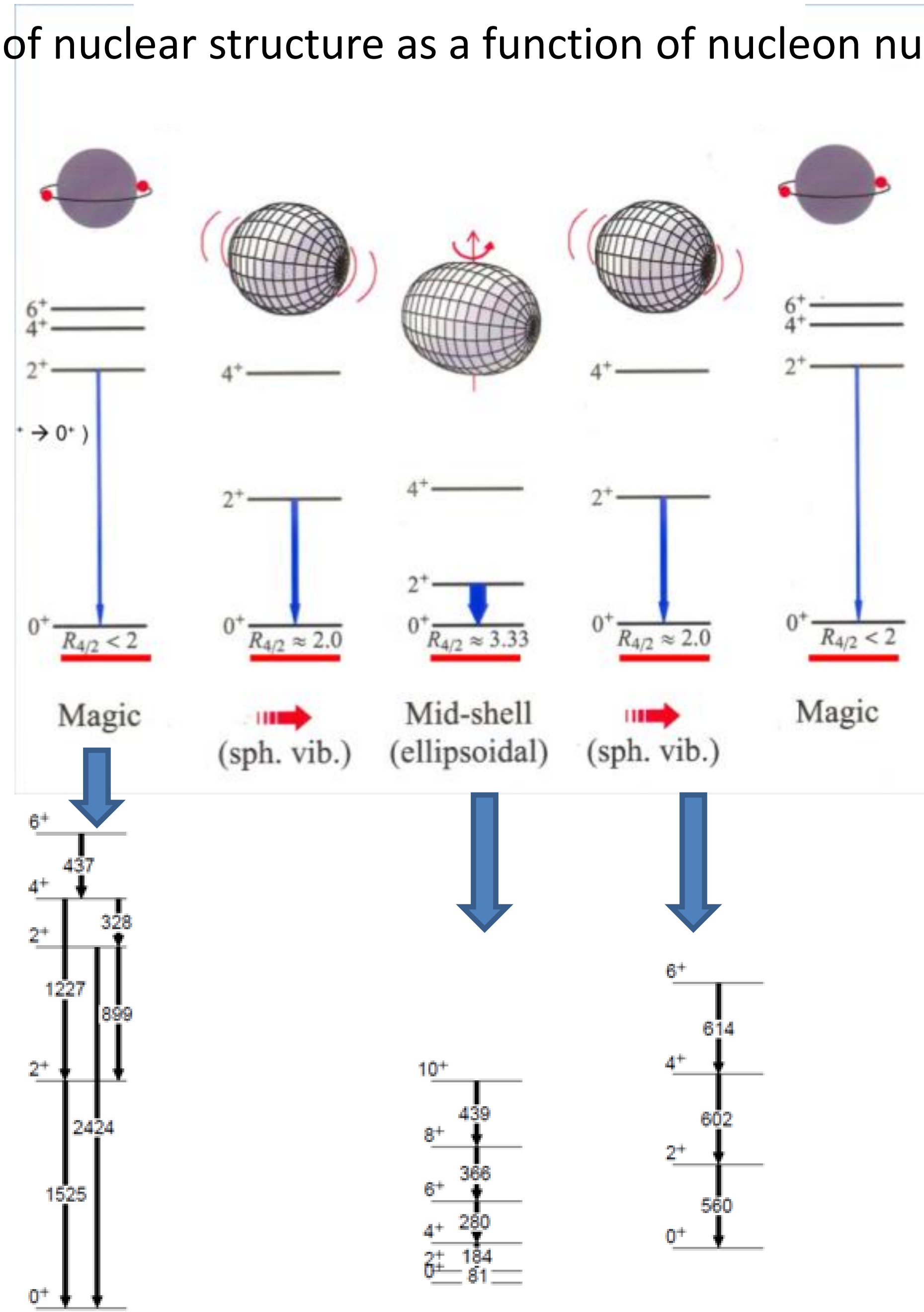
$$E_{vib}(I) = n(\hbar\omega)$$

$$E_{sp} = \left(N + \frac{3}{2}\right)\hbar\omega - K\hbar^2 l(l+1) + \xi\hbar^2 \left\{ \begin{array}{cc} -\frac{1}{2}l & j = l + \frac{1}{2} \\ \frac{1}{2}(l+1) & j = l - \frac{1}{2} \end{array} \right\}$$

$$\hbar\omega \simeq 41 A^{-1/3}\, MeV$$
$$\xi\hbar^2 \simeq 20 A^{-2/3}\, MeV$$
$$K\hbar^2 \simeq 0.1\, MeV$$

Evolution of nuclear structure as a function of nucleon number

$$R_{4/2} = \frac{E(4^+)}{E(2^+)}$$



^{42}Ca ^{166}Er ^{120}Te

Non-collective Level Scheme

- Complicated set of energy levels
- No regular features, e.g. band structures
- Some states are isomeric

Collective Level Scheme

- Large electric quadrupole moments Q_0
- Low-lying rotational bands ($E \propto I[I+1]$)
- The low-lying levels of deformed even-even nuclei which lie far from closed shells form a regular sequence of levels that are much lower in energy than the pairing energy. This arises from rotation.

The origin of deformation lies in the long range component of the nucleon-nucleon residual interaction: a quadrupole-quadrupole interaction gives increased binding energy for nuclei which lie between closed shells if the nucleus is deformed. In contrast, the short range (pairing) component favours sphericity

Ground State (I^{π})

- All even-even nuclei have a ground state with $I^{\pi} = 0^{+}$, a consequence of nuclear pairing
- For odd-mass nuclei (near closed shells) the low lying excited states map out the single-particle spectrum of states around the Fermi level

Types of Excitation

- Closed-shell nuclei are spherical and first excited nuclear states can only be formed by breaking pairs of nucleons or by vibrations.
- The energy difference between the first excited and ground states is a rough measure of the pairing energy
- ‘Deformed’ nuclei exhibit regular rotational bands:

Rotational Energy in Classical & Quantum Expression

The classical expression for rotational energy is

$$E_R = \frac{1}{2}\mathcal{J}\omega^2$$

Where $\mathcal{J}$ is the moment of inertia.

If we have angular momentum L, then it is related to $\mathcal{J}$ by:

$$L = \mathcal{J}\omega$$

Then $\omega = L / \mathcal{J}$

We have $$E_R = \frac{L^2}{2\mathcal{J}}$$

The expression for L^2 in QM $L^2 = I(I+1)\hbar^2$

Hence, we have the expression for QM rotor:

$$\boxed{E_R = \frac{\hbar^2 I(I+1)}{2\mathcal{J}}}$$

The moment of inertia is somewhere between that predicted by the nucleus being fluid, and that being a rigid body.

$$\boxed{\mathcal{J}_{fluid} < \mathcal{J}_{nuclear} < \mathcal{J}_{rigid}}$$

Moment of Inertia

- There are three types of moment of inertia used to describe high-spin rotational structures, the static ($\mathcal{J}^{(0)}$), kinematic ($\mathcal{J}^{(1))}$) and dynamic ($\mathcal{J}^{(2)}$). The static moment of inertia is defined by the simple relation**,**

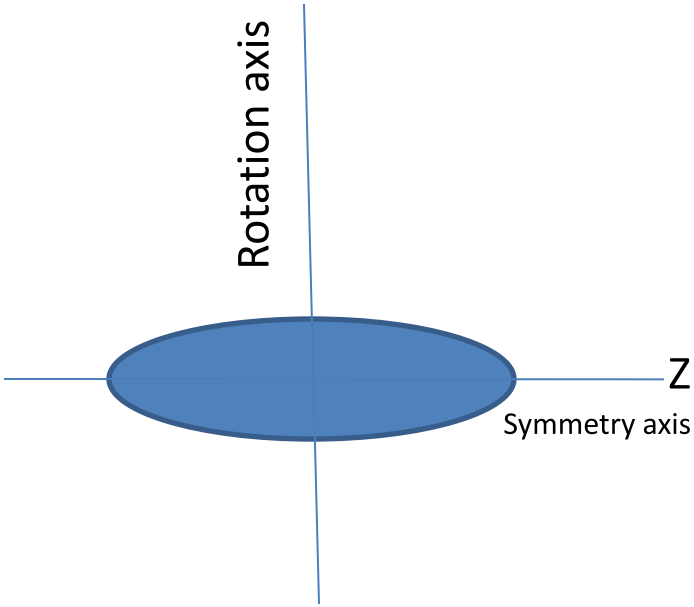
$$E_{rot}(I) = \frac{\hbar^2}{2\mathcal{J}^{(0)}(I)} I(I+1)$$

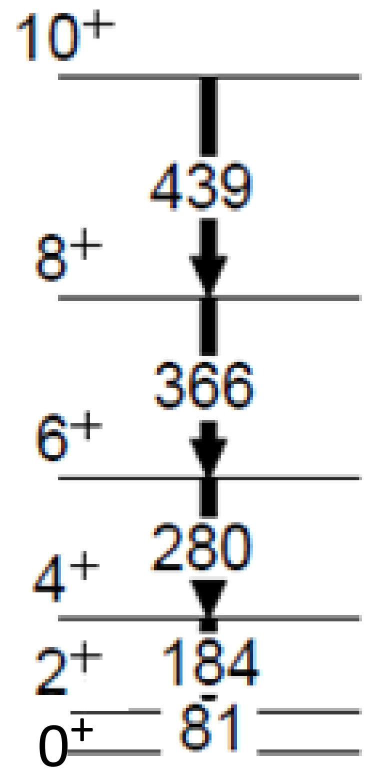
- The kinematic moment of inertia is given by

$$\mathcal{J}^{(1)}(I) = \frac{I}{\omega}$$

- while the dynamic moment is given by $$\mathcal{J}^{(2)} = \frac{dI}{d\omega} \approx \frac{4\hbar}{\Delta E_\gamma}$$

Rotational bands in even-even nuclei de-excite by cascades of γ-ray transitions, between adjacent states





$$E_R = \frac{\hbar^2 I(I+1)}{2\mathcal{J}}$$

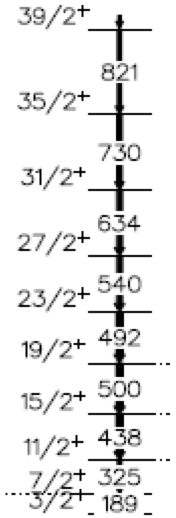
$$\frac{E(4^+)}{E(2^+)} = \frac{265}{81} = 3.27$$

Note that rotation cannot take place about the symmetry (z) axis

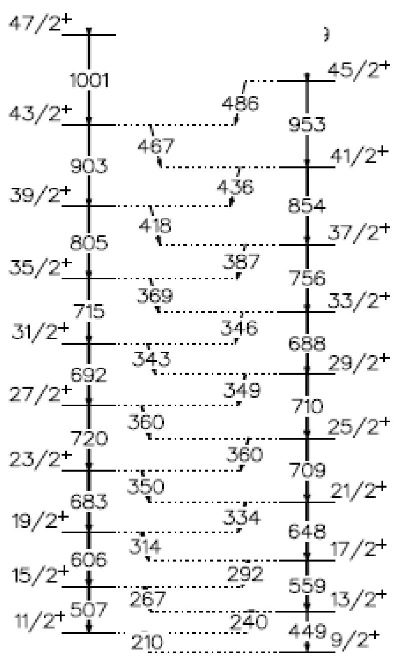
Rotations of a Deformed Odd-A Nuclei

The Hamiltonian is: $$H_{rot} = \frac{\hbar^2}{2\Im}\underline{R}^2 = \frac{\hbar^2}{2\Im}(\underline{I} - \underline{J})^2 \qquad \{ \underline{R} \equiv \vec{R} \}$$

Where $\Im$ is the moment of inertia and J is additional angular momentum generated by, e.g. the odd particle in an odd-A nucleus.

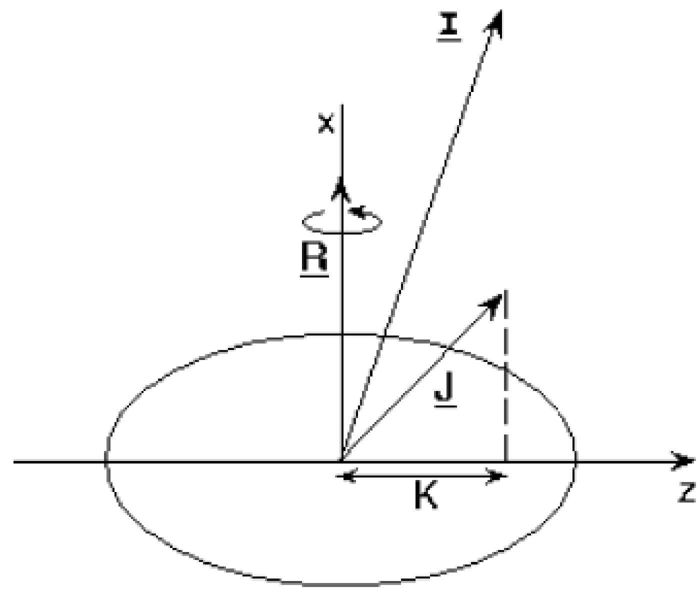


Low K band



High K band

^{123}La



Nuclear spins

Coriolis Coupling

- The rotational Hamiltonian can be expanded:

$$\underline{R}^2 = (\underline{I} - \underline{J})^2 = \underline{I}^2 - 2\underline{I}.\underline{J} + \underline{J}^2$$
$$= I^2 + J^2 - 2K^2 - (I_+J_- + I_-J_+)$$

where $I_\pm = I_x \pm i\, I_y$, $J_\pm = J_x \pm i\, J_y$ and $J_z = I_z = \pm K$

- The quantity K is the projection of I along the rotation axis
- The coupling term $(I_+J_- + I_-J_+)$ corresponds to the Coriolis force and couples $\underline{J}$ to $\underline{R}$

- The operators $I_{\pm}$ link states with K differing by ±1

- The term $(I_+J_- + I_-J_+)$ can be ignored if rotational bands with $\Delta K = 1$ lie far apart and the particular band does not have $K = ½$

- The excitation energies then become:

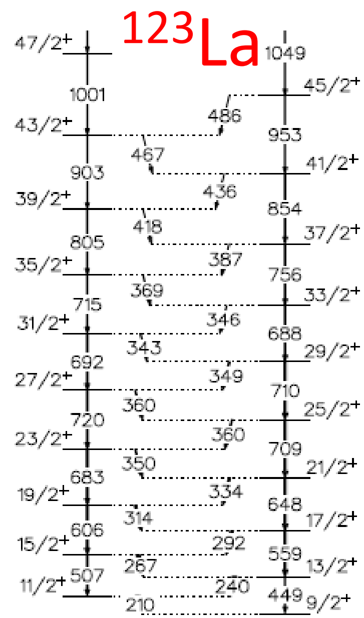
$$E_{rot} = (\hbar^2/2\Im)[I(I+1) + J(J+1) - 2K^2]$$

with I = K, K+1, K+2…

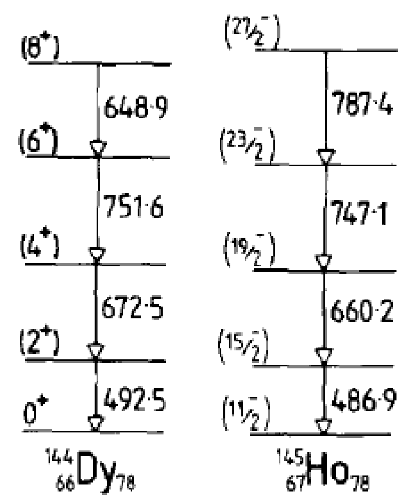
- K is a constant of the motion

- Then: $E_{rot} = E_K + (\hbar^2/2\Im)I(I+1)$

where E_K is the energy of the lowest band level



High K



K=1/2

Collective Motion in Nuclei

Molecules:

electronic motion fastest, vibrations 10^2 times slower, rotations 10^6 times slower

These different motions have very different time scales, so the wave function separates into a product of terms

In nuclei

the timescales are much closer

Collective and single-particle modes can perhaps be separated but they will interact strongly !

- The deformed wave function incorporates the two aspects of intrinsic motion (single particles) and rotational motion (even-even core).

- The latter is specified in terms of the well-known rotational D matrices, the former in terms of the wave functions χ_K.

- The adiabatic assumption of the separability of rotational and intrinsic motions leads to a product wave function in *D and* χ_K

$$\psi_{IK} = \left(\frac{2I+1}{16\pi^2}\right)^{\frac{1}{2}} \left[D_{IMK}\ \chi_K + (-1)^{I-K}\ D_{IM-K}\ \chi_K \right]$$

- Note that for *K = 0, only even I values are allowed, so wave function collapses* to a single term

Energy of a state & energy of γ-rays

$$E(I) = \frac{\hbar^2}{2\mathcal{J}} I(I+1)$$

$$\boxed{E_s(I) \propto I^2}$$

$$E_\gamma(I) = E(I) - E(I-2) = \frac{\hbar^2}{2\mathcal{J}}[I(I+1) - (I-2)(I-1)]$$

$$E_\gamma(I) = \frac{\hbar^2}{2\mathcal{J}}(4I-2)$$

The energy of γ-rays should increase linearly with angular momentum (I)

$$\boxed{E_\gamma(I) \propto I}$$

Determining Nuclear Quadrupole Deformation from Lifetimes of E2 Transitions

- Lifetime of a state in a stretched E2 cascade (rotational band) can allow the deduction (in a model dependent way) of the nuclear deformation.

- Lifetime is related to the quadrupole moment

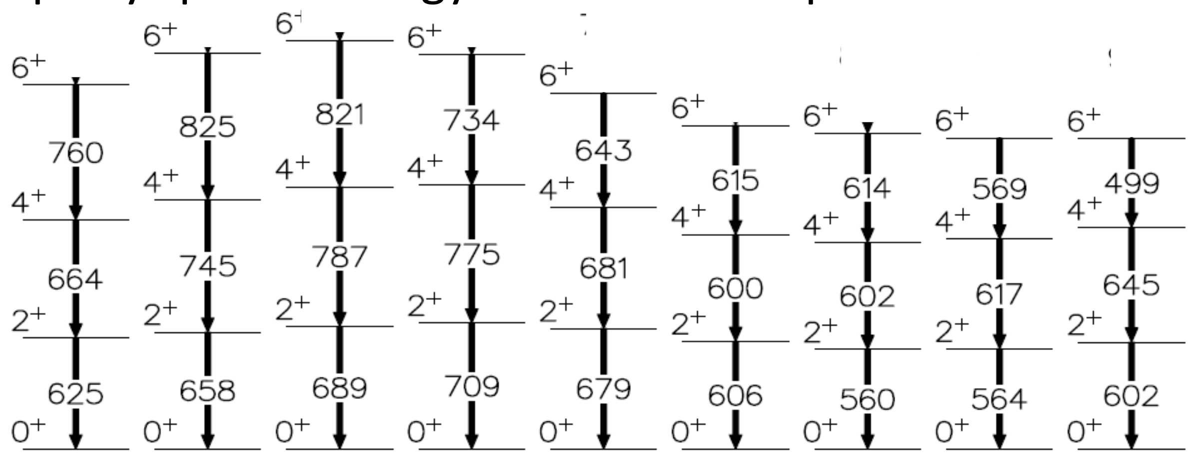
$$\frac{1}{\tau} = 1.223 E_\gamma^5 \frac{5}{16} Q_o^2 |< J_i K 20 | J_f K >|^2$$

- Quadrupole moment can be related to the quadrupole deformation parameter β_2

$$Q_0 = \frac{3}{\sqrt{5\pi}} Z R^2 \beta_2 \left(1 + \frac{1}{8}\sqrt{\frac{5}{\pi}}\beta_2 \ldots\right)$$

A vibration can be any distortion in the nuclear shape

Equally spaced energy levels for each phonon of vibration



Te A:	108	110	112	114	116	118	120	122	124
$E(4^+)/E(2^+)$	2.06	2.13	2.14	2.09	2.00	1.99	2.07	2.09	2.07

The ratio of 4^+ energy levels to 2^+ energy levels gives an indication of whether a nucleus is a vibrator or a rotor. In the vibrational limit, this energy ratio is equal to 2.0 ($E \propto n$), while in the rotational limit it is equal to 3.33 ($E \propto I(I+1)$).

Assignment

1. Draw the expected emitting γ-ray spectrum from exited nuclei by assuming the Classical & Quantum rotor expression.
2. How are the energy of a state & energy of γ-rays related with angular momentum?
3. Why are only even I (spin) values allowed for rotational bands in even-even nuclei ?
4. How is the ratio of 4^+ energy levels to 2^+ energy levels gives an indication of whether a nucleus either a vibrator or rotor?
5. How do you make a difference that 2^+ state generated either through noncollective or collective behaviour of nuclei
6. Draw rotational bands of a odd-A nuclei for K=7/2 and K=1/2.
7. Draw excited states of a deformed even-even nuclei if rotate about the symmetry axis.
8. The nucleus ^{24}Mg has a 2^+ first excited state at 1.369 MeV and a 4^+ second excited state at 4.123 MeV. Which model would be most likely to provide an accurate description of these states? Calculate the energy of fourth excited state.

9. The levels of ^{174}Hf show with energies given as follows (in MeV):

$E(0^+)$	$E(2^+)$	$E(4^+)$	$E(6^+)$	$E(8^+)$	$E(10^+)$
?	?	?	0.608	1.010	1.486

Calculate the energy of 0^+, 2^+ and 4^+ states.

10. The levels of ^{174}Hf show two similar rotational bands, with similar energies and different lifetime given as follows (in ps):

	$E(0^+)$	$E(2^+)$
Band 1	10	1
Band 2	100	0.1

Which band is correspond to large deformation of nuclei?

REFERENCES FOR ADDITIONAL READING

books written by: K. S. Krane, R.F Casten, A Bohr & B Mottelson and R.R Roy & B.P. Nigam