⟹ Unbalenced Transportation problem :— As we have already discussed, if total supply at the origin is equal to the total demand at the destination, the problem is known as balenced transportation problem. On the other hand, if total supply is not equal to the total demand, the problem is known as unbalenced transportation problem.

In unbalenced transportation problem if the total supply is more than the total demand then we introduce an additional column which will indicate the surplus supply with transportation cost zero. Similarly, if the total demand is more than the total supply an additional row is introduced in the transportation table which indicates unsatified demand with zero transportation cost.

Let us consider following unbalenced transportation problem.

Problem!

Plant \ Warehouses	W_1	W_2	W_3	
P_1	20	17	28	400
P_2	10	10	20	500
Demand	400	400	500	900 / 1300

Solution :- In this problem the total demand is 1300 ~~and~~ whereas the total supply is 900. Therefore this is a unbalenced transportation problem. Now, we

introduce an addition row with zero transportation cost denoting unsatisfied demands.
Hence the modified transportation table is;

	W_1	W_2	W_2	
P_1	20	17	25	400
P_2	10	10	20	500
Unsatisfied demand	0	0	0	400
	400	400	500	1300 / 1300

Now we can see the above table is a balanced transportation problem, so we can solve this problem as discussed in the previous problem.

⇒ Degenerate transportation problem:- In transportation problem, if a basic feasible solution with m origins and n destinations has less than m+n-1 positive occupied cells ($x_{ij} < m+n-1$), then the problem is said to be degenerate transportation problem.
The degeneracy problem does not cause any serious difficulty, but it can cause computational problem while determining the optimal minimum solution.

Therefore it is important to identify a degenerate problem as early as begning and take the necessary action to avoid any computation dificulty. The degeneracy

canbe identified through the following results:

In a transportation problem, a degenerate basic feasible solution exists if and only if some partial sum of supply (row) is equal to a partial sum of demand (column). For example the following transportation problem is degenerate.

	W_1	W_2	W_3	supply (a_i)
P_1	20	17	25	400
P_2	10	10	20	500
unsatisfied demand	0	0	0	400
Demand(b_j)	400	400	500	1300 / 1300

Becaus in this problem $a_1 = 400 = b_1$

$a_2 + a_3 = 900 = b_2 + b_3$

the problem is degenerate.

There is a technique called perturbation, which helps to solve degenerate problems.

⇒ Perturbation Technique :- The degeneracy of the transportation problem can be avoided if we ensure that no partial sum of supply (a_i) and demand (b_j) is equal. We setup a new problem where

$$a_i = a_i + d \qquad i = 1, 2 \cdots m$$

$$b_j = b_j \qquad j = 1, 2 \cdots n-1$$

$$b_n = b_n + m \cdot d \qquad d > 0$$

This modified problem is constructed in such a way

that no partial sum of a_i is equal to the b_j. Once the problem is solved, we substitute $d=0$ leading to the optimum solution of the original problems.

Problem: Let us consider the previous problem

	W_1	W_2	W_3	supply
P_1	20	17	25	400+d
P_2	10	10	20	500+d
Unsatisfied demand	0	0	0	400+d
	400	400	500+3d	1300+3d / 1300+3d

Now this modified problem can be solved by any of three methods.