Bohr's Correspondence Principle

Dr. R.K. Shukla
Dept. of Physics
University of Lucknow

Bohr's Correspondence Principle :- This principle connects classical theory and quantum theory.

In the limiting situation, the results from classical theory as well as from quantum theory must agree i.e.

$$\lim_{n \to \infty} \text{quantum physics} = \text{classical physics}.$$

where n is the principal quantum number.

The dynamics of a system, as described by quantum theory must agree with the classical description of that system in the limit of very large quantum numbers.

For example, according to Bohr's model the quantum condition for emission of radiation is

$$h\nu = E_n - E_p$$

and Maxwell's classical theory says that an electron revolving with orbital frequency must radiate light waves of frequency

calculation show that for quantum number 'n' as large as 10,000, the difference in ν is less than 0.015%.

Thus Bohr's Correspondence principle is established.

Proof :- we know that when any electron is moving in a circular orbit then, it take 'T' time as given by

$$T = \frac{2\pi r}{v}$$

where r is the radius of the orbit
v is the velocity of the electron

we know $\nu = \frac{1}{T}$

Hence $\nu = \frac{1}{T} = \frac{v}{2\pi r}$ —(1)

Now in case of hydrogen atom

$mvr = n\frac{h}{2\pi}$; (Bohr postulates)

$v = n\frac{h}{2\pi m r}$ —(3)

from Bohr's atomic model

Coulomb force = centripetal force

$\frac{1}{4\pi\epsilon_0}\frac{e\,e}{r^2} = \frac{mv^2}{r}$

$v^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$ —(2)

using Eq^n (2) and (3) we get

$\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = \frac{n^2h^2}{4\pi^2m^2r^2}$

$\Rightarrow \boxed{r = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}}$ —(4)

for Hydrogen $Z=1$

$\boxed{r = \frac{\epsilon_0 n^2 h^2}{\pi m e^2}}$ —(5)

using (3) & (4), we get

$v = \frac{nh}{2\pi m}\cdot\frac{\pi m Z e^2}{\epsilon_0 n^2 h^2}$

for Hydrogen $Z=1$

$\boxed{v = \frac{e^2}{2\epsilon_0 n h}}$ —(6)

Substituting Eq^n (5) & (6) in Eq(1), we get

$\nu = \frac{1}{2\pi}\cdot\frac{e^2}{2\epsilon_0 n h}\cdot\frac{\pi m e^2}{\epsilon_0 n^2 h^2}$

$\boxed{\nu = \frac{me^4}{4\epsilon_0^2 n^3 h^3}}$ —(7)

This frequency of radiation is obtained from Classical physics.

Frequency of radiation emitted by electron is calculated by using Quantum mechanics.

The energy of n^{th} orbit is given by

$$E_n = -\frac{mZ^2e^4}{8\epsilon_0^2 n^2 h^2}$$

In Case of hydrogen atom $Z=1$

$$\therefore \quad E_n = -\frac{mZ^2e^4}{8\epsilon_0^2 n^2 h^2}$$

Wavelength of radiation corresponding to the transition of electron from n^{th} orbit to the p^{th} orbit is given by

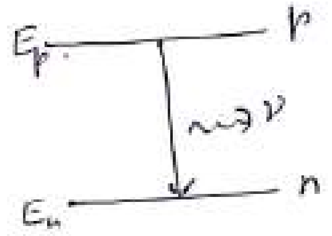
$$\Delta E = h\nu = E_n - E_p$$

or

$$\nu = \frac{1}{h}\left[E_n - E_p\right]$$

$$= \frac{1}{h}\left[-\frac{mZ^2e^4}{8\epsilon_0^2 n^2 h^2} + \frac{mZ^2e^4}{8\epsilon_0^2 p^2 h^2}\right] \quad \text{for } H_2;\ Z=1$$

$$= \frac{me^4}{8\epsilon_0^2 h^3}\left[\frac{1}{p^2} - \frac{1}{n^2}\right]$$



p is taken as $p = n+1$ in above eqn

$$\nu = \frac{me^4}{8\epsilon_0^2 h^3}\left[\frac{1}{(n+1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{me^4}{8\epsilon_0^2 h^3}\left[\frac{n^2-(n+1)^2}{n^2(n+1)^2}\right]$$

$$= \frac{me^4}{8\epsilon_0^2 h^3}\left[\frac{n^2-n^2-1-2n}{n^2(n+1)^2}\right]$$

$$= -\frac{me^4}{8\epsilon_0^2 h^3}\left[\frac{(2n+1)}{n^2(n+1)^2}\right]$$

If n is very large, then $n >> 1$; $2n+1 \simeq 2n$, $n+1 \simeq n$

$$\therefore \quad \nu = \frac{me^4}{8\epsilon_0^2 h^3}\,\frac{2n}{n^2 \cdot n^2} = \frac{me^4}{8\epsilon_0^2 h^3}\,\frac{2}{n^3}$$

$$\boxed{\nu = \frac{me^4}{4\epsilon_0^2 n^3 h^3}} \quad \text{—— (8)}$$

This frequency of radiation is obtained by.

Hence Eq^n (7) & (8) are identical for large value of 'n'.

$\therefore$ Bohr's correspondence principle is verified.

When 'n' is large, the frequency of revolution of an electron in its orbit is equal to the frequency of the radiation emitted due to the transition in neighbouring states. Thus in the limit characterised by large quantum numbers, the classical physics and the quantum physics lead to the same predictions.
This is known as the Bohr's correspondence principle.
The correspondence principle has proved to be of great value in the computation of the intensity of the spectral radiation and in the formulation of selection rules. The selection rules put a limit on the large number of possible transitions among the different energy levels in the atom and restrict the number of spectral lines emitted by the atom.