UNIT - 3

Numerical Differentiation

Branch- EC. 2nd year

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Numerical Differentiation

Newton's forward difference formula to get the derivative

x ——— x

We know that Newton's forward difference interpolation formula

$$y = f(x) = f(a) + P\Delta f(a) + \frac{P(P-1)}{2!}\Delta^2 f(a) + \frac{P(P-1)(P-2)}{3!}\Delta^3 f(a) + \cdots \quad \text{—— ①}$$

where $P = \frac{x-a}{h}$

Differentiating (1) w.r.t to P we get

$$\frac{dy}{dP} = 0 + \Delta f(a) + \frac{2P-1}{2!}\Delta^2 f(a) + \frac{3P^2-6P+2}{3!}\Delta^3 f(a) + \cdots$$

$P = \frac{x-a}{h}$, $\qquad \frac{dy}{dx} = \frac{dy}{dP}\cdot\frac{dP}{dx}$

$\frac{dP}{dx} = \frac{1}{h}$ $\qquad \frac{dy}{dx} = \frac{1}{h}\cdot\frac{dy}{dP}$ —— ②

$$\boxed{\frac{dy}{dx} = \frac{1}{h}\left[\Delta f(a) + \frac{2P-1}{2}\Delta^2 f(a) + \frac{3P^2-6P+2}{6}\Delta^3 f(a) + \cdots\right]} \quad \text{—— (3)}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{h}\cdot\frac{dy}{dP}\right) = \frac{d}{dP}\left(\frac{1}{h}\cdot\frac{dy}{dP}\right)\frac{dP}{dx}$$

$$= \frac{1}{h^2}\frac{d}{dP}\left(\frac{dy}{dP}\right)$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 f(a) + (P-1)\Delta^3 f(a) + \frac{6P^2-18P+11}{11}\Delta^4 f(a) + \cdots\right]} \quad \text{—— (4)}$$

Example Given that

x	1.0	1.1	1.2	1.3
y	0.841	0.891	0.932	0.963

find $\frac{dy}{dx}$ at $x = 1.0$

Sol. Newton's forward difference table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1.0	0.841			
		0.050		
1.1	0.891		−0.009	
		0.041		−0.001
1.2	0.932		−0.010	
		0.031		
1.3	0.963			

Here $x = 1$, $a = 1$, $h = 0.1$ $\quad p = \frac{x-a}{h} = \frac{1-1}{0.1} = 0$

first derivative of Newton's forward diff. interpolation formula.

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta f(a) + \frac{2p-1}{2}\Delta^2 f(a) + \frac{3p^2-6p+2}{6}\Delta^3 f(a)\right]$$

$$= \frac{1}{h}\left[\Delta f(a) - \frac{1}{2}\Delta^2 f(a) + \frac{1}{3}\Delta^3 f(a)\right]$$

$$= \frac{1}{0.1}\left[0.05 - 0.5(-0.009) + \frac{1}{3}(-0.001)\right]$$

$$= 0.542 \quad \text{Ans.}$$

first derivative of Newton's Backward diff interpolation form

$$\boxed{\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{2p+1}{2}\nabla^2 y_n + \frac{3p^2+6p+2}{6}\nabla^3 y_n + \cdots\right]}$$

Example: Find the first and second derivatives of the function given below at the point $x = 1.2$:

x	1	2	3	4	5
y	0	1	5	6	8

Sol. Newton's forward difference table

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	0				
		1			
2	1		3		
		4		-6	
3	5		-3		10
		1		4	
4	6		1		
		2			
5	8				

$x = 1.2$, $a = 1$ $\quad h = 1$ $\quad x = a + ph$ $\quad p = 0.2$

$$\frac{dy}{dx} = \frac{1}{1}\left[1 + \frac{2\times 0.2 - 1}{2}(3) + \frac{3(0.2)^2 - 6(0.2) + 2}{6}(-6) + \frac{4(0.2)^3 - 18(0.2)^2 + 22}{24} \ldots -6\right.$$

$$= -1.7733$$

$$\frac{d^2y}{dx^2} = \frac{1}{1^2}\left[3 + (0.2 - 1)(-6) + \frac{6(0.2)^2 - 18(0.2) + 11}{12}(10)\right]$$

$$= 3 + 4.8 + \frac{0.24 - 3.6 + 11}{12}(10)$$

$$= 14.1667$$

Ans

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