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Subject – Network Analysis
Topic – Network Function and Synthesis
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Definition:

A type of frequency that depends on two parameters, one is the " σ" which controls the magnitude of the signal and the other is "w", which controls the rotation of the signal ; is known as "complex frequency".

A complex exponential signal is a signal of type

$$X(t) = X_m e^{St} \longrightarrow (1)$$

where X_m and s are time independent complex parameter.
and

$$S = \sigma + jw$$

where Xm is the magnitude of X(t)

Network Functions

As the name suggests, in **theory of network synthesis** we are going to study about the synthesis of various networks which consists of both the active (Energy source) and passive elements (resistor, inductors and capacitors).

what is a network function?

In the frequency domain, **network functions** are defined as the quotient obtained by dividing the phasor corresponding to the circuit output by the phasor corresponding to the circuit input.

In simple words, **network functions** are the ratio of output phasor to the input phasor when phasors exists in frequency domain.

Network Functions

The general form of network functions are given below:

$$F(s) \quad \frac{a_p s^n - a_{n-1} s^{n-1} + \cdots + a_2 s^1 - a_0}{b_m s^m + b_{m-1} s^{m-} + \cdots - b_1 s^1 - b_1}$$

There are three main necessary conditions for the stability of these network functions and they are written below:

- The degree of the numerator of F(s) should not exceed the degree of denominator by more than unity. In other words (m - n) should be less than or equal to one.
- F(s) should not have multiple poles on the jω-axis or the y-axis of the pole-zero plot.
- F(s) should not have poles on the right half of the s-plane.

Poles and Zeros of Transfer Functions

- In general, the transfer function of a LTI system is a ratio of polynomials in the variable s and can be expressed as

$$G(s) = \frac{N(s)}{D(s)} \quad \begin{matrix} N(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 \\ D(s) = b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0 \end{matrix}$$

- Poles (finite): The m roots of D(s) are called the finite poles of the G(s)
- Zeros (finite): The n roots of N(s) are called the finite zeros of the G(s)
- The order of a transfer function is the value of the larger of the two integers m and n. Thus, if m > n, the transfer function is of order m. Most transfer function occurring in circuits are characterized by the condition m ≥ n.

Hurwitz Polynomial

If we have a stable network function then the denominator of the F(s) is called the **Hurwitz Polynomial**.

$$Let,\ F(s) = \frac{P(s)}{Q(s)}$$

Where, Q(s) is a **Hurwitz polynomial**.

Properties of Hurwitz Polynomials

- For all real values of s value of the function P(s) should be real.
- The real part of every root should be either zero or negative.
- Let us consider the coefficients of denominator of F(s) is b_n, $b_{(n-1)}$, $b_{(n-2)}$, . . . b_0. Here it should be noted that b_n, $b_{(n-1)}$, b_0 must be positive and b_n and $b_{(n-1)}$ should not be equal to zero simultaneously.

Transfer Functions:

Transfer Impedance Function:

- It is defined as the ratio of Laplace transform of voltage at one port and current at another port.

$$Z_{12}(s) = \frac{V_1(s)}{I_2(s)} \quad \text{and} \quad Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$$

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Driving point functions:

Driving point admittance functions

- The ratio of Laplace transform of current and voltage at port 1-1' or 2-2' is defined as driving point admittance function.
- Thus there are two driving point admittance functions.
- At port 1-1' denoted as Y11(s)

$$Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$$

- At port 2-2' denoted as Y11(s)

$$Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

Network functions for Two-Port Network are as follows:

1. Driving point functions:
   - Driving point impedance functions
   - Driving point admittance functions
2. Transfer Functions:
   - Voltage transfer functions
   - Current transfer functions
   - Transfer impedance functions
   - Transfer admittance functions