Ch.3 Superconductivity

Read one of the following references:

1. F. J. Owens and C. P. Poole, Jr., "The new superconductors", (QC611.95 .O89 2002 **ebook**).
2. V L Ginzburg & E A Andryushin, "Superconductivity" 2/e (QC611.92 .G56 2004 **ebook**)
3. G. Vidali, Superconductivity: the next revolution? (1993) (QC611.92V53)
4. P.F. Dahl, Superconductivity (QC611.92D34 1992)

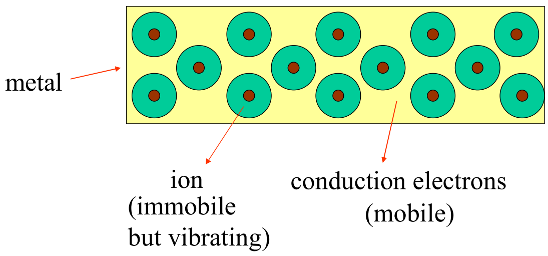
(I) Introduction

- Solids have different conductivity:

Solid materials
- conductor
  - Superconductor
  - metal
  - semimetal
- semiconductor
- insulator

- Superconducting materials: elements, alloys, compounds. (classified as Type I & Type II)
- Type I superconductor has properties:
  perfect conductor ($\rho = 0$)
  Meissner's effect ($B = 0$)
- Applications: superconducting magnet, Maglev, SQUID, …

Origin of conductivity in metal



Typically, the number of conduction electrons in a solid
$n \approx 10^{22}$ electrons/cm^3.
Note: At higher pressure, n (and thus conductivity) increase.
e.g. hydrogen can be forced to become metallic at
~ 1-10 megabars. (1 bar ≈ 1 atmospheric pressure)
(http://www.nature.com/news/1998/000622/full/news000622-6.html)

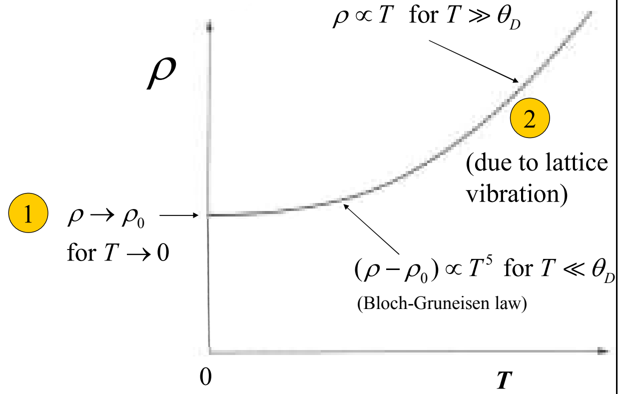
Motivation:
As $T \rightarrow 0$, gas becomes liquid, then solid in which atoms have only minimum thermal vibrations.
How about conduction electrons in metals at low temperature?

To answer this question, need QM.
Electron sees periodic potential and
has wave behavior (wave-particle duality).

By solving Schrödinger's equation (a wave equation), we can find allowed energy states for electrons and electron in these states can travel through the solid more freely!
Typical result (covered in PHY 4450) is shown in a graph:

Resistivity of a metal as a function of temperature:

(θ_D = Debye temperature)



Origin of resistance:

1. ρ_0 (residual resistivity) due to defects, impurities, etc.
2. temperature dependence of ρ depends on electron-ion scattering & lattice vibrations.

$\rho \propto T$ for $T \gg \theta_D$

$(\rho - \rho_0) \propto T^5$ for $T \ll \theta_D$

$\theta_D = 315K$ for Cu

$\theta_D = 230K$ for Pt

What happens when $T \rightarrow 0$?

If there are no defects and impurities then $\rho = \rho_0 = 0$ at $T = 0$.
Ideal periodicity ⇒ No resistance!
(impossible!)

Classical picture: At $T = 0$, all ions are standing still.
Quantum picture: Even at $T = 0$, electrons are still moving inside atoms while the atoms can almost stand still.

Conjectures near 1900:

James Dewar: $\lim_{T \to 0} \rho(T) = 0$
(no lattice vibration ⇒ very pure metal is a perfect conductor)

Lord Kelvin, Kamerlingh Onnes: $\lim_{T \to 0} \rho(T) = +\infty$
(freezing of electron gas)

No theory was known at that time!
Need experimental observation.

(II) The big chill: low temperature experiments.

1883 Olszewski & Wroblewski liquefied
oxygen (90.2 K) (boiling point at 1 bar) and
nitrogen (77.3 K).
1898 James Dewar liquefied hydrogen (20.5 K).
1908 K. Onnes liquefied helium (4.2 K),
obtained 1.7 K at reduced pressure.
Onnes obtained Nobel prize in 1913.

Lowest temperature recorded on Earth: in Vostok (Russian)
-89.2 ˚C (confirmed): July 21, 1983
-91 ˚C (unconfirmed): 1997 (colder than dry ice!)
The coldest temperature achieved in physics labs:
100 pK (in 2000)
(http://ltl.tkk.fi/wiki/LTL/World_record_in_low_temperatures)
Temperature in deep space ~ 2.735 K.

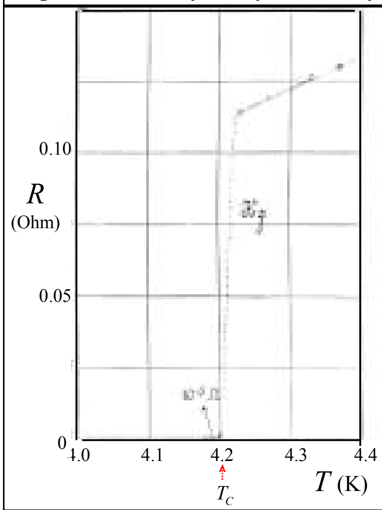
Onnes then measured the resistance of pure substances at these low temperatures using 4-point probe technique (taught in PHY2822).

Onnes's motivation:
1. to find out $\lim_{T\to 0}\rho(T) = ?$
2. to develop new thermometer using $R(T)$.
   (At that time, gas thermometers (bulky) were used for low temperature experiments.)

Results:
Smaller ρ_0 for purer samples, as expected.
He also showed that $\lim_{T\to 0}\rho(T) = +\infty$ is not correct.

Note: Exceptional cases were found in some metals,
$\frac{d\rho}{dT} < 0$ at very low temperatures.
They are Kondo effect (1964) and
weak localization effects (1978).

Onnes thought $\rho_0 = 0$ could be achieved by purification. He looked for very pure samples and limited his studies to pure metals only.
Then he studied Hg because it was easier to purify Hg. The experiment was done by G. Holst (a graduate student) and he found a rapid drop of R to zero at very low temperatures. Originally Holst thought it was due to short-circuit and thus modified his sample. The resistance drop was confirmed by a mistake on April 28, 1911. (J. de Nobel, "The discovery of superconductivity", Physics Today, Sept. 1996, pp.40-42.)



$T_C = 4.15$ K for Hg
- $10^{-5}\ \Omega$ is due to limit of instrument for measurements.
- Lower temperature can be reached by pumping liquid helium bath.
- More superconductors were found later by his group:
  In ($T_C = 3.4$ K)
  Sn ($T_C = 3.7$ K)
  Pb ($T_C = 7.2$ K)

Onnes called it supra-conductivity and later ***superconductivity***.

Nobel Prize (1913)

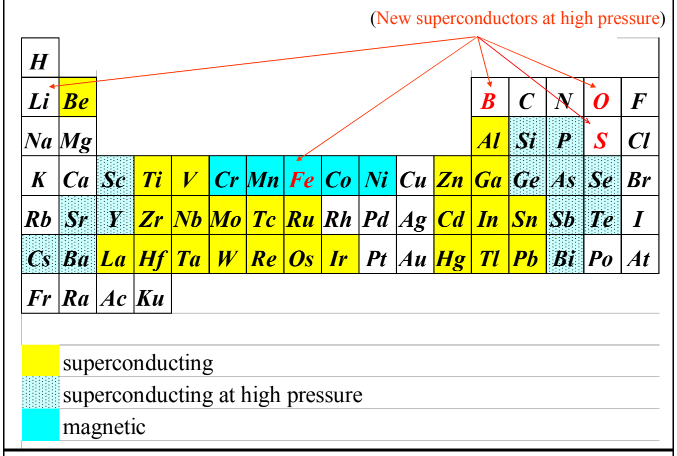
Heike Kamerlingh Onnes
(1853 - 1926)

"for his investigations on the properties of matter at low temperatures which led, *inter alia*, to the production of liquid helium"

For more than 4 decades since Onnes's discovery, superconductors were
- rare
- unpredictable
- poorly understood
- little connection with normal state properties.

1929 Hans Meissner found the barely metallic compound CuS to be superconducting whereas elemental Cu was not.
1930s Hans Meissner found superconducting intermetallic borides.
1950s John Hulm & Bernd Matthias discovered superconducting intermetallic alloys and compounds.

Superconducting elements known today:



Features:
- "Good metals"
  (e.g. Na, K, Cu, Ag, Au, Pt, etc)
  are <u>NOT</u> superconductors. **(How to check?)**

  (Exception: Li was found to be superconducting at high pressure in 2002. See p.12.)

  Recall: Electrons in "good metals" don't scatter much.
  ⇒ **Need bad conductivity to enhance superconductivity?**
  (No! Superconductivity needs appropriate lattice vibrations.)
- Magnetic elements are not superconducting.
  (How does magnetic field affect superconductivity?)
  (Exception: Fe was found to be superconducting at high pressure in 2001. See p.12.)

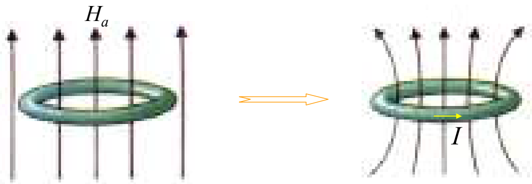
(III) How to make sure that $R = 0$?

For small resistance measurements, 4-point probe method must be used to eliminate the contact problems.

The smallest resistance that can be detected depends on the sensitivity of voltmeters or multimeters:
10 μV sensitivity in PHY2811/2822 lab
1 μV sensitivity in PHY3811/3822 lab.
Voltmeters with 1 nV or better sensitivity are commercially available. (www.keithley.com)
The most sensitive voltmeter is made of a superconducting device called SQUID (see p.10 & p.13).

A simple approach to test $R=0$ is to use a **superconducting ring** (first by Tuyn and Onnes).

The current in the ring can be induced by Faraday's law:



① First cool the ring to $T < T_C$ in a magnetic field H_a.

② Then remove the applied field H_a.

③ The induced current in the superconducting (SC) ring flows ***persistently***. (If the ring is made of ordinary metal, the induced current decays rapidly.) Tuyn and Onnes did not observe any decay of current in **24 h**.

This can be understood by Faraday's law.

The ring is equivalent to an *RL* circuit.

R = the total resistance of the ring (assuming $R \neq 0$ first).

L = the inductance of the ring

In an external magnetic field $\vec{B}_a = \mu_0 \vec{H}_a$,

the induced voltage in the ring is $V = -\dfrac{d\Phi_a}{dt} = L\dfrac{dI}{dt} + IR$

where $\Phi_a = \int_{ring} \vec{B}_a \cdot d\vec{a}$

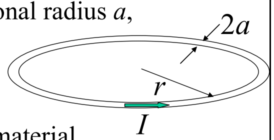
If $\dfrac{dB_a}{dt} = 0$, then $-\dfrac{d\Phi_a}{dt} = 0$ or $I(t) = I_0 e^{-(R/L)t} = I_0 e^{-t/\tau}$.

The decay time constant is $\tau = \dfrac{L}{R}$

If $R = 0$, then $I(t) = I_0$ (persistent current).

Can $R = 0$ be verified by the experiment on SC ring?

For a ring of radius r and cross-sectional radius a,



$$L = \mu_0 r[\log_e(8r/a) - 2]$$

and $R = 2r\rho/a^2$

where ρ is the resistivity of the ring material.

Thus $\rho\tau = \frac{1}{2}\mu_0 a^2[\log_e(8r/a) - 2]$ [1]

For a Cu ring at room temperature ($\rho \approx 1.56$ μΩ.cm, $a = 1.0$ mm and $r = 10$ cm), $\tau = 0.19$ ms.

For a superconducting ring with the same a and r, if, for over a period t, the supercurrent persists without appreciable change (assuming <1% of drop in I, which can be easily detected by a Hall effect sensor),

then τ is determined by $I(t) = I_0 e^{-t/\tau}$.

Assume $I(t) > 0.99 I_0$ in $t = 1$ year. Then $\tau > 3\text{x}10^9$ s and thus by Eq. [1], $\rho < 1\text{x}10^{-19}$ Ω.cm.

This is how we estimate an

upper limit of resistivity of superconductor.

One can never show $R = 0$ by experiment.

The measurement of $I(t)$ in a ring sample is thus the most sensitive method to set an upper limit of resistivity.

In 1962, the measured upper limits are

< $3\text{x}10^{-23}$ Ω-cm for conventional Type I superconductor (J. Appl. Phys, <u>33</u>, 748 (1962)),

< $2\text{x}10^{-19}$ Ω-cm for conventional Type II superconductor (Phy. Rev. Lett. <u>9</u>, 306 (1962)).

(The classification of superconductors will be given later.)

(IV) Magnetic property of superconductor

Onnes' Dream

Novel property ⇒ Novel application:

$R = 0$ in superconductor ⇒ we can pass very high current through a superconducting wire.

This suggests that we can use superconducting wire to build a magnet (**superconducting magnet**, **SM**) to generate strong and stable magnetic field. SM is just a solenoid of superconducting wire with high current.

Onnes received a grant to build a 10 T SM.

But it is not that simple! He never attained the goal because he worked only on elements (now classified as Type I superconductors). It took half a century for other scientists to achieve this goal.

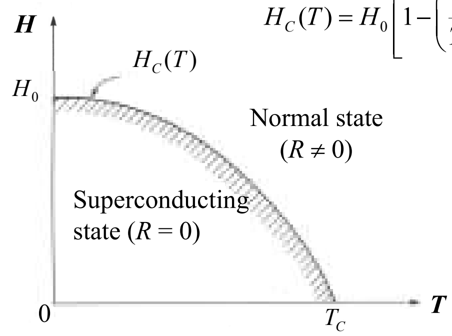
Instead he found **critical current** I_C and **critical field** H_C.

The superconducting state can be maintained only when $I < I_C$ and $H < H_C$.

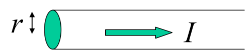
For all pure metals Onnes studied,

$H_C \sim$ few 10^2 Oe ($B_C \sim 10^{-2}$ T).

H_c depends on temperature: $H_C(T) = H_0\left[1 - \left(\dfrac{T}{T_C}\right)^2\right]$ [2]



Current-carrying wire generates magnetic field:



The magnetic field on the wire surface is $B = \dfrac{\mu_0 I}{2\pi r}$.

Assume there is no external magnetic field ($H_a = 0$).

In order to maintain superconducting state, the current must not exceed a critical value I_C so that the generated field on wire surface $B \le \mu_0 H_C$.

Critical current: $\dfrac{\mu_0 I_C}{2\pi r} \approx \mu_0 H_C \Rightarrow I_C \approx 2\pi r H_C$

We will show later that in a (Type I) superconductor, the supercurrent is confined to the surface.

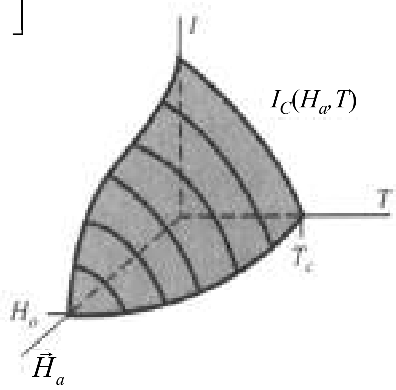
If $H_a \neq 0$ then we need $|\vec{H}_a + \underbrace{\vec{H}_I}_{\text{due to } I}| < H_c$

For applied magnetic field $H_a = 0$, $I_C \propto H_C$.

$$\therefore \quad I_C = I_0\left[1 - \left(\frac{T}{T_C}\right)^2\right]$$

For different applied field H_a, the superconductor has different $I_C(T)$ curve.

In general, the $I_C(H_a, T)$ surface depends on $\vec{H}_a$ direction.



(V) The Meissner effect

conductivity $\sigma = \infty$

- Is superconductor just a perfect conductor only?
  No! The evidence: Some alloys and even some amorphous metals (with low conductivity) are superconductors.
  Superconductor has peculiar magnetic property. It is now known as the Meissner effect.
- Meissner & Ochsenfeld (1933) used a **solid** Pb cylinder in order to
  1. look for any change in magnetic properties in association with the superconducting transition.
  2. find out whether the supercurrent shielded the magnetic field completely.

What is $\vec{B}$ inside SC?

For perfect conductor, $\sigma = \infty$.
Since $\vec{J} = \sigma\vec{E} < \infty$, $\vec{E} = 0$.
(Note: in electrostatics, $\vec{E} = 0$)

Faraday's law: $\nabla \times \vec{E} = -\dfrac{\partial B}{\partial t}$

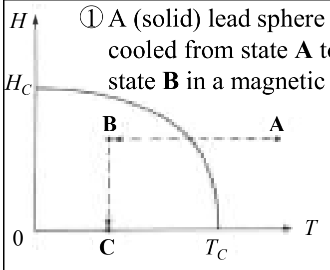
$$\therefore \frac{\partial B}{\partial t} = 0.$$

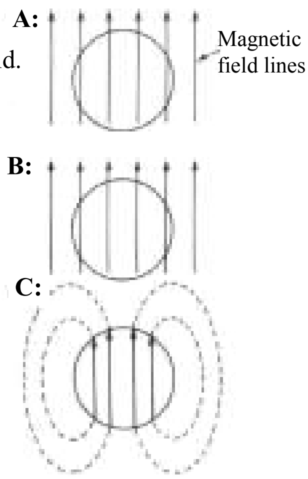
$\Rightarrow$ $\vec{B}$ inside a perfect conductor is time-independent.

But $\vec{B} = ?$
Are superconductors simply perfect conductors?

Now consider the cooling process:

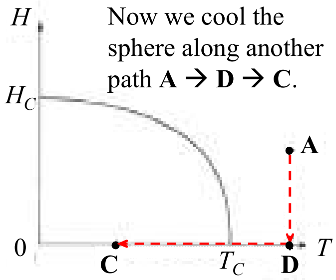
① A (solid) lead sphere is cooled from state **A** to state **B** in a magnetic field.

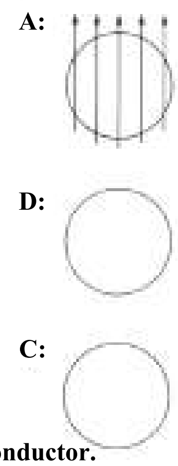




②
From state **B** to state **C**, the magnetic field is reduced to zero at the same *T*.
If a superconductor is just a perfect conductor, then by Faraday's law, the induced supercurrent flows persistently on the surface of sphere (similar to SC ring).

Now we cool the sphere along another path **A → D → C**.

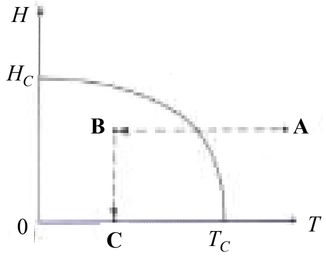


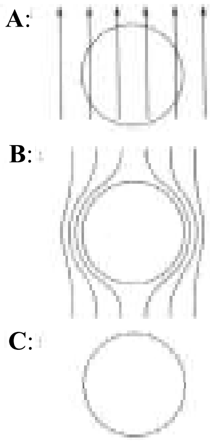


From state **A** to state **D**, the magnetic field is switched off. The induced current on the surface of sphere decays quickly (because $R \neq 0$).
No current is on the sphere in state **C**.

$\therefore$ The final state in state **C** is thus path-dependent & this is not acceptable.
$\Rightarrow$ **superconductor is not just a perfect conductor.**

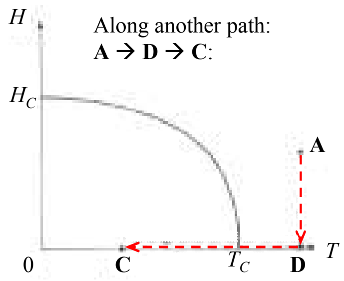
Meissner & Ochsenfeld 's discovery: $\vec{B} = 0$ inside SC

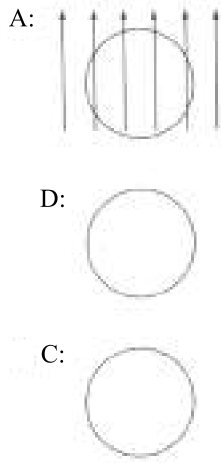




From state **A** to state **B**, all magnetic field inside superconductor is expelled.
Supercurrent is induced on the surface such that the total magnetic field inside the sphere is zero. From state **B** to state **C**, the current drops to zero in order to maintain $\vec{B} = 0$ inside superconductor.

Along another path:
A → D → C:





Now the state at **C** is path-independent.

The Meissner efect : $\vec{B} = 0$
inside (Type I) superconductor.

Supercurrent in superconductors:

The Meissner effect: $\vec{B} = 0$ inside (Type I) SC.

By Maxwell-Ampere's law, $\nabla \times \vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\dfrac{\partial \vec{E}}{\partial t}$

or $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed} + \mu_0\varepsilon_0 \dfrac{\partial}{\partial t}\int \vec{E} \cdot d\vec{a}$
(for a closed loop inside SC.)

Remember that we have shown that $\vec{E} = 0$ inside SC .
$\therefore$ $\vec{J} = 0$ At any point inside SC

Current in a (Type I) SC is confined to its surface.
Therefore if we put a SC in a magnetic field $H < H_C$,
then a supercurrent is induced on the SC surface to ensure that the total magnetic field $\vec{B}$ inside the SC is zero.
This current is called the screening current.

Screening current distribution is unique & can be calculated:

Consider a (Type I) superconducting ***sphere*** (at $T < T_C$)

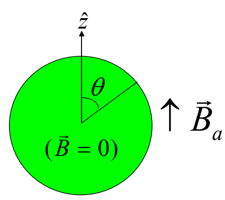
placed in a uniform magnetic field $\vec{B}_a = \mu_0\vec{H} = \mu_0 H\hat{z}$.

Because of the Meissner effect, a surface current is induced so that inside the superconductor $\vec{B} = 0$.

$$\vec{B} = \vec{B}_a + \vec{B}_i = 0$$

$\vec{B}_i$ is due to the surface current density

$$\vec{K} = -\frac{3}{2\mu_0} B_a \sin\theta\hat{\phi}$$



(Here we use spherical coordinates $(\hat{r}, \hat{\theta}, \hat{\phi})$.
For details, see Appendix 10.)

Penetration depth:

Meissner effect: $B = 0$ inside (Type I) superconductor.

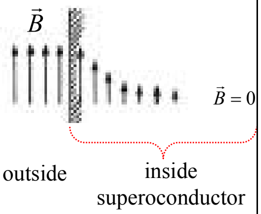
Actually the magnetic field can penetrate an **ultra-thin** depth underneath the surface of superconductor. (The thickness of this layer is characterized by a penetration depth λ).
The supercurrents also flow within this layer.

Penetration depth depends on T:

$$\lambda(T) = \lambda_0 \left[1 - \left(\frac{T}{T_C} \right)^4 \right]^{-1/2}$$

(λ_0: constant)

(Pb: λ_0 =370 Å)



(VI) Superconductors are diamagnetic.

By definition,

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

Inside SC, $\vec{B} = 0$

$\Rightarrow \vec{M} = -\vec{H}$ (diamagnetic)

Magnetization ($\vec{M}$) is the magnetic moment per unit volume: $\vec{M} = \lim_{V\to 0} \frac{\sum_i \vec{m}_i}{V}$

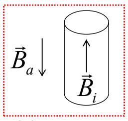
(H is the applied magnetic field.)

or $\vec{B} = \vec{B}_a + \vec{B}_i$

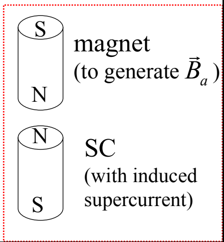
$\vec{B}_a$: applied field; $\vec{B}_i$: magnetization due to induced current

$\vec{B} = 0$ inside SC

$\Rightarrow \vec{B}_i = -\vec{B}_a$

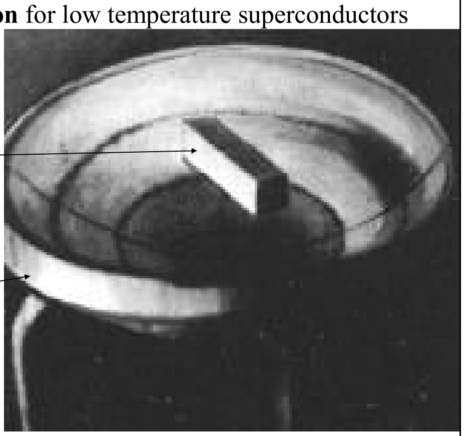


similar to



$\therefore$ SC can be levitated by a magnetic field! (a strong force)

Magnetic levitation for low temperature superconductors



(Note: We will see that magnetic levitation for high temperature superconductors is different.)

(VII) Summary

Superconductor possesses
1. infinite conductivity ($\rho = 0$)
2. perfect diamagnetism (the Meissner effect $\vec{B} = 0$)

provided that
- $T < T_C$ (critical temperature or transition temperature)
- $H < H_C$ (critical field)
- $I < I_C$ (critical current)

The transition is an example of ***critical phenomenon***.
Many properties change at T_C: specific heat, thermo-emf, Hall effect, absorption of ultrasound, absorption of infrared, ...

(VIII) Classification of superconductors

Type I superconductors: The superconductors that are metal elements except Nb & V. They have the properties listed in (VII).

Type II superconductors: Nb, V, alloys & compounds.

1930s: Schubnikow (Russian) discovered that some **Pb alloys** had H_C much higher than H_C(Pb). His paper was not noticed by the West due to the World War II.

1961: Scientists in Bell Lab discovered Nb_3Sn (an intermetallic compound) with high I_C and H_C.

These superconducting alloys and compounds have interesting properties: magnetic field can penetrate the material in a novel way.

(History: J.K. Hulm, J.E. Kunzler and B.T. Matthias, "The road to superconducting materials", Physics Today, Jan. 1981, pp.34-43.)

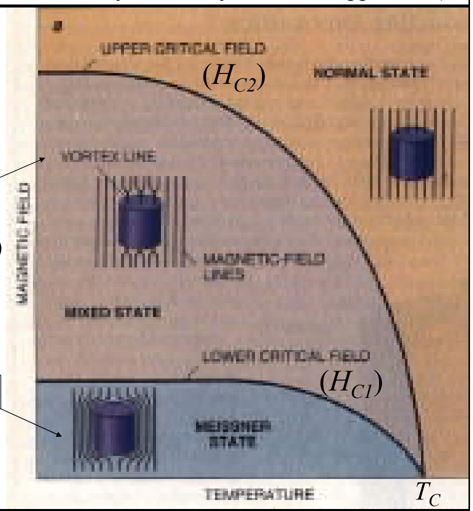
For Type II superconductor, there are two critical fields: $H_{C1}(T)$ & $H_{C2}(T)$

mixed state

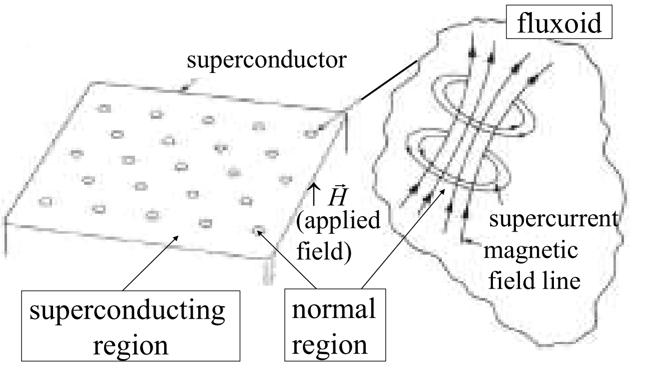
($H_{C1} < H < H_{C2}$) is neither fully superconducting nor fully normal.

Meissner state

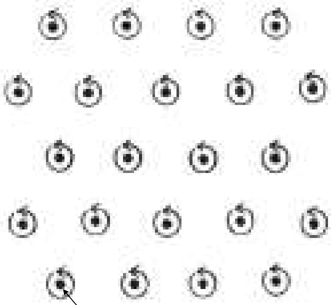
($0 < H < H_{C1}$) complete Meissner state



In mixed state, magnetic field penetrates the superconductor in a form of regular triangular array of fluxoids (magnetic field line bundles). Each fluxoid contains a normal core, surrounded by a vortex of supercurrent.



Vortices in mixed state when viewed along $\vec{H}$:



The magnetic flux of each fluxoid: $\phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15}$ T-m^2

For an average magnetic field B in the sample, the number of fluxoids per unit area is B/ϕ_0.
ϕ_0 depends on h; indicating that we need QM to understand its origin.

The mixed state allows very high $\boldsymbol{H_{C2}}$:

	$B_{C2}=\mu_0 H_{C2}$ at $T = 0$	T_C
Nb_3Ge	38 T	23 K
Nb_3Sn	24.5 T	18 K
V_3Si	23.5 T	16.9 K

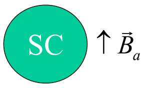
Tesla

Like Type I superconductor, Type II superconductors have small $\boldsymbol{H_{C1}}$.

Why is H_C of Type I superconductors or $\boldsymbol{H_{C1}}$ of Type II superconductors so small?

Recall: Magnetic field stores energy with energy density

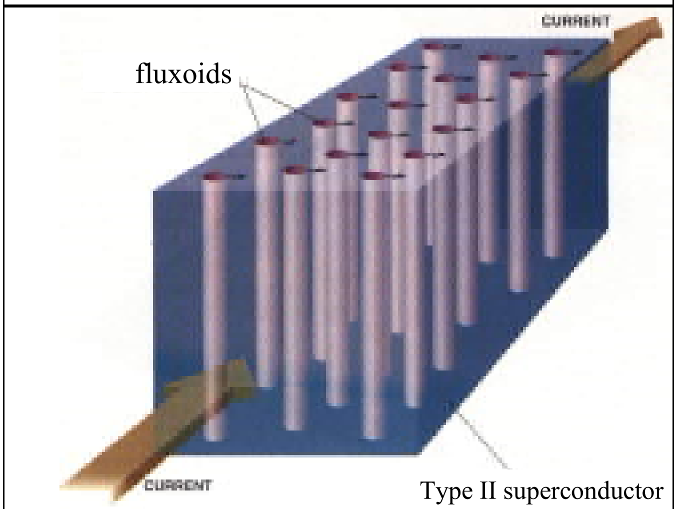
$$B^2/2\mu_0$$



Consider a superconductor (of volume V) in a uniform magnetic field $\vec{B}_a$.
In normal state, the energy of magnetic field within the superconductor is $(B_a^2/2\mu_0)V$.
In superconducting state (at $T < T_C$), $\vec{B} = 0$ inside SC.
$\Rightarrow$ Need to exclude energy $((\mu_0 H_C)^2/2\mu_0)V$ out of superconductor.

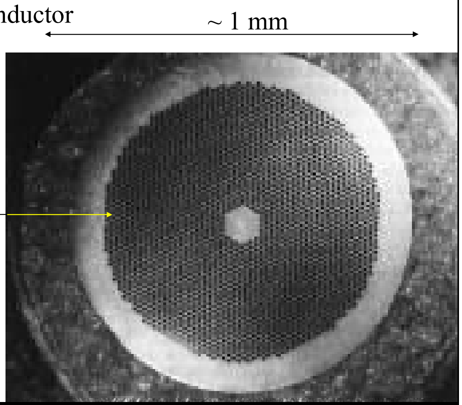
That explains why H_C & H_{C1} are so small (~ 10^2–10^3 Oe).

- **Fluxoids can move around**:
  When a current is passing through a superconductor, it may hit the fluxoids and interact with the magnetic field in the fluxoids. Or the magnetic field of this current interacts with the supercurrents surrounding the fluxoids. As a result, the fluxoids can be moved by Lorentz force, resulting in energy loss & very small I_C.
- **Flux pinning**:
  The fluxoids can be "pinned" (or fixed) by crystal imperfections (grain boundaries, defects, impurities, etc.), resulting in very high I_C.
- **Stable magnetic levitation demo** with high temperature superconductors (HTS) is due to strong pinning force.

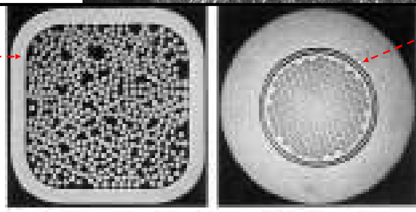


Immediately after superconductor Nb_3Sn was found in 1961, high field superconducting magnet (SM) (Onnes's dream) was built at General Electric using Nb_3Sn, and later in other labs using superconductor Nb-Ti.

Modern SMs are built using superconducting cable composed of Ni-Ti filaments embedded in pure Cu.



Cable (composed of many wires)



Wire (composed of many filaments)

filament

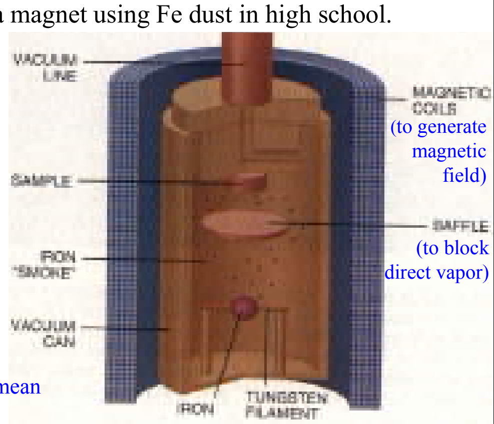
(The New Quantum Universe, Fig. 7.23)

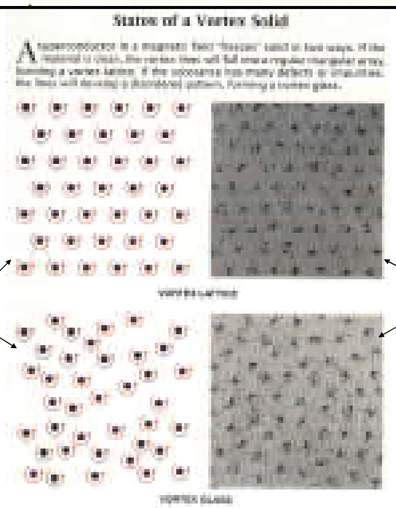
(IX) Decoration experiment (used to visualize the fluxoids) (first done in 1966): like mapping the magnetic field distribution of a magnet using Fe dust in high school.

The sample (superconductor) must be in mixed state first ($T<T_C$ & $H_{C1}< H <H_{C2}$).

Iron vapor ("smoke") is generated when the tungsten filament is heated.

Vacuum is required for large mean free path of vapor.





The flux quantum (ϕ_0)

The flux quantum $\phi_0 = h/2e$ is not just for Type II SC. It was measured by two groups using **Type I SC**:

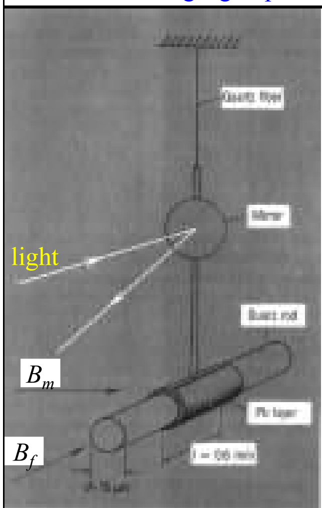
1. Doll & Näbauer (1961) used a Pb tube (actually a Pb coating on a quartz rod) (T_C(Pb) = 7.19 K) .
2. Deaver & Fairbank (1961) used a Sn tube (actually a Sn coating on a Cu wire) (T_C(Sn)= 3.72 K).

Both samples had diameter ≈ 10 μm.
Their experiments are quite similar.

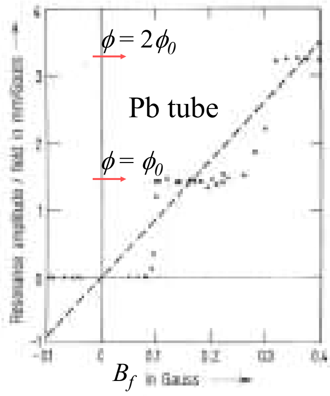
1. At $T > T_C$, a magnetic field B_f (called the freezing field) is applied along the axis of the tube (in normal state). Therefore the magnetic flux in the tube ≠ 0.
2. Then cooled the tubes in B_f to $T = T_0 < T_C$.
3. Finally removed B_f at T_0. As a result, a persistent current is induced on the tube's **inner** surface. (Why?)

The magnetic flux trapped in the tube is adjusted automatically to become quantized: $\phi = n\,\phi_0$, where n is an integer.
(Note: $\phi < B_f A$, where A is the cross-section area of the tube.)
In other words, the tube (in superconducting state) becomes a tiny magnet. The next step is to measure its magnetic moment, from which ϕ is determined. Note:

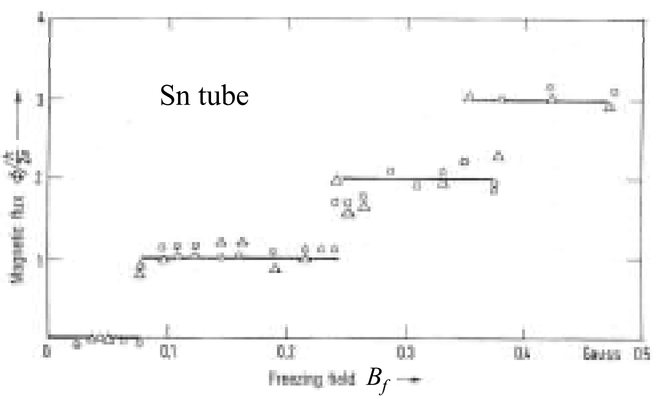
(1) The field is so small, even a Type II SC is used for this experiment, it is in Meissner state.
(2) We can estimate the required B_f for these experiments:
To get a flux = ϕ_0 = 2 x10^{-15} T.m^2,
the applied field is B_f = 2.5 x10^{-5} T.
Recall: Earth field ≈ 5 x 10^{-5} T. (1 T = 10^4 Gauss)
(3) In mixed state of type II superconductor, the magnetic flux of each fluxoid is ϕ_0 in order to minimize the total energy. (Why? Hint: treat each fluxoid as a tiny magnet and think about holding a group of tiny magnets together.)



Doll & Näbauer (Phys. Rev. Lett. 7, 51 (1961)): used torsional pendulum method to measure moment.



Deaver & Fairbank (Phys. Rev. Lett. 7, 43 (1961)):



(X) Theories for superconductivity (Type I & Type II)

Questions:

- Why T_C so low?
- Is superconductivity simply the state of a conductor with very high purity?
- Why are good metals not superconducting?
- Why is the transition so abrupt ?
- Do we need quantum mechanics (QM) developed in 20s-30s to explain superconductivity?
  Yes! Superconductivity is due to conduction electrons. Bloch (1928) used QM to show that conduction electrons in crystal move freely through the crystal like a wave (a quantum phenomenon).
- Why $R = 0$?

(a) Cooper pair (1956)

Cooper assumed a weak attractive force between two conduction electrons at 0 K, & then showed theoretically that the two electrons:

one with $\vec{p}\uparrow$ (momentum $\vec{p}$ and spin up),

one with $-\vec{p}\downarrow$ (momentum $-\vec{p}$ and spin down)

can form "bound" state (just like one proton and one electron form a bound state, the hydrogen atom).

The weakly bound electron pairs (now called Cooper pairs) in the superconductor are boson & can move cooperatively without resistance.

Note: Since the momentum of the pair is well defined, the Cooper pair must spread out in space. (Uncertainty principle)

Properties of Cooper pair:

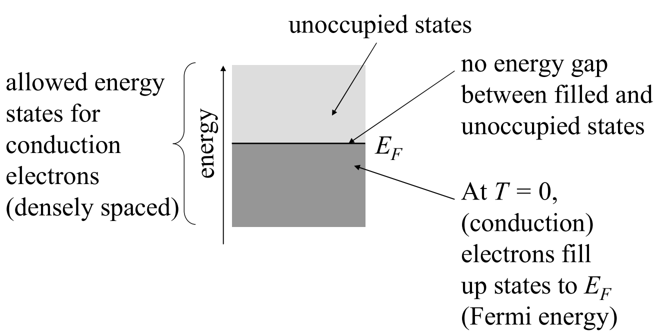
total charge = $2e$ (Note: $\phi_0 = h/2e$)
total momentum = 0
and total magnetic moment = 0.

Similar to two-fluid model, Cooper pairs appear at $T < T_C$ and increase in number as T is lowered. Only those electrons near E_F can form Cooper pairs.

The Cooper pair has a binding energy per electron ($\Delta(T)$, called the pairing energy) ~ 10^{-4} – 10^{-3} eV.
The energy required to break up a Cooper pair (to become two normal electrons) is $2\Delta(T)$.

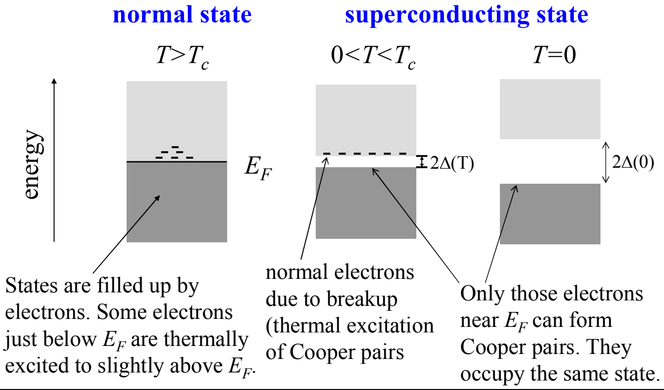
The two electrons in a Cooper pair are separated by few hundred Å. This size is much larger than the separation of two pairs. So all Cooper pairs overlap appreciably.
Cooper pair is the basics of the BCS theory.

Recall: **Energy diagram of electrons in normal metals**



The binding energy for superconductor introduces an energy gap $2\Delta(T)$ near the Fermi energy E_F. ~ eV

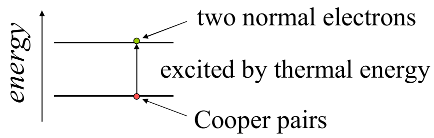
$2\Delta(0) \sim 10^{-4}\, E_F$



Low T_C is related to small energy gap $2\Delta(T)$:
In superconducting state, the thermal energy of lattice

$k_B T < 2\Delta(T)$,

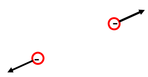
otherwise most Cooper pairs are destroyed by thermal energy.



Now $2\Delta(0) \sim 1$ meV (see data shown later)
and for estimation, we take $k_B T_C \sim 2\Delta(0)$
(More exact calculation gives $3.5 k_B T_C = 2\Delta(0)$)

$\Rightarrow \quad T_C \sim 12$ K.

What is the state of superconducting electrons in a superconductor?
Late 40s – early 50s, it was believed that electrons should have a new type of interaction in addition to the well-known repulsive Coulomb force.



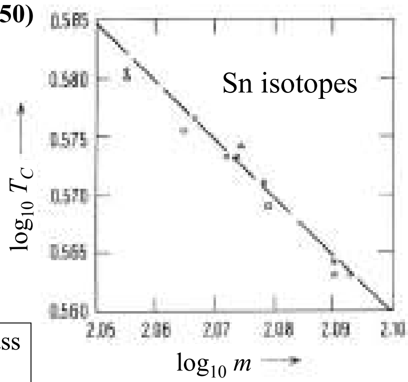
Fröhlich (1950) & Bardeen (1950) believed that the **lattice vibration** should play a major role. The idea was based on some experimental results.

E.g., **Isotope effect (1950)**

For superconducting element with more than one isotope

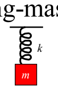
$T_C \propto m^{-1/2}$

where m is the atomic mass of the isotope.



Recall: For a spring-mass system,

$\omega = \sqrt{k/m}$



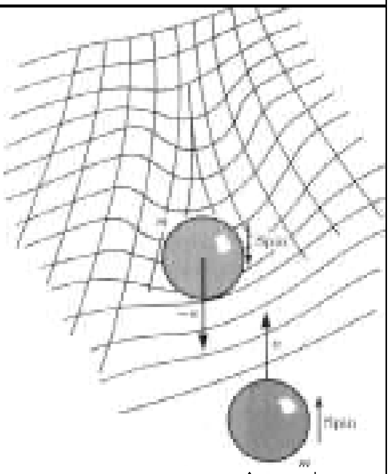
This confirmed the importance of lattice vibration and the electron-lattice interaction should be considered.

Why the electron-lattice interaction supplies the attractive force?

"The mattress effect": When a bowling ball falls onto a mattress, it distorts the mattress (床褥). Another bowling ball nearby may roll toward the first ball because of this distortion. It appears that the two bowling balls attract each other.

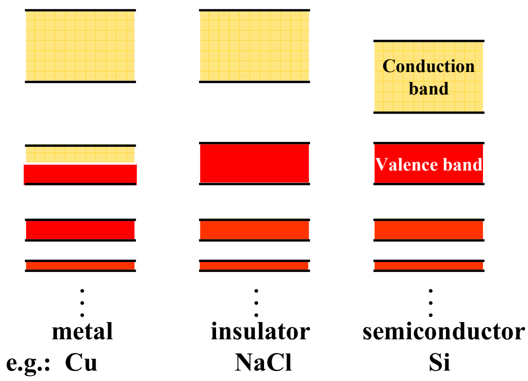
Now the analogy:
conduction electrons in superconductor ↔ bowling balls
crystal lattice of superconductor ↔ mattress

The crystal lattice is polarized by an electron, resulting in an effective positive charge to attract another electron. These two electrons form a bound state: the Cooper pair.

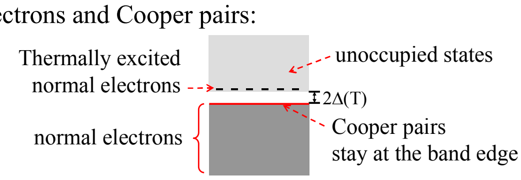


Cooper pair: $\{\vec{p}\uparrow, -\vec{p}\downarrow\}$

Why $R = 0$?
Recall:



Why $R = 0$? The following argument is quite tricky.
We use the following energy diagram for both normal electrons and Cooper pairs:



First, because all Cooper pairs (bosons) have the same binding energy, they are all gathering at the band edge.
If $R \neq 0$ (with power dissipation), then when we apply an electric field, the Cooper pairs in the current can scatter with the lattice & change their states.
But there is an energy gap above the original state.
$\Rightarrow$ Their states cannot be changed.
$\Rightarrow$ no scattering $\Rightarrow R = 0$.

(b) BCS theory of superconductivity

Many best minds in physics (including Einstein) tried to understand superconductivity, but only 40 years after Onnes's discovery, the theory was established.
BCS theory (1957) was derived using second quantization technique (taught in QMII):

$$H_{BCS} = \sum_k \varepsilon_k \left(c_k^+ c_k + c_{-k}^+ c_{-k} \right) - \sum_{kk'} V_{kk'} c_{k'}^+ c_{-k'}^+ c_{-k} c_k$$

They extended the Cooper model to all conduction electrons near E_F in the solid. The BCS theory explained successfully almost everything that was known about low-temperature superconductivity and provided the microscopic origin (i.e., the mechanism).

Nobel Prize (1972)

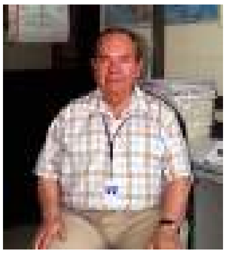
John Bardeen (1908-1991)	Leon Neil Cooper (1930 -)	John Robert Schrieffer (1931-)

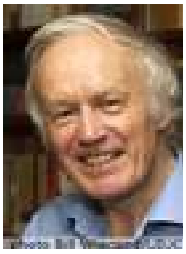


"for their development of a theory of superconductivity"

Nobel Prize (2003)

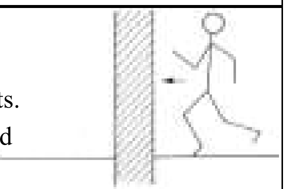
Alexei A. Abrikosov	Vitaly L. Ginzburg	Anthony J. Leggett





"for pioneering contributions to the theory of superconductors and superfluids"

(XI) Tunneling effect:



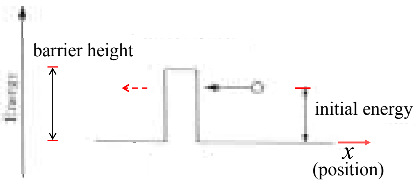
The energy gap 2Δ(T) can be determined by tunneling experiments.

Magician David Copperfield "walled through" the Great Wall.

(http://www.youtube.com/watch?v=jtG6t6LcNAU)

This is not allowed in classical mechanics.
Quantum mechanics: A particle can tunnel through the barrier even if its energy is lower than the barrier height.

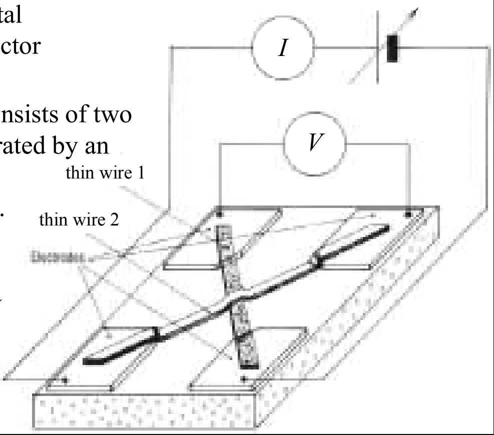
It is due to uncertainty principle.



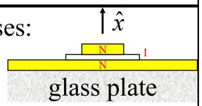
The tunneling effect can be observed using thin film samples: NIN, SIN or SIS junctions, where
N = normal metal
S = superconductor
I = insulator.

Each sample consists of two thin wires separated by an ultrathin insulating layer.

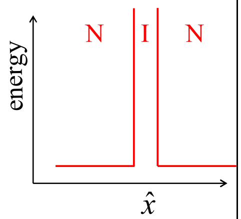
Each wire has a width < 1 mm.



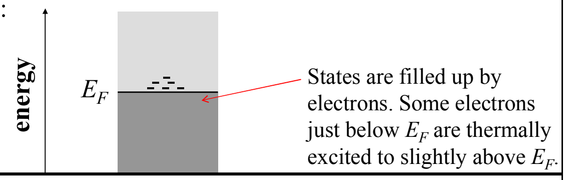
Consider **NIN junction** with layer thicknesses:
layer 1: normal metal (N) <1 μm
layer 2: oxide (insulator) (I) < 3nm
layer 3: normal metal (N) < 1 μm



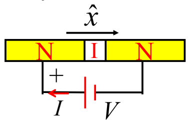
The potential barrier for electron:



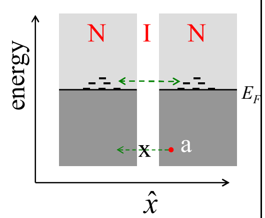
The energy of electrons in metal is limited (described by an energy diagram by free electron model or the conduction band in energy band diagram):



The metal layers are connected to a voltage source.



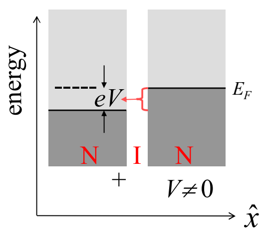
The energy diagram of the sample as a function of position x for $\boldsymbol{V} = \mathbf{0}$:

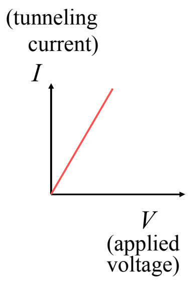


Not all electrons can tunnel through the barrier, e.g., electron **a** cannot tunnel because there is no available state at the same energy in the left layer.
(We will ignore the lower parts of the energy diagrams because no tunneling can occur in this region.)
Tunneling can occur for electrons above E_F but $I = 0$. Why?

When a voltage V is applied, the energy levels are shifted. (Why?)

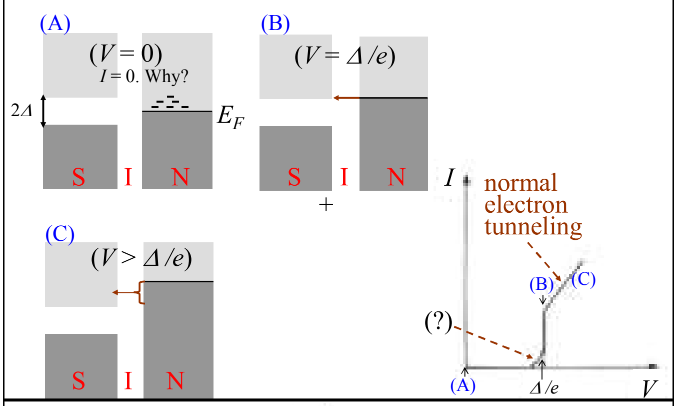
(Here $e > 0$, $V > 0$)



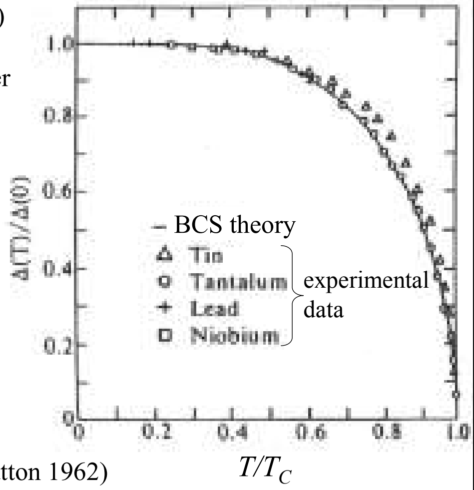


Electrons in the right layer with energy
$E_F - eV \le E \le E_F$
can tunnel through the insulating layer into the left layer.
Result: Tunneling current increases linearly with increasing V.
Tunneling is enhanced by electric field.

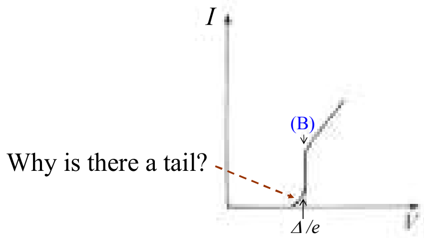
SIN junction: layer 1: superconductor (S)
layer 2: oxide (insulator) (I)
layer 3: normal metal (N)



Energy gap $2\Delta(T)$ can also be measured by other methods.
$\Delta(T)$ depends on temperature:



(Townsend & Sutton 1962)

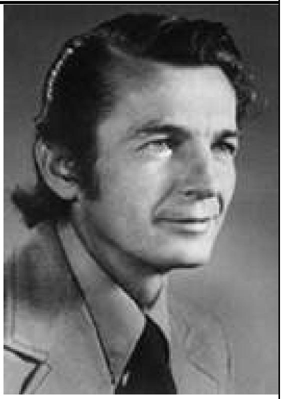


Recall: At $T > 0$, some electrons can be found with $E > E_F$ due to thermal excitation. The tunneling current in the tail is due to these electrons.
The tail is weaker at lower temperature.
See curves 2 & 3 of data for Al-Al_2O_3-Pb junction below.

Nobel Prize (1973)

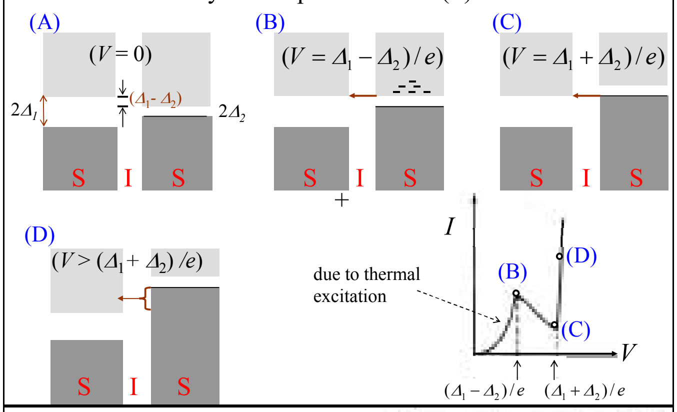
Ivar Giaever (1929 -)

"for his experimental discoveries regarding tunneling phenomena in superconductors"



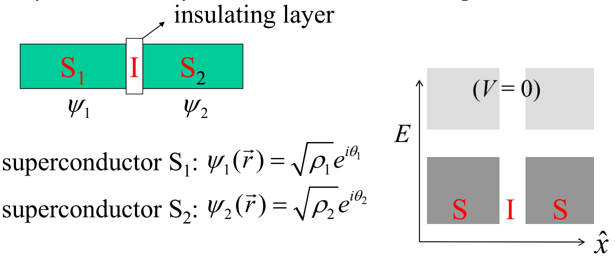
You will enjoy reading his Nobel lecture:
www.nobel.se/physics/laureates/1973/giaever-lecture.html

SIS junction: layer 1: superconductor (S)
layer 2: oxide (insulator) (I)
layer 3: superconductor (S)



(XII) The Josephson effect (1962)

Josephson further explored the theory for an SIS junction. The two superconductors are the same, i.e., $S_1 = S_2$ & $\Delta_1 = \Delta_2$.
At that time, people already knew that all Cooper pairs are in the same QM state, described by a single wavefunction (a complex number): $\psi(\vec{r}) = \sqrt{\rho}e^{i\theta}$
where ρ is the density of electrons and θ is the phase.

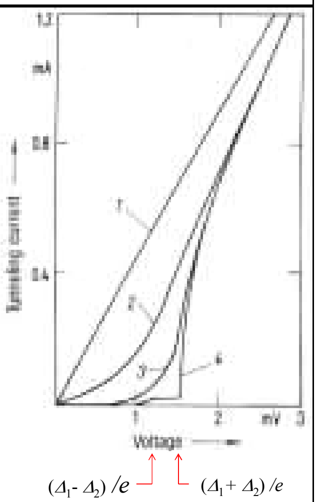


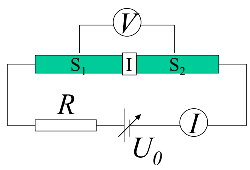
In superconductor S_1: $\psi_1(\vec{r}) = \sqrt{\rho_1}e^{i\theta_1}$

In superconductor S_2: $\psi_2(\vec{r}) = \sqrt{\rho_2}e^{i\theta_2}$

e.g. Al-Al_2O_3-Pb junction is an SIS junction.
T_C (Pb) = 7.2 K
T_C (Al) = 1.2 K

I-V curves were measured at different temperature T:
curve 1: T = 10 K (NIN junction)
curve 2: T = 4.2 K (SIN junction)
curve 3: T = 1.64 K (SIN junction)
curve 4: T= 1.05 K (SIS junction)
(Giaever & Megerle 1961)



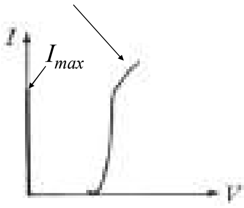


When a small U_0 is applied, a dc supercurrent I tunnels through the barrier with $V = 0$, and its direction depends on the polarity of U_0.

$$I = I_{max}\sin(\theta_2 - \theta_1)$$

I increases for higher U_0 until $I = I_{max}$. For even higher U_0, $V \neq 0$ and the *V-I* curve is

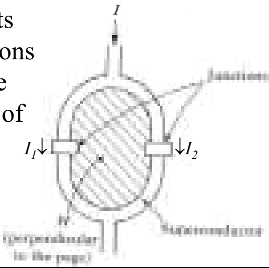
This is the dc Josephson effect.



(For a simple derivation, see The Feynman lectures on physics, vol. 3, p.21-14, 1963.)

Josephson also predicted the generation of a high frequency alternating current at the junction: the frequency $\nu_J = V/\phi_0$. This ac Josephson effect was confirmed by Giaever (1965). The Josephson effect is a quantum behavior seen in macroscopic object (the superconductor).

SQUID (**s**uperconducting **qu**antum **i**nterference **d**evice) is composed of two Josephson junctions. The two currents (I_1, I_2) passing through the two junctions can have very interesting interference behavior in magnetic field. Because of this exotic property, SQUID is an extremely sensitive magnetic field sensor.
(see applications)



(b) New materials

(i) Metallic hydrogen

H_2 is the most common element in the universe.

1898 liquefied hydrogen (20 K) } insulating
1899 solidified hydrogen (14 K)
1935 Wigner & Huntington predicted the existence of metallic hydrogen at 250 kbars pressure. (Scientific American, May 2000, pp.60-66.)
1960s Ashcroft predicted that metallic hydrogen would be a room temperature superconductor. (Physics World **8** (7), p.43 (1995).)

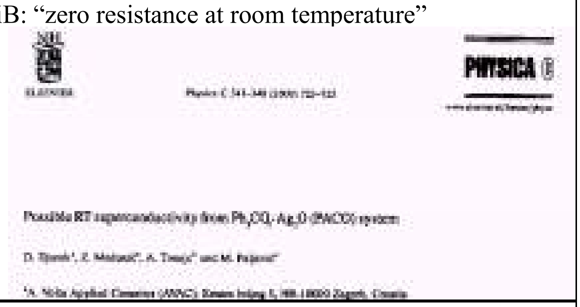
Nobel Prize (1973)

Brian David Josephson
(1940 -)

"for his theoretical predictions of the properties of a supercurrent through a tunnel barrier, in particular those phenomena which are generally known as the Josephson effects"

(ii) USO (unidentified superconducting objects)

1973 TTF-TCNQ (an organic crystal): "superconducting fluctuations at 60 K"
1978 CuCl (a semiconductor): "diamagnetic anomaly at high pressure at 150 K" (Rusakov & Chu)
1980 CdS: "flux exclusion at 77 K"
1980 TiB: "zero resistance at room temperature"
2000



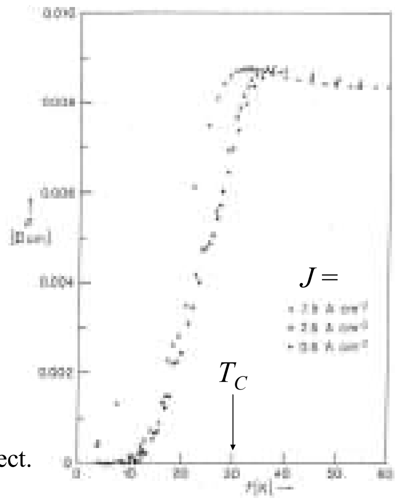
(XIII) Limit of T_C and new surprises

- BCS theory was further perfected so that given the energy spectrum of lattice vibration and the electronic properties of a material, one can predict the value of its T_C.
- An example is McMillan equation (1968) from which the maximum T_C was estimated to be about 30-40 K. Before 1986, the record T_C was found to be 23 K in Nb_3Ge.
- By 80s, people believed that superconductivity is a "mature" physics!
- Is high T_C superconductivity possible?
  Some people are optimistic.
  Theorists looked for new superconducting mechanisms.
  Experimentalists searched for new materials.

(iii) HTS
(High temperature superconductors)

J.G. Bednorz & K.A. Müller, "Possible high T_C superconductivity in the Ba-La-Cu-O system", Z. Phys. B64, p.289 (1986).

No data on Meissner effect.



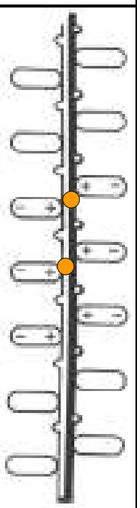
(a) New mechanisms

Little (1964) considered a hypothetical 1D organic molecule with side branches.
A conduction electron polarizes the side-branches and induces positive charges at the ends near the spine. A second electron is attracted to this region of positive charge and is therefore indirectly attracted to the first electron. The polarization is due to electron-electron interaction and thus

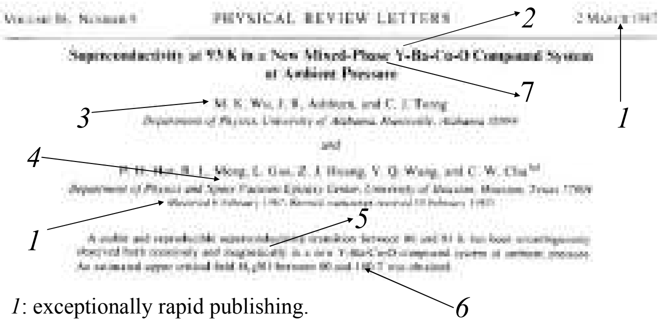
$T_C \sim m_e^{-1/2} \sim$ room temperature.

mass of electron << mass of ion

(Ref.: Scientific American, Feb 1965, p.21)

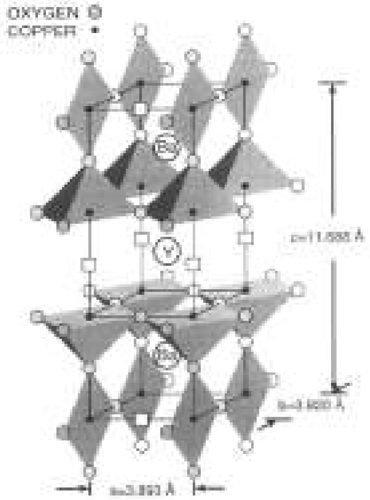


A few groups repeated their findings and achieved higher T_C at high pressure. This suggested new superconductors could be found by chemical substitution. Chu then became famous:



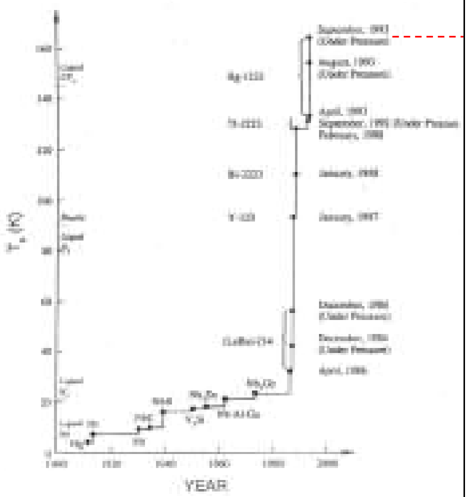
1: exceptionally rapid publishing.
2: Y or Yb?
3: Who (Wu or Chu) discovered YBCO?
(C.W. Chu, IEEE Tran. Appl. Sup. Vol. 7, p.80 (1997).

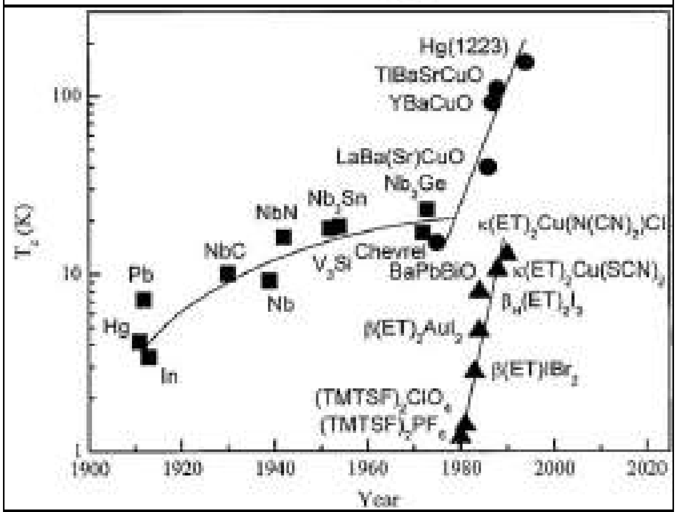
The 90 K superconductor was identified as $YBa_2Cu_3O_7$ with a perovskite crystal structure



Many more **HTS** were found.

Record T_C = 164 K
($HgBa_2Ca_2Cu_3O_8$ at 30 kbars hydrostatic pressure.)





Nobel Prize (1986)

J. Georg Bednorz (1950 -)

K. Alexander Müller (1927 -)

"for their important breakthrough in the discovery of superconductivity in ceramic materials"

(iv) New Surprises

(a) More superconducting elements at high pressures:
S (1997), O (1998), B (2001), Fe (2001), Li (2002)

(b) Ferromagnetic superconductors: Superconductivity is usually destroyed by high magnetic field but materials were discovered with coexistence of superconducting and ferromagnetic properties.
2001: Fe at high pressure (Nature 406, p.316)
2001: $ZrZn_2$ alloy (Nature 412, p.58)
2001: UGe_2, URhGe (Nature 413, p.613)
2000: κ-$(BEDT\text{-}TTF)_2Cu[N(CN)_2]Br$, magnetic-field-induced superconductivity in organic material. (Phys. Rev. Lett. 85, p.5420)

(c) Many semiconductors, oxides, hydrides, organic materials were found to be superconducting.

Figure 1 Superconductors under pressure. The colour code of this periodic table (adapted from ref. 13) shows elements that superconduct under normal atmospheric pressure conditions (purple) and those that superconduct when subjected to high pressure (orange). Shimizu et al.[1] confirm the superconductivity of lithium at high pressure, bringing the number of such elements to 23. Under normal pressure conditions, 29 elements are superconductors.

(d) New non-copper high temperature superconductors:

2001: MgB_2 (Jun Akimitsu)

- T_C = 39 K
- MgB_2 is available commercially.

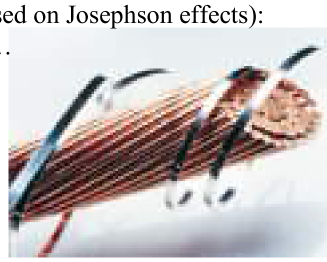
(Physics World, Jan.2002, p.29)

2008: Fe-based superconductors

$SmFeAsO_{1-x}F_x$
- T_C = 43 K

(XIV) Applications (This part will not be covered in final exam.)

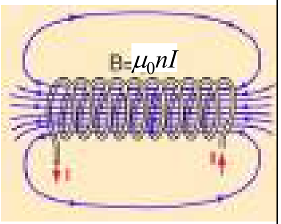
(1) High current applications (based on R = 0): electricity transmission, energy storage, …
(2) High field applications (based on superconducting magnet): MRI, Maglev, motor, generator, accelerators, research equipment, …
(3) Josephson applications (based on Josephson effects): SQUID, supercomputers, …



(Scientific American, Feb. 1989, p.45.)

(a) Superconducting magnet (SM)

Onnes' dream: to build a 10 T SM.
SM is a solenoid of SC wire.

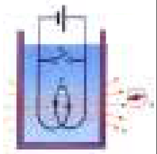


Features of SM:
1. generating a stable and uniform magnetic field
2. light weight

What SC is required?
1. Type II
2. with strong pinning force
3. with high T_C, high I_C, high H_{C2}
4. operated at $T << T_C$ (why?)
5. can be shaped into wires or tapes
6. with good mechanical properties

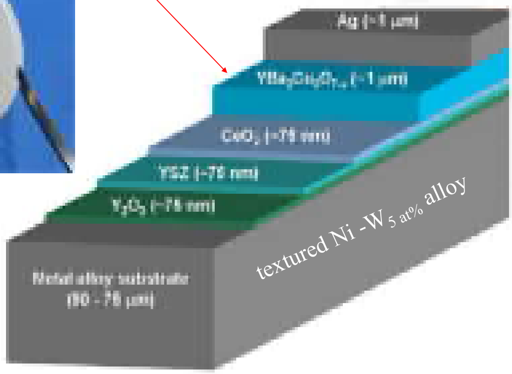
$B = \mu_0 nI$

10 T 10 turns/cm 8 kA



American Superconductor's RABiTS 2G **HTS** tape (ribbon-shaped YBCO tape) (2nd generation)

10 m spool

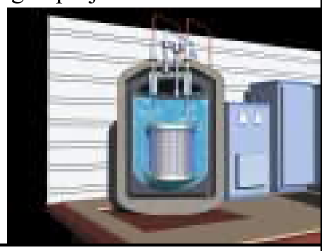


Commercial SM: e.g. 15 T, 5 cm bore

1968: RCA developed a 14 T magnet using their superconducting Nb_3Sn tapes.
(See also "Superconducting magnets above 20 Tesla", Physics Today, Aug. 2002, p.37)

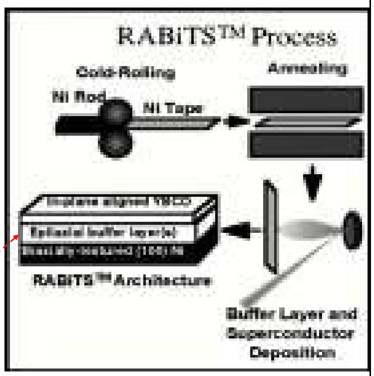
Note: Other high field magnets:

Bitter magnets (for B up to 33 T)
Hybrid magnets (Bitter + superconducting) (for B up to 45 T)
Pulsed magnets (for B up to 70 T)
For an introduction, visit http://www.magnet.fsu.edu/magnettechnology/research/magnetprojects/index.html



HTS cable or tape:

The most successful process for producing long-length tapes for high current and high field applications at 77 K is called **RABiTS**
(the **R**olling-**A**ssisted **Bi**axially **T**extured **S**ubstrate process)



ZrO_2

Applications of SM:
(i) Particle accelerator for high energy experiments:

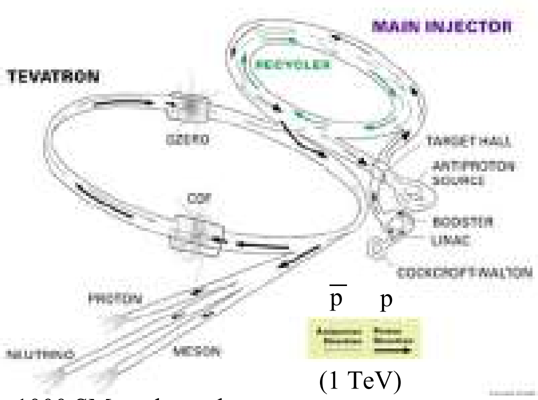
e.g. Tevatron (proton-antiproton collider) in Fermi Lab near Chicago in USA

Circumference of the circular accelerator is ~ 6.4 km.

(http://www.fnal.gov/)

Status of HTS cable/wire/tapes:

1995: The tapes were only 4 cm long .
JC ~ 106 A/cm2 (77 K)
(Science **268**, 644 (1995))
2002: The tapes could not be made longer than 1 m.
2006: I < 140 A at 77 K for wire 100 m long and 4 mm wide (http://www.amsuper.com)
2009: National High Magnetic Field Laboratory at Florida State University was awarded nearly US$ 3 million to build a high-temperature superconducting magnet (32 T with ~ 8 km of HTS cable).



Need about 1000 SMs to keep the high energy particles moving in the circular accelerator.

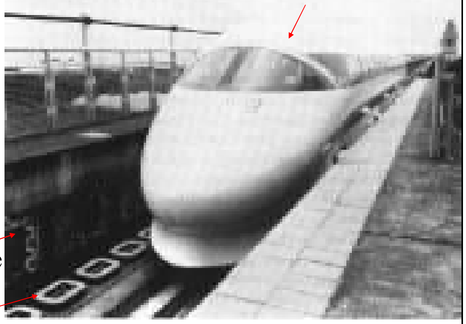
(ii) Maglev (磁浮火車) in Japan

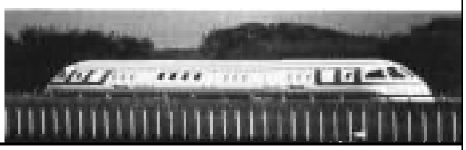
SMs on board

Speed record:
518 km/h

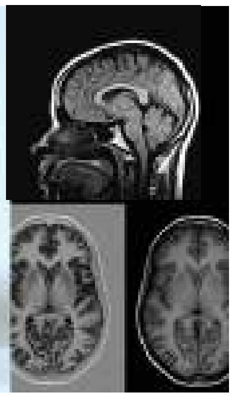
Coils for guidance

Coils for levitation using Faraday's law.





(iv) Magnetic Resonance Imaging (MRI) (1977)



Require a strong magnetic field (0.5 – 2 Tesla), usually generated by SM. (Physics World Dec. 2002, pp.31-35.)

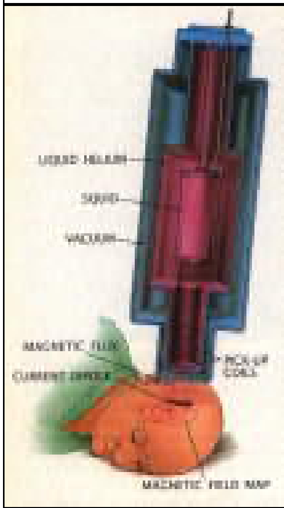
Stronger field ⇒ higher resolution of images
Here is a high field MRI (4 Tesla) (for research only):

(b) SQUID for brain research

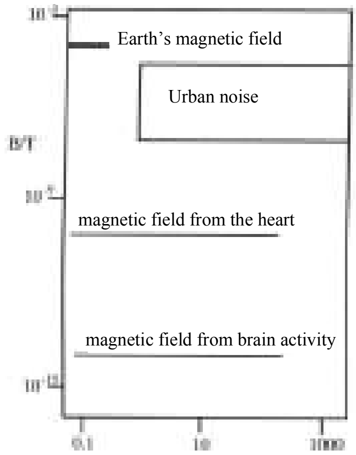
SQUID is a sensitive magnetic field sensor.

(magnetic field signal due to brain activity)

(Scientific American, Aug. 1994, p.36.)



Note:
Magnetic field level



frequency (Hz)