Index Numbers

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1.1 Introduction

Index numbers are devices for measuring changes in the magnitude of a group of related variables, over a period of time. These changes may have to do with the prices of the commodities, the physical quantity, changes of goods produced, marketed or consumed, or such concepts as intelligence beauty, or efficiency.

The comparisons may be between periods of time, between places, between like categories such as persons, schools, or objects. Thus, we may have index numbers comparing the cost of living at different times or in different countries or localities, the physical volume of production in different factories, or the efficiency of different school systems.

An index number is an aggregate measure of the relative change in a collection of presumably related items. For example a price index is an attempt to consider a particular market basket of commodities and services, the prices for each item in that market basket at two points of time, the relative importance of each item in the market basket, and to put all that information together in such a way that the general change in price for the total (or aggregate) market basket can be represented by one number.

1.2 Definition of an index number

Index numbers are statistical devices designed to measure the relative change in the level of a phenomenon with respect to time, geographical location or other characteristics. Such as incomes, level of production etc. such a study is of great importance for understanding economy, for the making of government policies and for fixing the wages etc. As a matter of fact, the index number is an economic barometer as it gives a measure of the economic pressure on the consumers, directly or indirectly.

John I. Griffin has describes it as "An index number is a quantity which by reference to a base period, shows by its variation, the changes in the magnitude over a period of time."

Index numbers are the numbers which express the value of a variable at any time (current period) as a percentage of the value of that variable at some reference period or base period.

Edge worth gave the definition of index numbers as- “Index Number shows by its variation the changes in a magnitude which is not susceptible of either accurate measurement in itself or of direct variation in practice”.

1.3 Construction of an index number

The methods of index number construction deal with the techniques of combining time series that can’t be added because they are not in comparable units.

E.g. the annual production of wheat can be measured by simply totaling the output of the individual producers or the total amount moving through the markets. Even though there may be different grades of the product, as is generally true for commodities, all types of wheat are nearly enough alike to make the total production a significant amount. In general, items of the same kind, when classified according to differences of kind may be added if all measurements are expressed in the same units.

Many situations arise, however, when it is desired to measure the composite changes in the production of a number of commodities that are not expressed in the same units and that cannot therefore be added in order to secure a total. Changes in the production of manufactured goods cannot be measured by totaling the production of all kinds of goods, since there is no common physical unit of measurement that can be used for all. When the production of the different items cannot be added, it would also be improper to average the prices of the items. Accordingly, when it is desired to measure the change in the average level of prices, it is necessary to use the methods of index number construction.

The basic device used in all methods of index number construction is to average the relative change in either quantities or prices, since relatives are comparable and can be added even though the data from which they are derived cannot themselves be added, Tons of cotton and Tons of wheat cannot properly be added; but if wheat-production was 110 % of the previous year's production and, cotton production was 106% it is valid to average these two percentages and to say that the volume of these commodities produced was 108% of the previous year. This assumes they are of equal importance, since each is given the same weight. But if cotton production is six times as important as wheat production, the percentages should be weighted 6 and 1. The average relative secured by this process is referred to as an index number.

Another distinguishing characteristic, of all index numbers is that they are expressed relative to some specified place, time or period called the base.

When data can be added and the single series reduced to a fixed base series of relatives, the relatives are called index numbers by some statisticians, others reserve the term 'index number' exclusively for an average of relatives derived from series that cannot properly be added. Since many relatives based on a single series are widely used as measures of business conditions, it seems simple to use the term index number to describe both simple relatives and average of relatives.

1.4 Price Relatives

One of the simplest examples of an index number is a price relative, which is the ratio of the price of a single commodity in a given period to its price in another period called the base period or reference period. Here we assume prices to be constant for any one period. If they are not, an appropriate average for the same can be taken to make this assumption valid.

If p_o and p_i denote the commodity prices during the base period and given period respectively, then by definition

P_{oi} = Price relative = p_i/p_o

and is generally expressed as a percentage by multiplying by 100. i.e. price relative = p_i/p_o x 100 More generally if p_i and p_j are prices of a commodity during periods i and j respectively, the price relative in period j with respect to period i is defined as p_j/p_i and is denoted by p_{ij}..

Properties of price relatives

If p_i p_j p_k,… denote prices in periods i, j, k respectively, the following properties exist for the associated price relatives.

1) Identity Property $P_{ii} = 1$

It is also evident as $P_{ii} = p_i/p_i = 1$ or 100%

This states that the price relative for a given period with respect to the same period is 1 or 100%.

2) Time Reversal Property $P_{ij} . P_{ji} = 1$ or $P_{ij} = 1/P_{ji}$

As $P_{ij} . P_{ji} = (p_i/p_j) . (p_j/p_i) = 1$

This states that if two periods are interchanged, the corresponding price relatives are reciprocals of each other. So price relatives follow time reversal property.

3) Cyclical or Circular Property According to this $P_{ij}. P_{jk}. P_{ki} = 1$

As $P_{ij}. P_{jk}. P_{ki} = P_i/P_j. P_j/P_k. P_k/P_i = 1$

Modified Cyclical or Circular Property $P_{ij}. P_{jk} = P_{ik}$

1.5 Quantity or volume relatives

Instead of comparing prices of a commodity, we may be interested in comparing quantities or volumes of the commodity, such as quantity or volume of production, consumption, exports, etc. In such cases we speak of quantity relatives or volume relatives. For simplicity, as in the case of prices, we assume that quantities are constant for any period. If they are not, an appropriate average for the period can be taken to make this assumption valid.

If q_0 denotes the quantity or volume of a commodity produced, consumed, exported, etc., during a base period, while q_i denotes the corresponding quantity produced, consumed, etc., during a given period, we define

Quantity or volume relative = q_i/q_o

which is generally expressed as a percentage.

As in the case of price relatives, we use the notation q_{ij} to denote the quantity relative in period j with respect to period i. The same remarks and properties pertaining to price relatives are applicable to quantity relatives.

1.6 Value relatives

If p is the price of a commodity during a period and q is the quantity or volume produced, sold, etc., during the period, then pq is called the total value. Thus if 1000 items are sold at 30 Rs. each the total value is Rs 30 x 1000 = Rs 30,000.

If p_o and q_o denote the price and quantity of a commodity during a base period while p_i and q_i denote the corresponding price and quantity during a given period, the total values during these periods are given by v_o and v_i respectively and we define

$$\text{Value relative} = v_i/v_o = p_iq_i/p_oq_o = (p_i/p_o)\,(q_i/q_o)$$

$$= \text{price relative x quantity relative}$$

The same remarks, notation and properties pertaining to price and quantity relatives can be applied to value relatives.

In particular if p_{ij} q_{ij} and v_{ij} denote the price, quantity and value relatives of period j with respect to period i then we have

$$v_{ij} = p_{ij}.q_{ij}$$

1.7 Link and chain relatives

It may often be desirable to express an index number not as a percentage of the original base (fixed or reference) but as a percentage of the preceeding period.

Such an index might employ any of the formulae utilizing weights pertaining to either both of the years or months being compared. Frequently these separate percentages are chained back to the original base by a process of successive multiplication. Such an index is known as a chain Index.

Chain base method or chain relative method consists in calculating a series of index numbers for each year with the preceding year as base, viz, P_{01}, P_{12}, P_{23}......... where P_{ij} represents the price index with i as base year and j as given year. These are known as link indexes (relatives). The basic index number is obtained by the successive multiplication of the link indexes (relatives).

P_{o1} = first link

$P_{o2} = P_{o1} \times P_{12}$

$P_{o3} = P_{o1} \times P_{12} \times P_{23} = P_{o2} \times P_{23}$

1.8 Utility of chain indexes

Over a number of years, various changes take place commodities shift considerably in their relative importances. Old commodities disappeared from scene are succeeded by new commodities models, styles. Grades of a commodity become obsolete and cease to be manufactured with new models, styles or grades taking their place; marketing centers shift, so that a price quotation at the new center must replace that at the old. Again, it rarely happens that comparable data are available for long periods; thus, the chain index is an extremely valuable tool.

(1) commodities may readily be dropped, if they are no longer relevant;

(2) new commodities may be introduced and

(3) weights may be changed.

Thus, account may readily be taken of basic changes in production, distribution, and consumption habits with the use of chain indexes.

The disadvantage of the chain index is that, while the percentage of previous year figures gives accurate comparisons of year to year changes, the long range comparisons of the chained percentages are not strictly valid. However, when the index number user wishes to make year to year comparisons, as is so often done by the businessman, the percentages of the preceding year provide a flexible and useful tool.

1.9 Problems involved in computation of Price Index Number

The problems which the statistician encounters in index-number construction are:-

1) **Definition-of the purpose for which the index is being compiled.**
2) **Selection of data for inclusion in index.**
3) **Selection of sources of data.**
4) **Collection of data.**
5) **Selection of base.**
6) **Method of combining data.**
7) **System of weighting.**

Not all of the seven problems listed above are of equal importance, nor are they always independent of one another. Thus, a simple system of weighting would require a different list of commodities for a price index than would a method that employs a separate weighting system for each subgroup of an index. Likewise, the weighting system to be used depends upon the method of combining the data. Let us discuss all in detail.

1) Definition of the purpose for which the index is being compiled

Before gathering data and making calculations, it is important to know what we are trying to measure, and also how we intend to use our measures. An index number properly designed for the purpose in hand is a most useful and powerful tool. If not properly compiled and constructed, it can be a dangerous one. If we wish to know changes in the cost of constructing private dwellings, we should not gather prices of heavy structure steel. Similarly, if we wish to measure the changes in family clothing costs, we should not gather prices of cotton by the bale. To measure the course of retail trade, we should use a sample of department store sales, and not data from wholesalers and factories.

(2) Selection of data for inclusion in index

Although the method of combining the variables is of considerable importance in constructing index numbers, it is insignificant when compared with the problem of selecting the data that are the raw materials of the index. Too much emphasis cannot be put upon this point. The data must be accurate and homogeneous; and the sample should be a good representative of the whole. A sample cannot be expected to be representative unless adequate number of items are included. To state the idea in other language: a sufficiently large sample of relevant items must be selected to obtain reliable index numbers.

As noted before, the commodities to be chosen for a price index, and the type of quotation to be selected, depend on what is being measured. A wholesale price index requires wholesale prices. An index of prices paid by consumers necessitates not only retail prices of food, but rents, gas and electric rates, clothing prices, transportation, medical care, and so forth, applying to the class of persons for whom the cost of living is to be ascertained.

(3) Selection of sources of data

When selecting the sources of data for index numbers, we may rely on regularly published quotations or obtain periodic special reports from the merchants, producers, exporters, or others, who posses the basic information needed. Under either circumstance, we must make sure that the data pertains strictly to the thing being measured. Thus, if retail food price changes are being measured, quotations should be from super markets, chain stores, independent stores and any other important outlets. These different sources should not be mixed indiscriminately, but should be appropriately weighted when combined. Neither should first-of-the-month quotations, middle-of-the-month quotations and end-of-the-month quotations ordinarily be combined in one index.

(4) Collection of data

Most index numbers are based on samples rather than on population data.

This, together with the fact that the proper choice of data is of great importance for the construction of index numbers, requires the following considerations to be borne in mind:

(i) Accuracy: Statistical data that appear in precise printed form are not necessarily accurate. In this connection, it may be remembered that by and large, the primary source data are relatively more accurate than the secondary source data. It is therefore the responsibility of the statistician to ascertain how the data are collected and to select his source with discrimination.

(ii) Comparability over space and time: When data are to be drawn from two or more sources, the reliability of each source must be considered and, in-addition, the user must be sure that the data from the different sources are comparable

(5) Selection of Base

Regardless of the formula employed for weighting and combining the data, it customary (although not necessary) combining to select some period of time as 100 per cent with which compare the other index numbers. A month is ordinarily too short a period to use as base period, since any one month is likely to be unusual on account of accidental or seasonal influences. A year is sometimes used. However, it is often true that no one year is sufficiently "normal" -to be a good basis of comparison. Business and prices are always advancing or receding with the business cycle.

For example comparisons of wage increases with cost of living increases depend heavily on the period selected as the base for the index. It is to be expected that, for a base year in labour management wage negotiations, labour union leaders would select a year when employees were relatively well paid. Representatives of management would select the year before the major wage concession. It is not surprising that both sides would prepare their own indexes and that they would arrive at widely varying conclusions. The selection of the base year can have a significant impact on the sympathy of Government and the general public toward the demands or positions of the two parties to the negotiations.

For these various reasons, the base periods for most indexes have been changed from time to time. As a general rule, and with notable exceptions, the base is no more than about ten to fifteen years prior to the current date. It is quite common for the base to be a single year, but many indexes have a three to five year period average as their base as an attempt to get a more normal period. Major changes in an index, such as the updating of the base, are accompanied by a recalculation of old index numbers to express them relative to the new base, and often by changes in the commodities and the quantity weights. Index numbers covering a period of many years are, at best only reasonably good estimates of relative prices.

(6) Method of combining Data

There are two methods of constructing index numbers:

(1) by computing aggregate values (2) by averaging relatives. Each of these basic procedures may be further described as simple (un weighted) or specifically weighted. Although only the specifically weighted procedures are logically defensible, the simple unweighted procedures are usually explained-because they are sometimes used, and their development leads naturally to the problems and effects of various weighting systems.

The aggregative method obtains the result directly, and produces a result that has a simple and clear meaning; the method employing relatives is more roundabout, and its meaning is more technical. Nevertheless, there are situations in which the aggregative method is not applicable, and recourse must then be had to the averaging of relatives.

(7) Systems of Weighting

In selecting the weights for an index, attention must be paid to the use that will be made of the result. If a price index is to measure the changes in the prices paid by consumers, it is necessary for the weights applied to the various commodities to reflect the importance of the individual items to consumers. Likewise, the weights for an index of farm prices should reflect the importance of the various commodities in the income of farmers. Frequently the commodities included in an index and the weights assigned to items do not give the information wanted for the solution of a particular problem. If weighted price indices are to be constructed, and if the quantity of each commodity marketed changed from year to year in the same proportion, it would make no difference to what period the weights referred, for the results would be identical. In fact, however, the relative importance of the different commodities is constantly changing, and this is due in part to the change in the relative prices of the different commodities, which in turn result from changes in supply and demand. Therein lies a great source of difficulty for which there is no completely satisfactory solution. The answer depends in part on, what the analyst thinks a price index is supposed to do.

One view is that such an index number measures the changing cost of a constant aggregate of goods. Another view concerns itself not with the goods level of analysis, but with the satisfactions level; an index number, according to this view, should measure the changing cost of aggregates of goods yielding the same utility or satisfaction at two periods, or two places. Thus suppose we compare the cost of living of two groups of similar persons at two periods (or places), these groups having at the two periods (or places) the same tastes and capacity for enjoyment, as well as an income that will p00.3.u rchase, and does purchase for the same amount of satisfaction. The commodities, of course, will be different but if the expenditures were Rs. 500 the first year and Rs.550 the second year, we may conclude that the cost of living has gone up by 10 percent. It goes without saying that no one has accurately made a measurement of this kind. Although it seems feasible to measure only the varying value of a fixed, aggregate of goods, yet the analyst should select a list of gods that will avoid the certainty of bias in a known direction with respect to the cost of obtaining equal satisfactions at different times.

As described earlier index numbers (or indices) are those measure, which indicate about relative changes in the value of a variable over a period of time, the most popular index number being the price index number. From the initial discussion, it is apparent that the index number is the ratio of two quantities with reference to two time periods. In index number is obtained for any given time period generally termed as current period in comparison with some references period known as base period. For example if one has to speak about price index for cotton prices in year 1995 with reference to1980 as reference year 1980 will be termed as base year while 1995 will be known as current year.

Notatation:

Before the discussion of constriction of index number, let us explain the notation and terminology used.

I_{oj} = Index number for the current year j with references to base year o

I_{o1} = Index number for the current year 1 with references to base year o

P_{o1}=Price Index number for the current period 1 as compared to base year

p_{ij} = Price of jth commodity in ith year

p_{oj} = Price of jth commodity in base year ‘O’

p_{1j} = Price of jth commodity in current year ‘1’.

q_{ij} = Quantity consumed of jth commodity in ith year.

q_{oj} = Quantity consumed of jth commodity in ‘o’ base year

q_{1j} = Quantity consumed of jth commodity in current year ‘1’.

(8) Calculation of Index Numbers

Broadly, the calculation of price index can be divided into two subgroups, namely

(a) Simple (unweighted) Aggregate method.

(b) Weighted Aggregates method.

Now, let us discuss both of these, one by one.

(a) Simple Aggregate method: This method consists of expressing aggregate of prices in any year as a percentage of their aggregate in base year.

Thus price index for the ith year as compare to base year (‘o’) in given as

$$P_{oi} = \left(\frac{\sum p_{ij}}{\sum p_{oj}} \right) \times 100$$

And

Quantity index –

$$Q_{01} = \left(\frac{\sum q_{ij}}{\sum q_{oj}} \right) \times 100$$

This formula is very simple for the purpose of calculation and so provides a quick measure of Index number when one has to obtain Index number for similar type of articles, e.g. crop items like wheat. rice, bajra, gram etc. But if prices of commodities under study have different units i.e. per kg, per meter, per ton, this formula is not of any use as different units cannot be summed us directly.

Thus merits and demerits of this formula are –

Merit: It is an easy and quick method and so come in handy for a quick overview of the situation.

Demerits: 1. It does not take into account the fact that articles or commodities whose prices are to be added have different importance. Thus prices of commodities which are not much important will affect the index number.

2. As said earlier, this formula cannot be used when commodities involved have different units for prices or for quantities.

During to these demerits, this formula is mot of much use for practical purposes.

(b) Weighted Aggregate method: In this method appropriate weights are assigned to different commodities to make them comparable and thus compatible for summation. Thus, a weighted index is –

$$P_{oi} = \frac{\sum p_{ij} w_j}{\sum p_{ij} w_j} \times 100$$

and

$$Q_{oi} = \frac{\sum q_{ij} w_j}{\sum q_{ij} w_j} \times 100$$

Where w_j is the weight assigned to jth commodity.

Thus in this method the commodities of higher importance are given higher weight and vice versa. In this way, each commodity selected for obtaining the index number influence it according to its weight (i.e. importance, literally). Or in other words, alottement of weights enables the commodities of greater importance to have more impact on index number.

Now, time to time various renowned statisticians and research workers have suggested different methods for allotment of weight (w_j) to the commodities. So a person interested in obtaining an index number has a large number of choices to select from. Some important formulae, which are treated as standard ones are given below.

(9) Important Formulae

Various methods have been suggested by different workers time to time. These methods give different formula on the basis of choice of different weights. Though the basic character of an index number does not changes and it gives the relative change yet choice of weight changes the utility and purpose of index number and vice versa.

(A). Laspeyre's Price Index or Base Year Method

French economist Laspeyere in 1871, suggested that quantities of commodities consumed in base year can be taken as weights for the purpose of calculating index numbers.

That is

$$P_{oi} = \left(\frac{\Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{ij}}\right) \times 100$$

Where notation bear their usual meaning. The upper suffix 'La' is added to distinguish this formula as 'Laspeyere's' Index number.

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(B). Paasche's Price Index or Given Year Method

By taking year quantities as weights, we get Paasche's formula. This formula is suggested by German statistician Paasche in 1874 and so it is named after him and while writing this formula, as in earlier case, we add upper suffic 'Pa'. Thus

$$P_{oi}^{Pa} = \left(\frac{\Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{ij}}\right) \times 100$$

(C). Marshall – Edgeworth Price Index number

The statistician Marshall and Edgeworth suggested that to obtain a better index number we should use simple average of base year and current year quantities as the weights.

i. e. $$w_{ij} = \frac{\left(q_{oj} + q_{ij}\right)}{2}$$

Hence, Marshall Edgeworth Price index no. is-

$$P_{oi}^{ME} = \frac{\Sigma p_{ij} \frac{\left(q_{oj} + q_{ij}\right)}{2}}{\Sigma p_{oj} \frac{\left(q_{oj} + q_{ij}\right)}{2}} \times 100$$

$$= \frac{\Sigma p_{ij} \frac{\left(q_{oj} + q_{ij}\right)}{2}}{\Sigma p_{oj} \frac{\left(q_{oj} + q_{ij}\right)}{2}} \times 100$$

(D). Irving Fisher's Index Number

Fisher's index number is the geometric mean of the Laaspeyere's and Paasche's formula.Mathematically –

$$P_{oi}^{F} = \left[P_{oi}^{La} \times P_{oi}^{Pa}\right]^{1/2}$$

$$= \left[\frac{\Sigma p_{ij} q_{oj}}{\Sigma p_{oj} q_{oj}} \times \frac{\Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{ij}}\right]^{1/2} \times 100$$

This formula holds the view that neither base year nor current year quantities are fully appropriate to use as weights, so a geometric mean of Laspeyere's and Paasche's formula can be more appropriately used as weight for obtain index number. We will further observe that this formula satisfies several tests and so it is taken as the best of all formulae suggested above. In fact, it is known as "**Fisher's Ideal Index Number".**

There are several other index numbers also, given my other research workers. Some known names are Dorbish – Bowley's index number and Walsch's index number. But only four formulae mentioned above are most commonly used in general Practice and so these are established standard formulae for calculating Price Indices.

10. Other Indices

On the basis of requirement of economic workers various formulae can be developed .For example we can have an index of global warming ,an index for depletion of ozone layer ,an index for change in literacy rates in different parts of the country etc. The thing is that the sky is the limit when we go for applications of index numbers .On the basis of different roles these indices play and on the basis of different factors these are based on these indices are known by different names.Some of the important formulae are listed below.

11. Quantity Index Numbers:

In the above formula we concentrated ourselves on price index numbers. By interchanging the price (p_{ij}) and quantities (q_{ij}) in the above formula we get the corresponding formulae for the calculation of quantity index numbers which reflect the change in the volume of quantity or production.

$$Q_{oi}^{Ea} = \left(\frac{\Sigma q_{ij} p_{oj}}{\Sigma q_{oj} p_{oj}} \right) \times 100$$

$$Q_{oi}^{Pa} = \left(\frac{\Sigma q_{ij} p_{ij}}{\Sigma q_{oj} p_{ij}} \right) \times 100$$

$$Q_{oi}^{ME} = \frac{\Sigma q_{ij} \left(p_{ij} + p_{oj} \right)}{\Sigma q_{oj} \left(p_{ij} + p_{oj} \right)} \times 100$$

Quantity Index Numbers study the changes in the volume of goods produced (manufactured), consumed or distributed, like the indices of agriculture production, industrial production, imports and exports etc.. They are extremely helpful in studying the level of physical output in an economy.

12. Value Index Numbers:

Value index numbers are given by the aggregate expenditure for any given year expressed as a percentage of the same in the base year.

Thus

$$V_{oi} = \left(\frac{\Sigma q_{ij} p_{ij}}{\Sigma q_{oj} p_{ij}} \right) \times 100$$

However these indices are not as common as price and quantity indices.

13. The Criteria of a Good Index Number

A number of mathematical test discussed below have been suggested for comparing various index numbers.

1) Unit Test: This test requires the index numbers to be independent of the units in which prices and quantities are quoted. This test is satisfied by all the formulae 1-4.

2) Time Reversal Test: This is one of the two very important test proposed by Irving Fisher as tests of consistency for a good index number. According to this ‘the formula for calculating an index number should be such that it will give the same ratio between one point of comparison and the other no matter which of the two is taken as base.

In notations we have –

$$P_{ij} \times P_{ji} = 1$$

Or $$P_{ij} = \frac{1}{P_{ji}}$$

For example if we take the Laspeyre’s formula

$$P_{oi}^{La} = \left(\frac{\Sigma p_{ij} q_{oj}}{\Sigma p_{oj} q_{oj}} \right) \times 100$$

$$P_{oi}^{La} = \left(\frac{\Sigma p_{oj} q_{ij}}{\Sigma p_{ij} q_{oj}} \right) \times 100$$

$$P_{oi}^{La} \times P_{oi}^{La} = \frac{\Sigma p_{ij} q_{oj}}{\Sigma p_{oj} q_{oj}} \times \frac{\Sigma p_{oj} q_{ij}}{\Sigma p_{ij} q_{oj}} \neq 1$$

Hence Laspeyre’s formula does not satisfy Time Reversal Test. Similarly it can be seen that Paasche’s formula also does not satisfy this test.

For the Fisher’s ideal formula.

$$P_{oi}^{F} = \left[\frac{\Sigma p_{ij} q_{oj}}{\Sigma p_{oj} q_{oj}} \times \frac{\Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{ij}} \right]^{1/2}$$

$$P_{io}^{F} = \left[\frac{\Sigma p_{oj} q_{ij}}{\Sigma p_{ij} q_{ij}} \times \frac{\Sigma p_{oj} q_{oj}}{\Sigma p_{ij} q_{oj}} \right]^{1/2}$$

$$P_{oi}^{F} \times P_{io}^{F} = 1$$

Hence Fisher’s ideal index satisfies Time Reversal Test.

Factor Reversal Test

This is the second test of consistency suggested by I. Fisher. In his words: "Just as our formula should permit the interchange of two items without giving inconsistent results, so it ought to permit interchanging the prices and quantities without giving inconsistent results – i.e. the two results multiplied together should give the true value ratio, except for a constant of proportionality".

Symbolically we should have

$$P_{oi} \times Q_{oi} = \frac{\Sigma v_{ij}}{\Sigma v_{oj}} = \frac{\Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{oj}}$$

For example,

$$P_{oi}^{F} = \left[\frac{\Sigma p_{ij} q_{oj}}{\Sigma p_{oj} q_{oj}} \times \frac{\Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{ij}} \right]^{1/2}$$

and

$$Q_{oi}^{F} = \left[\frac{\Sigma q_{ij} p_{oj}}{\Sigma q_{oj} p_{oj}} \times \frac{\Sigma q_{ij} p_{ij}}{\Sigma q_{ij} p_{ij}} \right]^{1/2}$$

$$P_{oi}^{F} \times Q_{oi}^{F} = \frac{\Sigma p_{ij} q_{ij}}{\Sigma p_{oj} q_{oj}}$$

Hence Fisher's ideal index satisfies Factor Reversal Test. It may be pointed out that none of the other formulae satisfies the factor reversal test.

So we see that Fisher's index satisfies both time reversal and factor reversal test. Hence it is termed as 'Ideal Index Number'.

Simple aggregate method of obtaining index number consists in expressing the aggregate of prices in any year (under consideration) as a percentage of their aggregate in base year. But in this method relative importance of the various commodities is neglected and if the group of articles carries different units we cannot use this method.

In Laspeyre's index number base year quantities are used as weight and in Paasche's index number current year quantities are used as weights.

Fisher's index number is the geometric mean of the Laspeyre's and Paasche's index number. It is also termed as ideal index number as it satisfies both time reversal and factor reversal tests.

14. Consumer Price Index

As we have studied earlier, price index number is the measure of relative changes in the prices of some commodity over a period of time. In practice these indices are used for various different purposes. One very important use of the theory of index number is in obtaining consumer price index or alternatively also called cost of living index number.

It is a well known fact that the prices of the commodities required for day to day living go on increasing, e.g. prices of food items like wheat, rice, oil etc. are different in different years. This increase (or decrease, if there is any) in prices of commodities directly hit the purchasing power of consumer. A consumer price index is, therefore, devised to measure the over all changes in the purchasing power of the consumer.

A **consumer price index** or cost of living index, is a measure which indicates the relative changes in the prices of a group of items, necessary for the living for a selected group of consumers. In a way, it tells us about what should be the increase in the wages of consumer so that they are able to maintain some standard of living in two time periods. For this purpose, the total expenditure of a household are categorized like food, clothing, rent, electricity, entertainment, education, medicines, miscellaneous etc.

Consumer Price Index number or Cost of living index number measures the effect of changes in the prices of the described basket of goods and services on the purchasing power of a particular class of people during current period as compared with some base period. Change in the cost of living of an individual between two periods means the change in his money income, which will be necessary for him to maintain the same standard of living in both periods. There the cost of living index numbers are intended to measure the average increase in the cost of maintaining the same standard in a given year as in the base year.

15. Construction of Consumer Price Index Number

Thus the consumer price index numbers, also known as cost of living index numbers are generally intended to represent average change over time in the prices paid by the ultimate consumer of a specified group, goods and services. The need for constructing consumer price indices arises because the general index numbers fail to give an exact idea of effect of the change in the general price level on the cost of living of different classes of people, since a given change in the level of prices aces affects different classes of people in different manners. Different classes of people consume different types of commodities and even the same types of commodities are not consumed in the same proportion by different classes of people. For example, the consumption pattern of rich, poor and middle class people varies widely. Not only this, the consumption habits of the people of the same class differ from place to place. For example, the mode of expenses true of a lower division clerk living in Delhi, may differ widely from that of another clerk of the same category living in, say, Mumbai. The consumer price index helps us in determining the effect of rise and fall in prices on different classes of consumers living in different areas, The construction of such an index is of great significance because very often the demand for a higher wage is based on the cost of living index and the wages and salaries in most countries are adjusted in accordance with the consumer price index.

It should be carefully noted that the cost of living index does not measure the actual cost of living nor the fluctuations in the cost of living due to causes other than the change in the price level; its object is to find out how much the consumers of a particular class have to pay more for a certain basket full of goods and services in a given period compared to the base period. To bring out clearly this fact, the Sixth International Conference of Labor Statisticians recommended that the term 'cost of living index' should be replaced in appropriate circumstances by the terms 'price of living index', 'cost of living price index' or 'consumer price index'. At present, the three terms, namely, cost of living index, consumer price index and retail price index are in use in different countries with practically no difference in their connotation.

It should be clearly understood at the very outset that two different indices representing two different geographical areas cannot be used to compare actual living costs of the two areas. A higher index for one area than for another with the same period is no indication that living costs are higher in the one than in the other. All it means is that as compared with the base periods, prices have raised more in one area than in another. But actual costs depend not only on the rise in prices as compared with the base period, but also on the actual cost of living for the base period which will vary for different regions and for different classes of population.

16. Computation of Consumer Price Index No.

For computing consumer Price Index no. or cost of living index no. we need to have prices and quantity consumed for different categories of items in base a year and current year.

Let p_{oj} = Price of jth commodity in 0 year (base year)

p_{ij} = Price of jth commodity in I year (current year).

q_{oj} = Quantity consumed for j th commodity in 0 year (base year)

Now, consumer Price Index no. is derived as the weighted average of the price relatives, the weight being the values of the quantities consumed in the base year.

Price relative

$$P_j = \frac{p_{ij}}{p_{oj}} \times 100$$

And Weight

$$W_j = p_{oj} . q_{oi}$$

Then,

Consumer Price Index

$$= \sum_j \frac{W_j P_j}{\sum_j W_j}$$

Construct the cost of living index for the year 1982 (Base 1980 = 100).

Item	Unit	Price (1981)	Price (1983)	Weight
A	kg./	0.50	0.75	10%
B	Litre	0.60	0.75	25%
C	Dozen	2.00	2.40	20%
D	kg.	0.80	1.00	40%
E	One pair	8.00	10.00	5%

The consumer price index is obtained by the method of weighted price relatives.

COMPUTATION OF COST OF LIVING INDEX NUMBER

Item	Price in Rupees		Price Relatives (base 1980)	Weight W	PW
	1981 (P_o)	1983 (P_1)	$P = 100 \times \frac{P_1}{P_o}$		
A	0.50	0.75	$100 \times \frac{0.75}{0.50} = 150$	10	1,500
B	0.60	0.75	$100 \times \frac{0.75}{0.60} = 125$	25	3,125
C	2.00	2.40	$100 \times \frac{2.40}{2.00} = 120$	20	2,400
D	0.80	1.00	$100 \times \frac{1.00}{0.80} = 125$	40	5,000
E	8.00	10.00	$100 \times \frac{10.00}{8.00} = 125$	5	625
Total				100	12,650

$$\text{Cost of Living Index} = \frac{\sum PW}{\sum W} = \frac{12{,}650}{100} = 126.5$$

17. Steps in construction of Consume Price Index.

The main steps which are required for construction of CPI are described below.

(l) Decision about the class of people for whom the index is meant. It is absolutely essential to decide clearly the class of people for whom the index is meant i.e. whether it relates to industrial workers, teachers, officers, etc. The scope of the index must be clearly defined. For example, when we talk to teachers, we are referring to primary teachers, middle class teachers, etc. or to all the teachers taken together. Along with the class of people it is also necessary to decide the geographical area covered by the index.

(2) Conducting, family budget enquiry. Once the scope of the index is clearly defined the next step is to conduct a family budget enquiry covering the population group for whom the index is to be designed. The object of conducting a family budget enquiry is to determine the-amount that an average family of the group, included in the index spends on different items of consumption. While conducting such an enquiry, the quantities of commodities consumed and their prices are taken into account. The consumption pattern can thus be easily ascertained. It is necessary that the family budget enquiry amongst the class of people to whom the index series is applicable should be conducted during the base period. The Sixth International Conference of Labour Statisticians held in Geneva suggested that the period of enquiry of the family budgets and the base periods should be identical as far as possible.

The enquiry is conducted on a random basis. By applying lottery method some families are selected from the total number and their family budgets are scrutinized in detail.

(3) Deciding on the items. The items on which the money is spent are classified into certain well–accepted groups. One of the choicest and most frequently used classification is –

(i) Food

(ii) Clothing

(iii) Fuel and lighting

(iv) House Rent

(v) Miscellaneous

Each of these groups is further divided into sub–groups. For example, the broad group 'food' may be divided into wheat, rice, pulses, sugar etc. The commodities included are those which are generally consumed by people for whom the index is meant. Through family budget enquiry an average budget is prepared which is the standard budget for that class of people. While constructing the index only such commodities should be included as are not subject to wide variations in quality or to wide seasonal alterations in supply and for which regular and comparable quotations of prices can be obtained.

(4) Obtaining price quotations. The collection of retail prices is very important and at the same time, very tedious and difficult task also. That is because such prices may vary from place to place, shop to shop and person to person. Price quotations should be obtained from the localities in which the class of people concerned reside or from where they usually make their purchases. Some of the principles recommended to be observed in the collection of retail price data required for purposes of construction of cost of living indices are described below:

(a) The retail prices should relate to a fixed list of items and for each item., the quality should be fixed by means of suitable specifications.

(b) Retail prices should be those actually charged to consumers for cash sales.

(c) Discount should be taken into account if it is automatically given to all customers.

(d) In a period of price control or rationing, where illegal prices are charged openly, such prices should be taken into account along with the controlled prices.

The most difficult problem in practice is to follow principle (a) i.e., the problem of keeping the weights assigned and qualities of the basket of goods and services constant with a view to ensuring that only the effect of price change is measured. To confirm to uniform qualities, the accepted method is to draw up detailed descriptions or specifications of the items priced for the use of persons furnishing or collecting the price quotations.

Since prices form the most important component of cost of living indices, considerable attention has to be paid to the methods of price collection and to the price collection personnel. Prices are collected usually by special agents or through mailed questionnaire or in some cases through published price lists. The greatest reliance can be placed on the price collection through special agents as they visit the selected retail out-lots and collect the prices from them. However, these agents should be properly selected and trained and should be given a manual of instructions as well as manual of specifications of items to be priced. Appropriate methods of price verification should be followed such as 'check pricing' in which price quotations are verified by means of duplicate prices obtained by different agents or purchase checking in. which actual purchases of goods are made.

(5) Working on CPI. After quotations have been collected from all retail outlets an average price for each of the items included in the index has to be worked out. Such averages are first calculated for the base period of the index and later every month if the index is maintained on a monthly basis. The method of averaging the quotations should be such as-to yield unbiased estimates of average prices as being paid by the group as a whole. This of course, will depend upon the method of selection of retail outlets and also the scope of the index.

In order to convert the prices into index numbers the prices or their relatives must be weighted. The need for weighting arises because the relative importance of various items for different classes of people is not the same. For this reason, the cost of living index is always a weighted index. While conducting the family budget enquiry the amount spent on each commodity by an average family is decided and these constitute the weights. Percentages of expenditure on the different items constitute the individual weights, allocated to the corresponding price relative and the percentage expenditure on the five groups constitute the group weight.

18. Uses and limitations of consumer price index

(1) These indices are compiled for different groups or classes of people (such as low income, middle income, clerical, labor class, etc.) and are useful to assess the general price movement of the commodities consumed by them.

(2) Cost of living index numbers indicate whether the real wages are

raising or falling, money wages remaining unchanged. In other words they are used for the calculation of real wages and for determining the change in the purchasing power of the money.

(3) Cost of living indices are used for the regulation of dearness allowance or the grant of bonus to the workers so as to enable them to meet the increased cost of living.

(4) These indices are also used for deflation of income and value series in national accounts.

(5) By itself, cost of living index number does not throw much light on the inflationary or deflationary trend on the soundness of an economy but in conjunction with other tools such as the indices of wholesale prices, wages, profits, production, employment etc., it serves as an economic indicator for the analysis of price situation.

19. Base Shifting of Index Numbers.

Base shifting means the changing of the given base period (year) of a series of index numbers and recasting them into a new series based on some recent new base period. This step is quite often necessary under the following situations :

(i) When the base year is too old or too distant from the current period to make meaningful and valid comparisons.

(ii) If we want to compare series of index numbers with different base periods, to make quick and valid comparisons both the series must be expressed with a common base period.

Base shifting requires the recompilation of the entire series of the index numbers with the new base. However, this is a very difficult and time consuming job. A relatively much simple, though approximate method consists in taking the index number of the new base year as 100 and then expressing the given series of index numbers as a percentage of the index number of the time period selected as the new base year. Thus, the series of index numbers, recast with a new base is obtained by the formula :

Recast Index No. of any year

$$= \frac{\text{Old Index No. of the year}}{\text{Index No. of new base year}} \times 100$$

$$= \left[\frac{100}{\text{Index No. of new base year}} \right] \times \text{Old Index Number of the year}$$

In other words, the new series of index numbers is obtained on multiplying the old index numbers with a common factor :

$$= \frac{100}{\text{Index No. of New Base Year}}$$

If arithmetic mean or median is used for averaging the price relatives then the usual method of base shifting consists in calculating the price relatives for each individual item w.r.t. the new base and then averaging their totals, i.e., the whole of the series is to be reconstructed. However, in practice, even in these cases the approximate method described above gives result which are fairly close to those obtained otherwise.

20. Splicing Two Index Numbers Series.

In order to obtain continuity in the comparison of two or more overlapping series of index numbers, we combine or splice them into a single continuous series. For example, suppose an index number series 'A' with base period 'a' is discontinued in 'period' 'b' due to certain reasons and a new series 'B' of index numbers is computed with base period 'b' (and the same items). In order to compare the series 'B' with 'A' we splice the series B to A to obtain a continuous series from 'a' onwards. The process is very much alike to that of base shifting and is illustrated below :

SPLICING OF TWO INDEX NUMBER SERIES

Year	Series I Base 'a'	Series II Base 'b'	Series II (Base 'a') spliced to series I	Series I spliced to series II (Base 'b')
a	100		100	$\frac{100}{a_k} \times 100$
a+1	a_1		a_1	$\frac{100}{a_k} \times a_1$
a+2	a_2		a_2	$\frac{100}{a_k} \times a_2$
⋮	⋮		⋮	⋮
b–1	a_{k-1}		a_{k-1}	$\frac{100}{a_k} \times a_{k-1}$
b	a_k	100	a_k	100
b+1		b_1	$\frac{a_k}{100} \times b_1$	b_1
b+2		b_2	$\frac{a_k}{100} \times b_2$	b_2
b+3		b_3	$\frac{a_k}{100} \times b_3$	b_3
⋮		⋮	⋮	⋮

Explanation. When series II is spliced to series I to get a continuous series with base 'a'.

100 of II series becomes a_k

=> b_1 of II series becomes $\frac{a_k}{100} \times b_1$,

and b2 of II series becomes $\frac{a_k}{100} \times b_2$, and so on. Thus multiplying each index of the series II with a constant factor $\frac{a_k}{100}$, we get the new series of index numbers spliced to series I (Base 'a'). In this case series I is also said to be spliced forward.

If we splice series I to series II to get a new continuous series with base 'b' then,

a_k of first series becomes 100

$$a_{k-1} \text{ of first sereis becomes } \frac{100}{a_k} \times a_{k-1},$$

=>

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$a_2 \text{ of first series becomes } \frac{100}{a_k} \times a_2, \text{ and so on.}$$

Thus the new series of index numbers with series I spliced to series II (Base 'b') obtained on multiplying each index of series I by new constant factor ($100/a_k$). In this case we say that series is spliced backward.

Given below are two price index series. Splice them on the base 1974=100. By what per cent did the price of steel rise between 1970 and 1975?

Year	**Old price index for Steel Base (1965=100)**	**New price index for Steel Base (1974=100)**
1970	141.5	
1971	163.7	
1972	158.2	
1973	156.8	99.8
1974	157.1	100.0
1975		102.3

SPLICING OF OLD PRICE INDEX TO NEW PRICE INDEX

Year	**Old price index for Steel Base (1965=100)**	**New price index for Steel Base (1974=100)**
1970	141.5	$\frac{100}{157.1} \times 141.5 = 99.06$
1971	163.7	$\frac{100}{157.1} \times 163.7 = 104.19$
1972	158.2	$\frac{100}{157.1} \times 158.2 = 100.69$
1973	156.8	$\frac{100}{157.1} \times 156.8 = 99.80$
1974	157.1	100.0
1975		102.3

The percentage increase in the price of steel between 1970 and 1975 is

$$\frac{102.30 - 90.06}{90.06} \times 100 = 0.1359 \times 100 = 13.59$$

Hence, the required increase is 13.59%.

21. **Deflating the Index Numbers**

Deflating means "making allowance for the effect of changing price levels". The increase in the prices of consumer goods for a class of people over a period of years means a reduction in the purchasing power for the class. For example the increase in price of a particular commodity from Rs. x in base year 'a' to Rs. 2x in the year 'b' implies that in 'b' a person can buy only half the amount of the commodity with Rs. x which he was spending on it in 'a'. Thus the purchasing power of a rupee in only 50 paise in 'b' as compared to 'a'.

The idea of "the purchasing power of money" or "a measure of the real income" for a class of people is obtained on deflating the wage series by dividing each item by an approximate price index e.g., the cost of living index. The real wages so obtained my be converted into index number if desirable. More precisely,

$$\text{Real wage} = \frac{\text{Money or Nominal Wages}}{\text{Price Index}} \times 100$$

The real income is also known as deflated income. This technique is extensively used to deflate value series or value indices, rupee sales, inventories, income wages and so on.

22. Index of Industrial Production.

The index of industrial production is aimed at reflecting changes (increase or decrease) in the volume of industrial production (i.e., production of non-agricultural commodities) in a given period compared to some base period. These indices measure, at regular intervals, the general movement in the quantum of industrial production. Such indices are useful for studying:

(i) The progress of general industrialization of a country, and

(ii) The effect of tariff on the development of particular industries.

These indices of industrial activity are of great importance in the formulation and implementation of industrial plans. For the construction of the indices of industrial production, the data about production of various industries are usually collected under the following heads:

(i) Textile Industries: Cotton, silk, woolen, etc.

(ii) Metallurgical Industries: Iron and steel, etc.

(iii) Mining Industries: Coal, pig-iron and Ferro-alloys, petrol, kerosene, copper (virgin metal), etc.

(iv) Mechanical Industries: Locomotives, sewing machines, aero-planes, etc.

(v) Industries subject to excise duty: Tea, sugar, cigarettes and tobacco, distilleries and breweries, etc.

(vi) Electricity, gas and steam; Electric lamps, electric fans, electrical apparatus and appliances, etc.

(vii) Miscellaneous: glass, paints and varnish, paper and paper-board, cement chemicals etc.

Usually, the data (figures of output) are obtained for various industries on monthly basis and the indices of industrial production are obtained as the weighted arithmetic mean (or sometimes geometric mean) of the production (quantity) relatives by the formula:

$$I_{oj} = \frac{\sum Q_j W_j}{\sum W_j}$$

Where Q_j = production relative = q_{ij}/q_{oj},

And W_j is the weight assigned to jth them (industry).

The weights may be assigned to various industries on the basis of, say, capital invested, net output, production etc. The concept of 'value added by manufacture' is the most commonly used criterion for determining the weights to be assigned to different industries.