Experimental Verifications for Characteristics of Nuclear Force

Spin dependence

The Nucleon – Nucleon Interaction is Strongly Spin Dependent

This observation follows from the failure to observe a singlet bound state of the deuteron and also from the measured differences between the singlet and triplet cross sections. What is the form of an additional term that must be added to the potential to account for this effect? Obviously the term must depend on the spins of the two nucleons, $\mathbf{s}_1$ and $\mathbf{s}_2$, but not all possible combinations of $\mathbf{s}_1$ and $\mathbf{s}_2$ are permitted. The nuclear force must satisfy certain symmetries, which restrict the possible forms that the potential could have. Examples of these symmetries are *parity* ($\mathbf{r} \rightarrow -\mathbf{r}$) and *time reversal* ($t \rightarrow -t$). Experiments indicate that, to a high degree of precision (one part in 10^7 for parity and one part in 10^3 for time reversal), the internucleon potential is invariant with respect to these operations. Under the parity operator, which involves spatial reflection, angular momentum vectors are unchanged. This statement may seem somewhat surprising, because upon inverting a coordinate system we would naturally expect all vectors defined in that coordinate system to invert. However, angular momentum is not a true or polar vector; it is a pseudo- or axial vector that does not invert when $\boldsymbol{r} \to -\boldsymbol{r}$. This follows directly from the definition $\boldsymbol{r} \times \boldsymbol{p}$ or can be inferred from a diagram of a spinning object. Under the time-reversal operation, all motions (including linear and angular momentum) are reversed. Thus terms such as $\boldsymbol{s}_1$ or $\boldsymbol{s}_2$ or a linear combination $A\boldsymbol{s}_1 + B\boldsymbol{s}_2$ in the potential would violate time-reversal invariance and cannot be part of the nuclear potential; terms such as $\boldsymbol{s}_1^2$, $\boldsymbol{s}_2^2$, or $\boldsymbol{s}_1 \cdot \boldsymbol{s}_2$ are invariant with respect to time reversal and are therefore allowed. (All of these terms are also invariant with respect to parity.) The simplest term involving both nucleon spins is $\boldsymbol{s}_1 \cdot \boldsymbol{s}_2$. Let's consider the value of $\boldsymbol{s}_1 \cdot \boldsymbol{s}_2$ for singlet and triplet states. To do this we evaluate the total spin $\boldsymbol{S} = \boldsymbol{s}_1 + \boldsymbol{s}_2$

$$\boldsymbol{S}^2 = \boldsymbol{S} \cdot \boldsymbol{S} = (\boldsymbol{s}_1 + \boldsymbol{s}_2) \cdot (\boldsymbol{s}_1 + \boldsymbol{s}_2)$$
$$= \boldsymbol{s}_1^2 + \boldsymbol{s}_2^2 + 2\boldsymbol{s}_1 \cdot \boldsymbol{s}_2$$

Thus

$$\boldsymbol{s}_1 \cdot \boldsymbol{s}_2 = \tfrac{1}{2}(\boldsymbol{S}^2 - \boldsymbol{s}_1^2 - \boldsymbol{s}_2^2)$$

To evaluate this expression, we must remember that in quantum mechanics all squared angular momenta evaluate as $\boldsymbol{s}^2 = \hbar^2 s(s+1)$;

$$\langle \boldsymbol{s}_1 \cdot \boldsymbol{s}_2 \rangle = \tfrac{1}{2}\left[S(S+1) - s_1(s_1+1) - s_2(s_2+1)\right]\hbar^2$$

With nucleon spins s_1 and s_2 of $\frac{1}{2}$, the value of $\boldsymbol{s}_1 \cdot \boldsymbol{s}_2$ is, for triplet ($S = 1$) states:

$$\langle \boldsymbol{s}_1 \cdot \boldsymbol{s}_2 \rangle = \tfrac{1}{2}\left[1(1+1) - \tfrac{1}{2}(\tfrac{1}{2}+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)\right]\hbar^2 = \tfrac{1}{4}\hbar^2$$

and for singlet ($S = 0$) states:

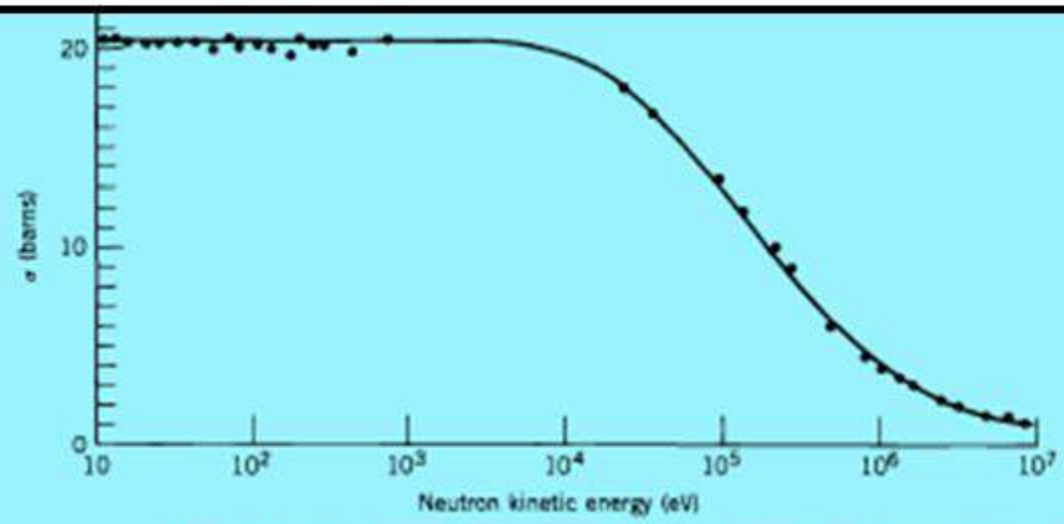
$$\langle \boldsymbol{s}_1 \cdot \boldsymbol{s}_2 \rangle = \tfrac{1}{2}\left[0(0+1) - \tfrac{1}{2}(\tfrac{1}{2}+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)\right]\hbar^2 = -\tfrac{3}{4}\hbar^2$$

Thus a spin-dependent expression of the form $\boldsymbol{s}_1 \cdot \boldsymbol{s}_2 V_s(r)$ can be included in the potential and will have the effect of giving different calculated cross sections for singlet and triplet states. The magnitude of V_s can be adjusted to give the correct differences between the singlet and triplet cross sections and the radial dependence can be adjusted to give the proper dependence on energy.

We could also write the potential including V_c and V_s as

$$V(r) = -\left(\frac{\boldsymbol{s}_1 \cdot \boldsymbol{s}_2}{\hbar^2} - \frac{1}{4}\right)V_1(r) + \left(\frac{\boldsymbol{s}_1 \cdot \boldsymbol{s}_2}{\hbar^2} + \frac{3}{4}\right)V_3(r)$$

where $V_1(r)$ and $V_3(r)$ are potentials that separately give the proper singlet and triplet behaviors.



The neutron–proton scattering cross section at low energy. Data taken from a review by R. K. Adair, *Rev. Mod. Phys.* **22**, 249 (1950), with additional recent results from T. L. Houk, *Phys. Rev. C* **3**, 1886 (1970).

where 1 barn (b) = 10^{-28} m^2. This result suggests that the cross section should be constant at low energy and should have a value close to 4–5 b.

Figure shows the experimental cross sections for scattering of neutrons by protons. The cross section is indeed constant at low energy, and it decreases with E at large energy but the low-energy cross section, 20.4 b, is not in agreement with our calculated value of 4–5 b.

For the solution to this discrepancy, we must study the relative spins of the incident and scattered nucleons. The proton and neutron spins (each $\frac{1}{2}$) can combine to give a total spin $\boldsymbol{S} = \boldsymbol{s}_p + \boldsymbol{s}_n$ that can have magnitude either 0 or 1. The $S = 1$ combination has three orientations (corresponding to z components $+1, 0, -1$) and the $S = 0$ combination has only a single orientation. For that reason, the $S = 1$ combination is called a *triplet* state and the $S = 0$ combination is called a *singlet* state. Of the four possible relative spin orientations, three are associated with the triplet state and one with the singlet state. As the incident nucleon approaches the target, the probability of being in a triplet state is 3/4 and the probability of being in a singlet state is 1/4. If the scattering cross section is different for the singlet and triplet states, then

$$\sigma = \tfrac{3}{4}\sigma_t + \tfrac{1}{4}\sigma_s$$

where σ_t and σ_s are the cross sections for scattering in the triplet and singlet states, respectively. In estimating the cross section we used parameters obtained from the deuteron, which is in a $S = 1$ state. We therefore take $\sigma_t = 4.6$ b and using the measured value of $\sigma = 20.4$ b for the low-energy cross section, we deduce

$$\sigma_s = 67.8 \text{ b}$$

This calculation indicates that there is an enormous difference between the cross sections in the singlet and triplet states—that is, *the nuclear force must be spin dependent*.

Tensor Potential/Non Central Potential

The Internucleon Potential Includes a Noncentral Term, Known as a Tensor Potential

Evidence for the tensor force comes primarily from the observed quadrupole moment of the ground state of the deuteron. An s-state ($\ell = 0$) wave function is spherically symmetric; the electric quadrupole moment vanishes. Wave functions with mixed ℓ states must result from noncentral potentials. This tensor force must be of the form $V(\boldsymbol{r})$, instead of $V(r)$. For a single nucleon, the choice of a certain direction in space is obviously arbitrary; nucleons do not distinguish north from south or east from west. The only reference direction for a nucleon is its spin, and thus only terms of the form $\boldsymbol{s} \cdot \boldsymbol{r}$ or $\boldsymbol{s} \times \boldsymbol{r}$, which relate $\boldsymbol{r}$ to the direction of $\boldsymbol{s}$, can contribute. To satisfy the requirements of parity invariance, there must be an even number of factors of $\boldsymbol{r}$, and so for two nucleons the potential must depend on terms such as $(\boldsymbol{s}_1 \cdot \boldsymbol{r})(\boldsymbol{s}_2 \cdot \boldsymbol{r})$ or $(\boldsymbol{s}_1 \times \boldsymbol{r}) \cdot (\boldsymbol{s}_2 \times \boldsymbol{r})$. Using vector identities we can show that the second form can be written in terms of the first and the additional term $\boldsymbol{s}_1 \cdot \boldsymbol{s}_2$, which we already included in $V(r)$. Thus without loss of generality we can choose the tensor contribution to the internucleon potential to be of the form $V_T(r)S_{12}$, where $V_T(r)$ gives the force the proper radial dependence and magnitude, and

$$S_{12} = 3(\boldsymbol{s}_1 \cdot \boldsymbol{r})(\boldsymbol{s}_2 \cdot \boldsymbol{r})/r^2 - \boldsymbol{s}_1 \cdot \boldsymbol{s}_2$$

which gives the force its proper tensor character and also averages to zero over all angles.

Charge Symmetric & Charge Independent

The Nucleon – Nucleon Force Is Charge Symmetric

This means that the proton–proton interaction is identical to the neutron–neutron interaction, after we correct for the Coulomb force in the proton–proton system. Here "charge" refers to the character of the nucleon (proton or neutron) and not to electric charge. Evidence in support of this assertion comes from the equality of the pp and nn scattering lengths and effective ranges. Of course, the pp parameters must first be corrected for the Coulomb interaction. When this is done, the resulting singlet pp parameters are

$$a = -17.1 \pm 0.2 \text{ fm}$$

$$r_0 = 2.84 \pm 0.03 \text{ fm}$$

These are in very good agreement with the measured nn parameters ($a = -16.6 \pm 0.5$ fm, $r_0 = 2.66 \pm 0.15$ fm), which strongly supports the notion of charge symmetry.

The Nucleon – Nucleon Force Is Nearly Charge Independent

This means that (in analogous spin states) the three *nuclear* forces nn, pp, *and* pn are identical, again correcting for the pp Coulomb force. Charge independence is thus a stronger requirement than charge symmetry. Here the evidence is not so conclusive; in fact, the singlet np scattering length (−23.7 fm) seems to differ substantially from the pp and nn scattering lengths (−17 fm)

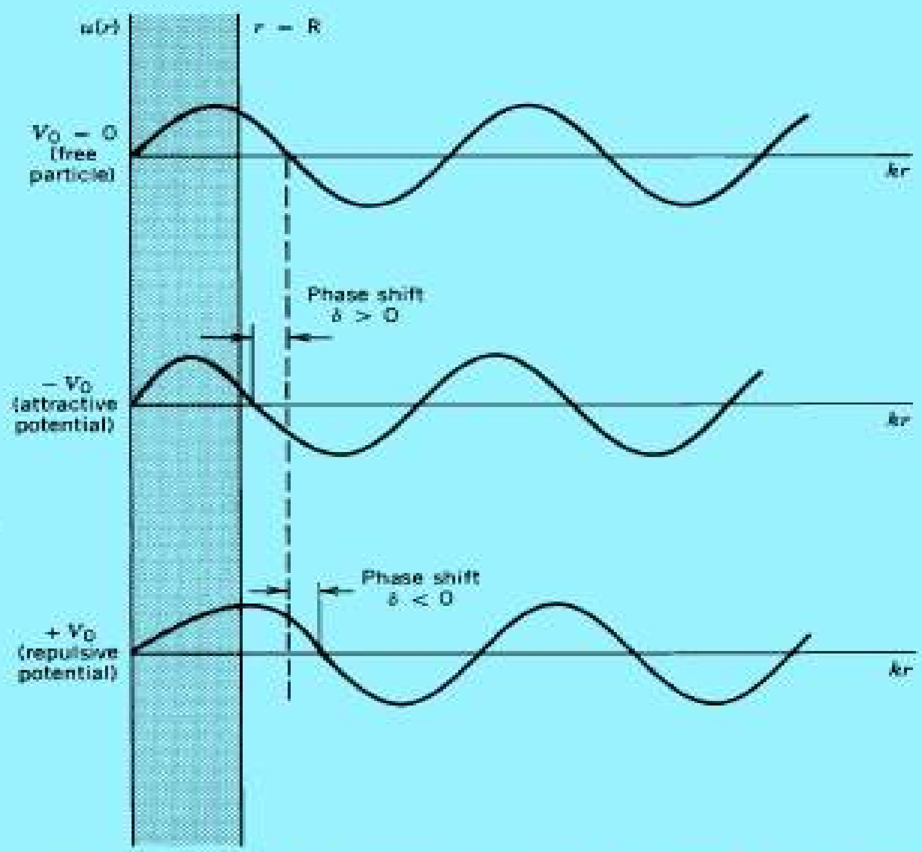
However, we see that large negative scattering lengths are extraordinarily sensitive to the nuclear wave function near $r = R$, and a very small change in ψ can give a large change in the scattering length. Thus the large difference between the scattering lengths may correspond to a very small difference (of order 1%) between the potentials, which (as we see in the next section) is easily explained by the exchange force model.

Repulsive at very short distance (Existence of Hard Core)

The s-wave phase shifts can be easily extracted from the differential scattering measurements of $d\sigma/d\Omega$ vs θ because they do not depend on θ.) At about 300 MeV, the s-wave phase shift becomes *negative*, corresponding to a change from an attractive to a repulsive force. To account for the repulsive core, we must modify the potentials we use in our calculations. For example, again choosing a square-well form to simplify the calculation, we might try

$$\begin{aligned} V(r) &= +\infty & r &< R_{\text{core}} \\ &= -V_0 & R_{\text{core}} \le r &\le R \\ &= 0 & r &> R \end{aligned}$$

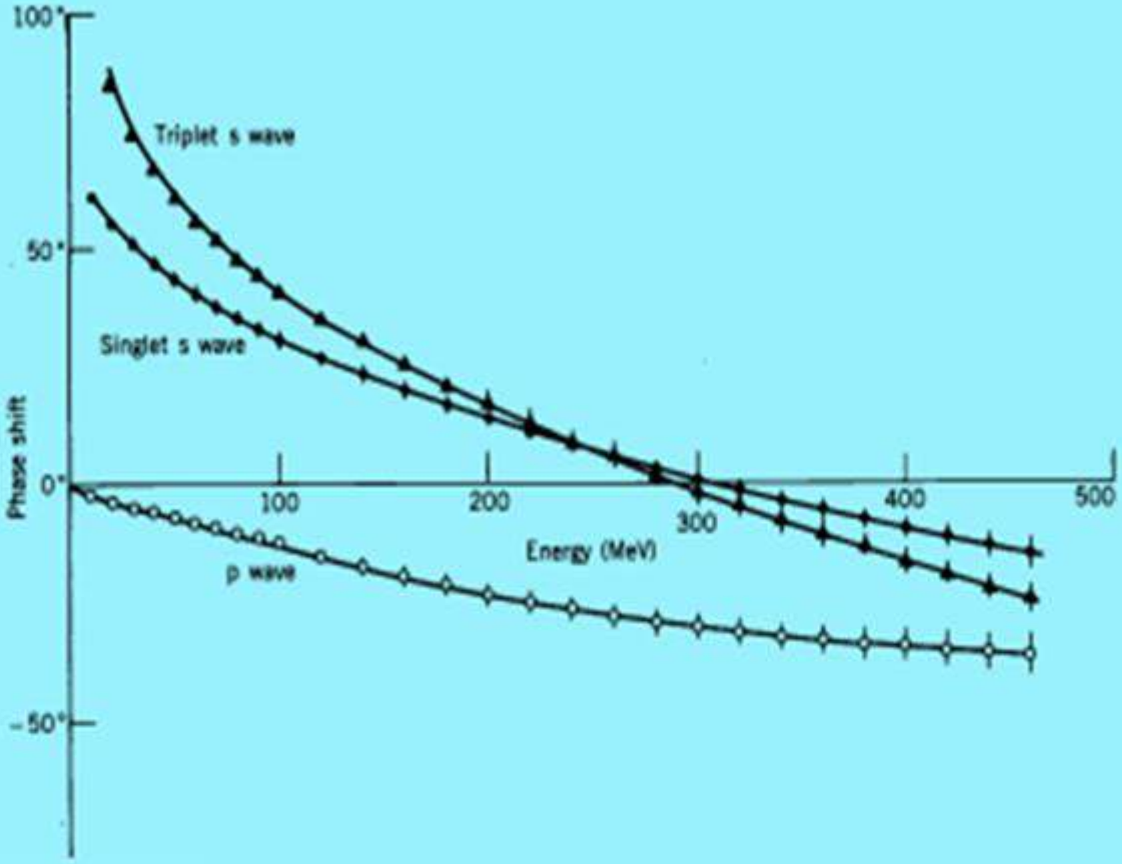
and we can adjust R_{core} until we get satisfactory agreement with the observed s-wave phase shifts. The value $R_{\text{core}} \simeq 0.5$ fm gives agreement with the observed phase shifts.



The effect of a scattering potential is to shift the phase of the scattered wave at points beyond the scattering regions, where the wave function is that of a free particle.

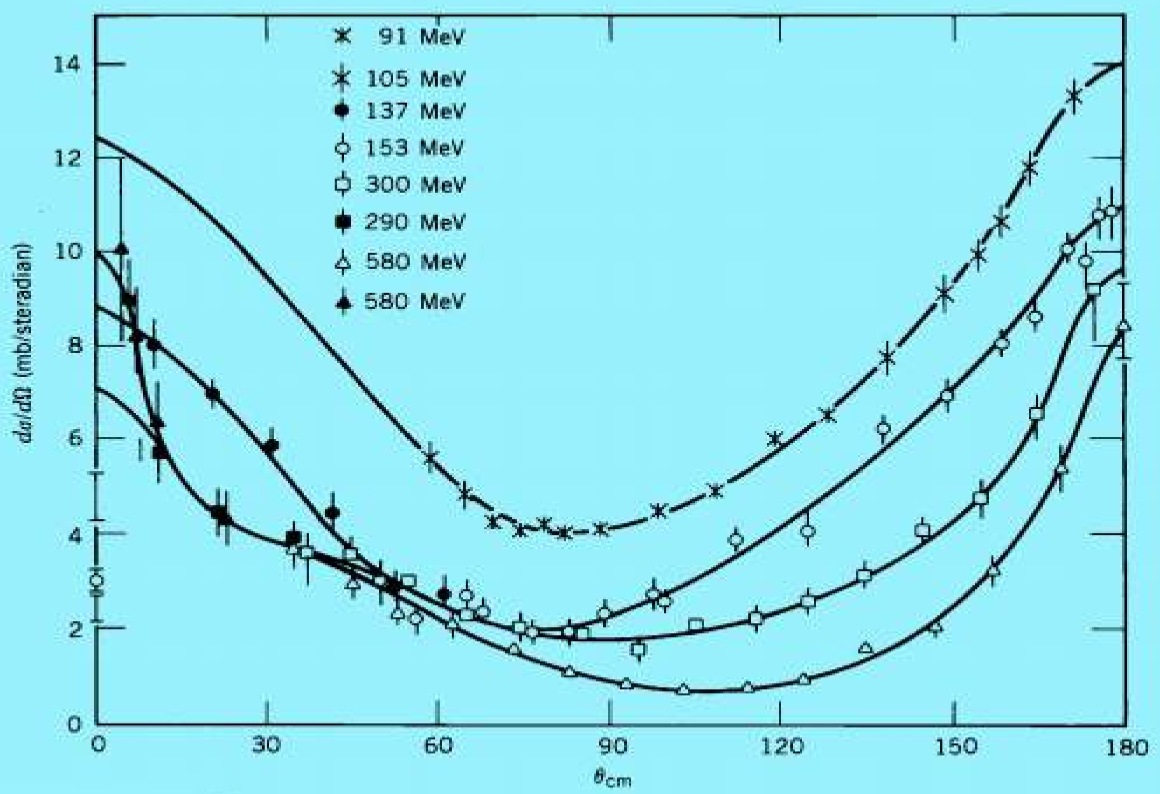
The Nucleon – Nucleon Interaction Becomes Repulsive at Short Distances

This conclusion follows from qualitative considerations of the nuclear density: as we add more nucleons, the nucleus grows in such a way that its central density remains roughly constant, and thus something is keeping the nucleons from crowding too closely together. More quantitatively, we can study nucleon–nucleon scattering at higher energies. Figure shows the deduced singlet s-wave phase shifts for nucleon–nucleon scattering up to 500 MeV. (At these energies, phase shifts from higher partial waves, p and d for example, also contribute to the cross



The phase shifts from neutron–proton scattering at medium energies. The change in the s-wave phase shift from positive to negative at about 300 MeV shows that at these energies the incident nucleon is probing a repulsive core in the nucleon–nucleon interaction. ▲, $^3S_1$; ●, $^1S_0$; O, $^1P_1$. Data from M. MacGregor et al., *Phys. Rev.* **182**, 1714 (1969).

Exchange character



The neutron–proton differential cross section at medium energies. The strong forward-scattering peak (near 0°) is expected; the equally strong backward peak (near 180°) is evidence for the exchange force. From R. Wilson, *The Nucleon–Nucleon Interaction* (New York: Wiley-Interscience, 1963).

In view of the preceding discussion, it is natural to associate the "something" that is exchanged in the nucleon–nucleon interaction with quanta of the nuclear field. For a spin-$\frac{1}{2}$ neutron to turn into a spin-$\frac{1}{2}$ proton, it is clear that the exchanged particle must have integral spin (0 or 1) and must carry electric charge. In addition, if we wish to apply the same exchange-force concepts to nn and pp interactions, there must also be an uncharged variety of exchanged particle. Based on the observed range of the nuclear force, we can estimate the mass of the exchanged particle. Let us assume that a nucleon (which we denote by N, to include both neutrons and protons) emits a particle x. A second nucleon absorbs the particle x:

$$N_1 \rightarrow N_1 + x$$
$$x + N_2 \rightarrow N_2$$

How is it possible for a nucleon to emit a particle of mass energy $m_x c^2$ and still remain a nucleon, without violating conservation of energy? It is not possible, unless the emission and reabsorption take place within a short enough time Δt that we are unaware energy conservation has been violated. Since the limits of our ability to measure an energy (and therefore to determine whether energy is conserved) are restricted by the uncertainty principle, if $\Delta t < \hbar/(m_x c^2)$, we will be unaware that energy conservation has been violated by an amount $\Delta E = m_x c^2$. The maximum range of the force is determined by the maximum distance that the particle x can travel in the time Δt. If it moves at speeds of the order of c, then the range R can be at most

$$R = c\Delta t = \frac{\hbar c}{m_x c^2} = \frac{200 \text{ MeV} \cdot \text{fm}}{m_x c^2}$$

where we have used a convenient approximation for $\hbar c$. Equation 4.53 gives a useful relationship between the mass energy of the exchanged particles and the range of the force. For nuclear forces with a range of about 1 fm, it is clear that we must have an exchanged particle with a mass energy of the order of 200 MeV.

Such particles that exist only for fleeting instants and allow us to violate conservation of energy (and momentum—the emitting and absorbing nucleons do not recoil) are known as *virtual* particles. We can observe the force that results from the exchange of virtual particles, but we cannot observe the particles themselves during the exchange. (Exchanged virtual particles can be identical with ordinary particles, however. According to field theory, the Coulomb interaction between electric charges can be regarded as the exchange of virtual photons, which have properties in common with ordinary real photons.)

The exchanged particles that carry the nuclear force are called *mesons* (from the Greek "meso" meaning middle, because the predicted mass was between the masses of the electron and the nucleon). The lightest of the mesons, the π-meson or simply *pion*, is responsible for the major portion of the longer range (1.0 to 1.5 fm) part of the nucleon–nucleon potential. To satisfy all the varieties of the exchanges needed in the two-nucleon system, there must be three pions, with electric charges of +1, 0, and −1. The pions have spin 0 and rest energies of 139.6 MeV (for $\pi^{\pm}$) and 135.0 MeV (for π^0). At shorter ranges (0.5–1.0 fm), two-pion exchange is probably responsible for the nuclear binding; at much shorter ranges (0.25 fm) the exchange of ω mesons ($mc^2 = 783$ MeV) may contribute to the repulsive core whereas the exchange of ρ mesons ($mc^2 = 769$ MeV) may provide the spin-orbit part of the interaction.

The differing masses for the charged and neutral pions may explain the possible small violation of charge independence we discussed previously. The single pion that is exchanged between two identical nucleons must be a π^0:

$$n_1 \rightarrow n_1 + \pi^0 \qquad \pi^0 + n_2 \rightarrow n_2$$

or

$$p_1 \rightarrow p_1 + \pi^0 \qquad \pi^0 + p_2 \rightarrow p_2$$

Charged pion exchange will not work:

$$n_1 \rightarrow p_1 + \pi^- \qquad \text{but } \pi^- + n_2 \rightarrow ?$$

$$p_1 \rightarrow n_1 + \pi^+ \qquad \text{but } \pi^+ + p_2 \rightarrow ?$$

because there are no nucleons with charges -1 or $+2$. (There are *excited states* of the nucleon with these charges, as we discuss in Chapters 17 and 18, but these high-energy states are unlikely to contribute substantially to the low-energy experiments we have discussed in this chapter.) However, the neutron–proton interaction can be carried by charged as well as neutral pions:

$$n_1 \rightarrow n_1 + \pi^0 \qquad \pi^0 + p_2 \rightarrow p_2$$

$$n_1 \rightarrow p_1 + \pi^- \qquad \pi^- + p_2 \rightarrow n_2$$

This additional term in the np interaction (and the difference in mass between the charged and neutral pions) may be responsible for the small difference in the potential that produces the observed difference in the scattering lengths.

The meson-exchange theory of nuclear forces was first worked out by Yukawa in 1935; According to the exchange theory of nuclear force the Meson exchange can be represented by a potential in the basic form of $r^{-1}e^{-r/R}$, where R is the range of the force ($R = \hbar/m_\pi c = 1.5$ fm for pions). A more detailed form for the one-pion exchange potential (called OPEP in the literature) is

$$V(r) = \frac{g_\pi^2(m_\pi c^2)^3}{3(Mc^2)^2\hbar^2}\left[s_1 \cdot s_2 + S_{12}\left(1 + \frac{3R}{r} + \frac{3R^2}{r^2}\right)\right]\frac{e^{-r/R}}{r/R}$$

Here g_π^2 is a dimensionless coupling constant that gives the strength of the interaction (just as e^2 gives the strength of the electromagnetic interaction) and M is the nucleon mass. This particular potential describes only the long-range part of the nucleon–nucleon interaction; other aspects of the interaction are described by other potentials.

The exchange-force model enjoyed a remarkable success in accounting for the properties of the nucleon–nucleon system. The forces are based on the exchange of virtual mesons, all of which can be produced in the laboratory and studied directly. The pion is the lightest of the mesons and therefore has the longest range. Exploring the nucleus with higher energy probes (with shorter de Broglie wavelengths) allows us to study phenomena that are responsible for the finer details of the nuclear structure, such as those that occur only over very short distances. These phenomena are interpreted as arising from the exchange of heavier mesons. Studying the spatial and spin dependence of these detailed interactions allows us to deduce the properties of the hypothetical exchanged meson. On the other hand, particle physicists are able to observe a large variety of mesons from high-energy collisions done with large accelerators. Among the debris from those collisions they can observe many varieties of new particles and catalog their properties. Nuclear physicists are then able to choose from this list candidates for the mesons exchanged in various details of the nucleon–nucleon interaction. This slightly oversimplified view nevertheless emphasizes the close historical relationship between nuclear physics and elementary particle physics.

Reference Book

Introductory Nuclear Physics by K. S. Krane