The group of symmetries of a square:-

We take a square cardboard marked as a, b, c d, e, f, g, h as shown in the fig. which can rotate about its perpendicular axis passing through center of cardboard. This card boad is hanged on blackboad or wall at its centre point. Mark 1, 2, 3, 4, 5, 6, 7, 8 be made on wall or blackboard. as shown in fig 1

If a rotation through an angle $2\pi/n$ above some axis leave the system invariant, the axis is called an n-fold symmetry axis of the system which is denoted as C_n. the there are k successive operation then it is represented as C_n^k. Reflection in the plane can be represented as m or σ.

It is interesting to note that there are eight transformation correspond to eight different ways which is shown in fig. 2

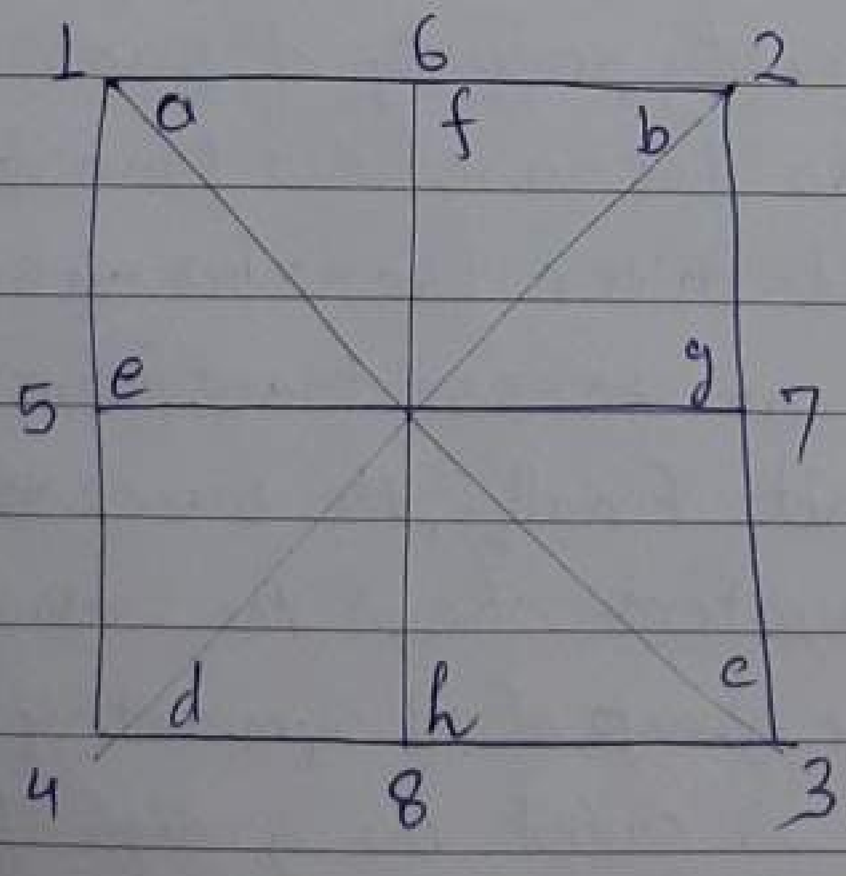


Fig 1: The axis and planes of symmetry of a square

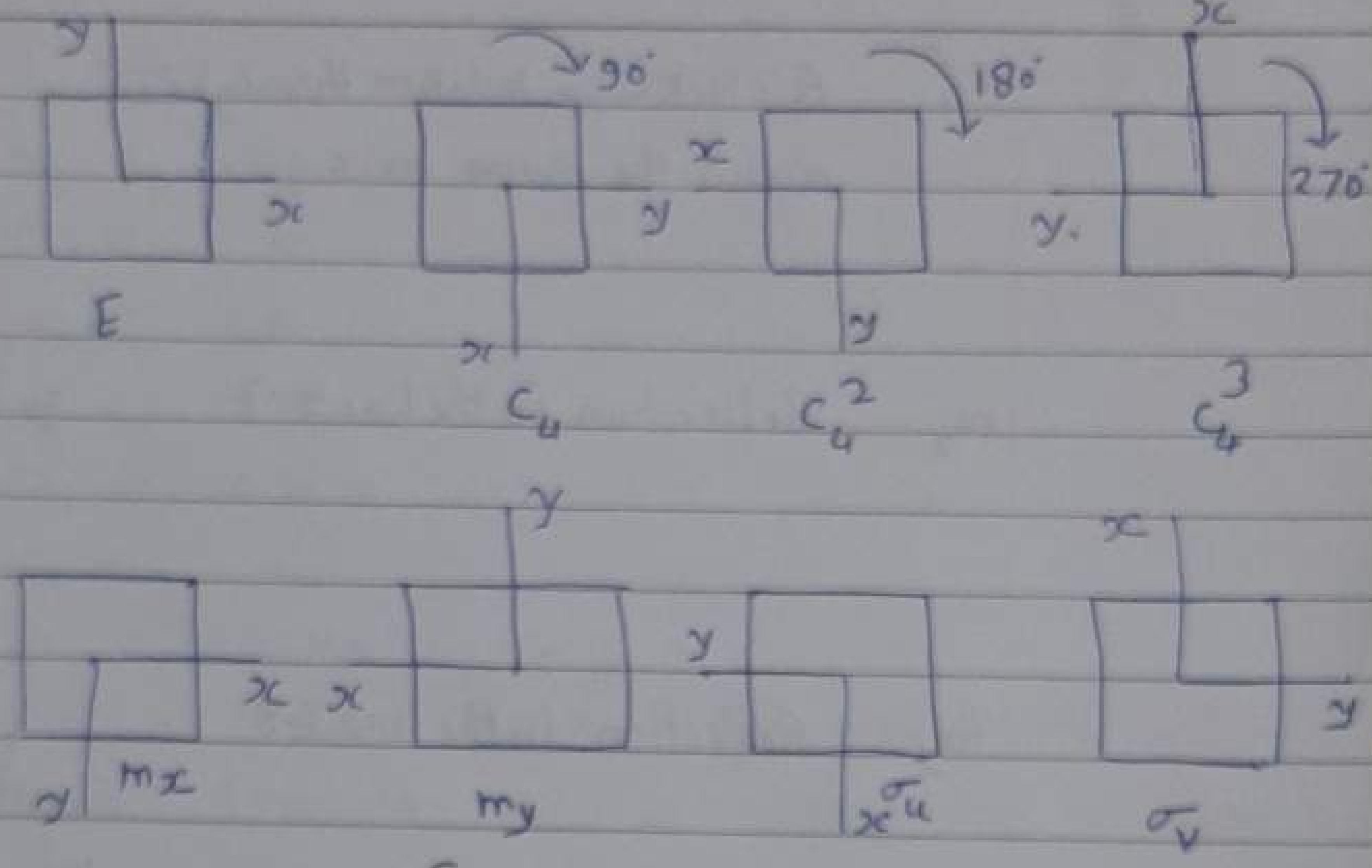


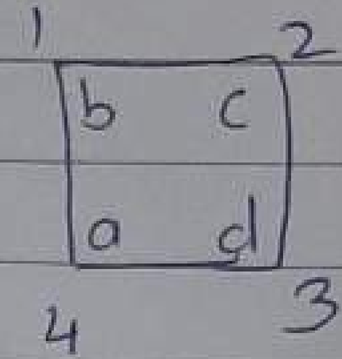
Fig. 2.

The equivalent of the transformations of a square with those of a cartesian co-ordinate system

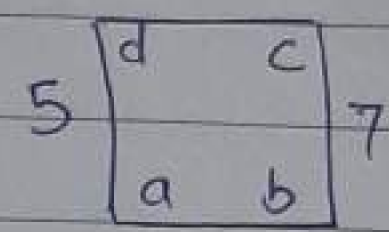
Table: 1. Symmetry Transformations of a square

Symbol	Operation	Results.
E	The identity	$\begin{array}{cccc}1&&&2\\&a&b&\\&d&c&\\4&&&3\end{array}$
C_4	A clockwise rotation through 90° about an axis normal to the square and passing through its centre	$\begin{array}{cccc}1&&&2\\&d&a&\\&c&b&\\4&&&3\end{array}$
C_4^2	A rotation through 180° about the axis	$\begin{array}{cccc}1&&&2\\&c&d&\\&b&a&\\4&&&3\end{array}$

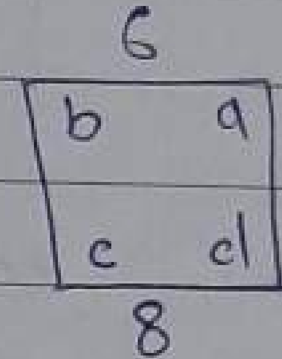
C_4^3 A clockwise rotation through 270° about the same axis



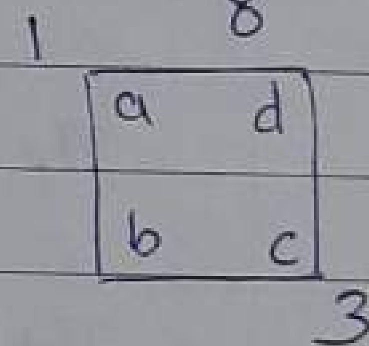
m_x Reflection in the line 5-7



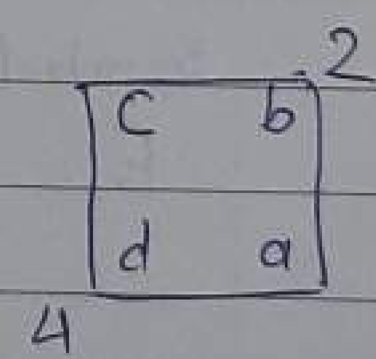
m_y Reflection in the line 6-8



σ_u Reflection in the line 1-3



σ_v Reflection in the line 2-4



The set of all above operation is also a group under binary multiplications which be proved as follows:-

(i) Closure Property:-

$$C_4 E = C_4$$
$$C_4^3 C_4 = C_4^4 = E$$
$$C_4^2 C_4 = C_4^3$$