that the group elements are distinct.
Since no element can occur more than once in a row or in a column, and since the number of places to be filled in each row or each column is equal to the order of the group, each element must occur once and only once in each row and in each column. Then the theorem proves.

Generators of a finite group:-

The elements of the smallest set of elements whose powers and products generate all the elements are called the generators of the group.

Example:-1 Generate a group starting from an elements A subject only to the relation $A^n = E$ such that n is the smallest positive integer satisfying this relation.

Solution:- Now here is a new element A of the group. Then next new elements are generated as $A^2, A^3, A^4, \ldots A^n = E$, The higher powers of A do not give us new elements because $A^{n+k} = A^k$. Thus desired group is thus $[A, A^2, A^3, \ldots A^{n-1}, A^n = E]$ whose order is n.

Ex2:- Generate a group from two elements A and B subject only to the relation $A^2 = B^3 = (AB)^2 = E$.

Solution:-

The group must contain the elements $E, A, B, B^2,$

since $A^2 = E$ and $B^3 = E$. But then it must also contain all the products of A, B and B^2 among themselves. Hence we get two new elements of the group, AB and BA. But it do not commute because $(AB)^2 = E$

$$E = ABAB = A^2B^2 = B^2 \qquad \text{Since } A^2 = E$$

which is not true. Therefore AB and BA are distinct elements. Thus it contains six elements of the group i.e E, A, B, B^2, AB, BA.

It can be shown that the set is a group.

(i) closure property:-

$$(AB).A = AB^2 \text{ belongs to set.}$$

Now from the relation $(AB)^2 = E$

$$(AB)^{-1} = AB$$

$$B^{-1}A^{-1} = AB$$

$$B^{-1}A = AB \qquad \text{since } A^2 = E \text{ or } A = EA^{-1} = A^{-1}$$

But $B^3 = E$. we have

$$B^{-1} = B^2.$$

Hence $AB = B^2A$. Thus

Similarly it can be verified that the inverse of each elements of the set also belongs to the set. Hence the desired group is $[E, A, B, B^2, AB, BA]$ whose order is six.

Conjugate elements and its classes:-

Consider a relation such as

$$A^{-1}BA = C$$

where A, B and C are elements of a group. When such a relation exists between two elements B and C, they are said to be conjugate elements. The operation is called a similarity transformation of B by A. It is clear that

$$ACA^{-1} = B$$

<u>Example:-</u> Prove that the following pairs are conjugate to each other

(i) (m_x, m_y) with C_4

(ii) (C_4, C_4^3) with C_4

(iii) (σ_u, σ_v) with C_4

Proof: (i) The group of symmetry operations is

$[E, C_4, C_4^2, C_4^3, m_x, m_y, \sigma_u, \sigma_v]$.

The condition of conjugate elements is

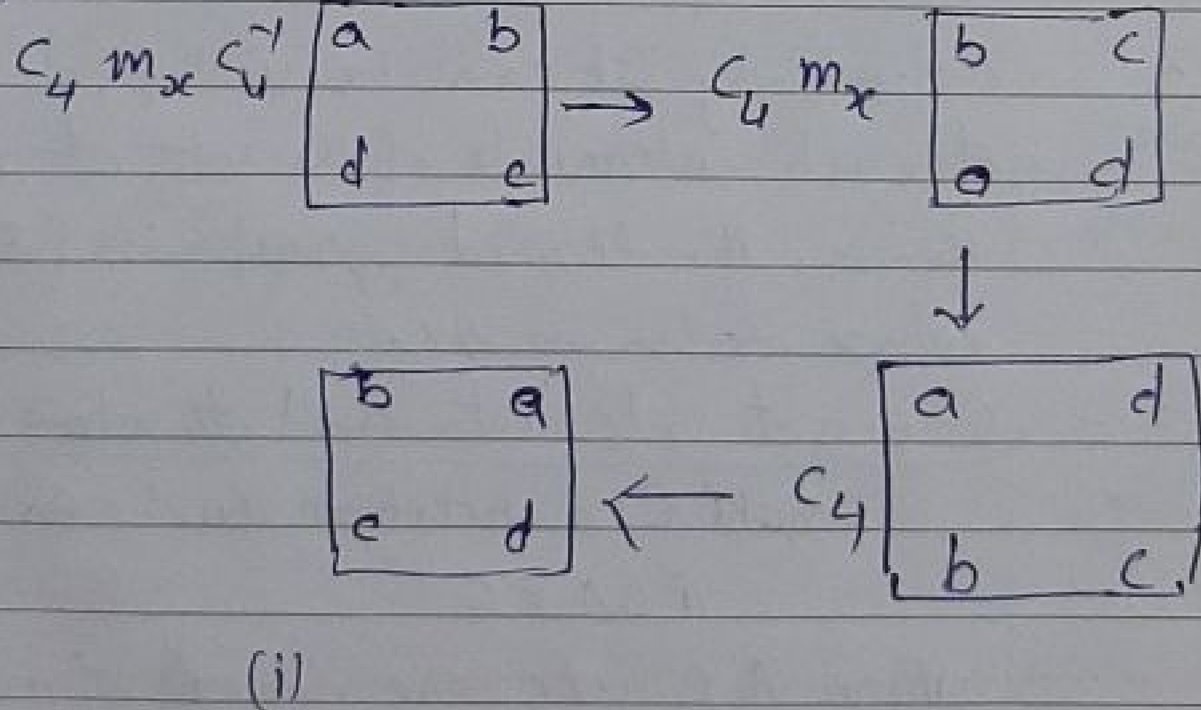
$$ABA^{-1} = C$$

Let $A = C_4$, $B = m_x$, $C = m_y$

We have to prove

$$C_4\, m_x\, C_4^{-1} = m_y$$

L.H.S



(i)