FOET
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Subject – Network Analysis

Topic – Two port Network

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TWO PORT NETWORKS

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TWO – PORT NETWORKS

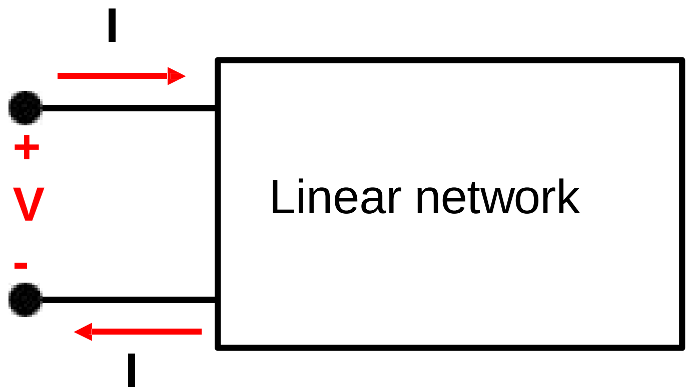
A pair of terminals through which a current may enter or leave a network is known as a *port*.

Two terminal devices or elements (such as resistors, capacitors, and inductors) results in one – port network.

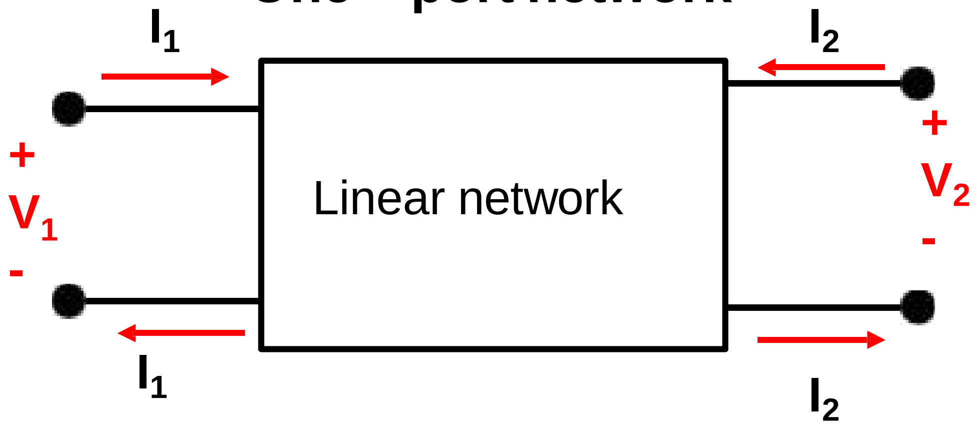
Most of the circuits we have dealt with so far are two – terminal or one – port circuits.

A two – port network is an electrical network with two separate ports for input and output.

It has two terminal pairs acting as access points. The current entering one terminal of a pair leaves the other terminal in the pair.



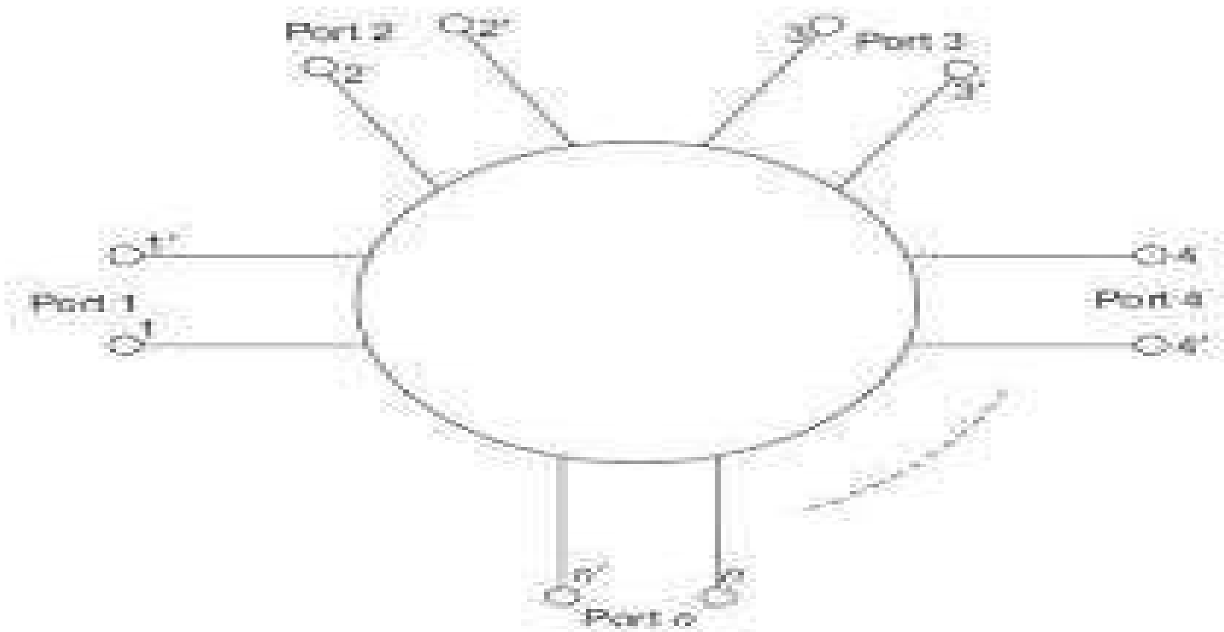
One – port network



Two – port network

N PORT NETWORK :-

A network having N numbers of ports ; is called N port network.



Two (2) reason why to study two port – network:

- Such networks are useful in communication, control system, power systems and electronics.

- Knowing the parameters of a two – port network enables us to treat it as a “black box” when embedded within a larger network.

From the network, we can observe that there are 4 variables that is I_1, I_2, V_1 and V_2, which two are independent.

The various term that relate these voltages and currents are called *parameters*.

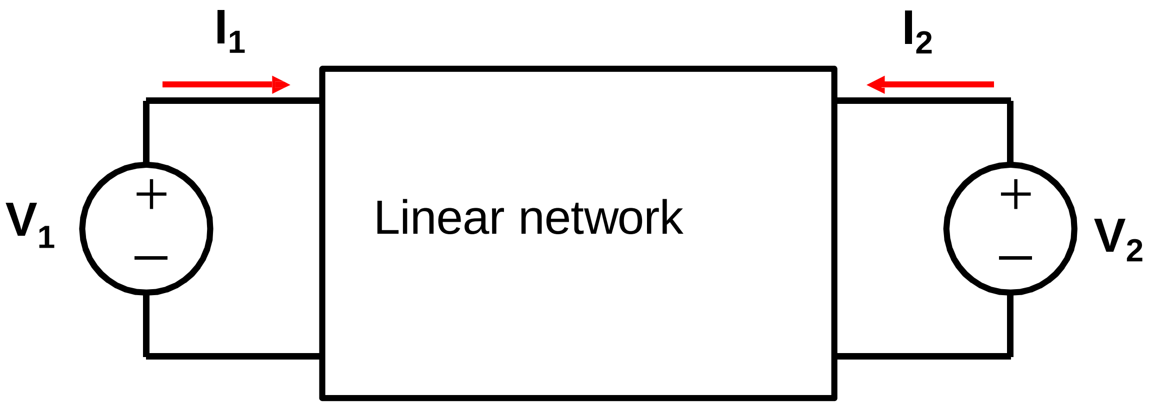
Z – PARAMETER

Z – parameter also called as impedance parameter and the units is ohm (Ω)

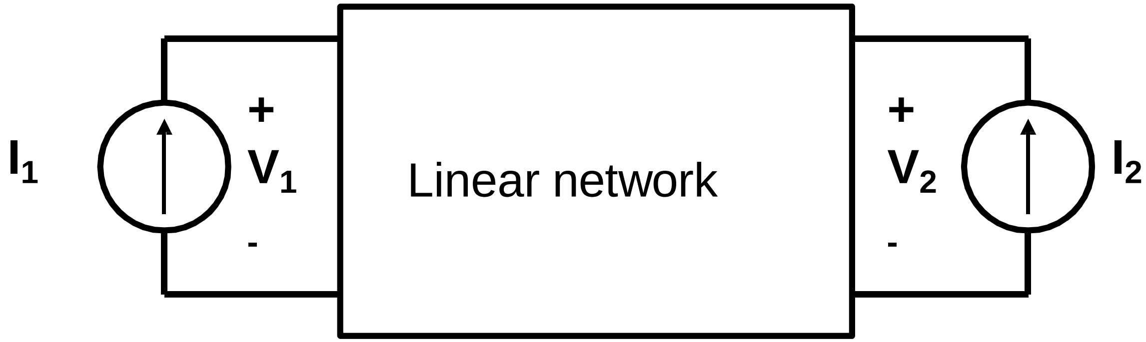
Impedance parameters is commonly used in the synthesis of filters and also useful in the design and analysis of impedance matching networks and power distribution networks.

The two – port network may be voltage – driven or current – driven.

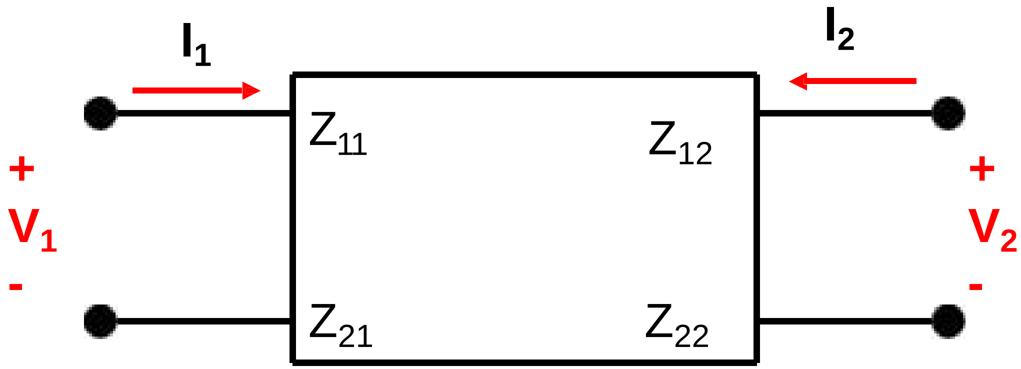
Two – port network driven by voltage source.



Two – port network driven by current sources.



The "black box" is replace with Z-parameter is as shown below.



The terminal voltage can be related to the terminal current as:

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \text{(1)}$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \text{(2)}$$

In matrix form as:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

The Z-parameter that we want to determine are z_{11}, z_{12}, z_{21}, z_{22}.

The value of the parameters can be evaluated by setting:

1. $I_1 = 0$ (input port open – circuited)
2. $I_2 = 0$ (output port open – circuited)

Thus,

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Where;

z_{11} = open – circuit input impedance.

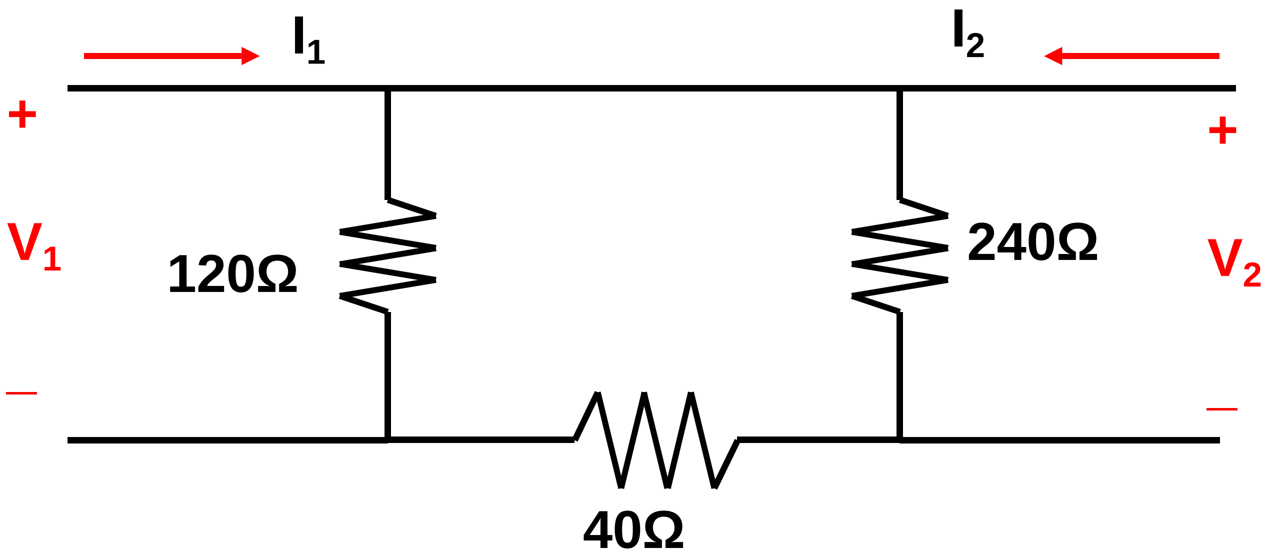
z_{12} = open – circuit transfer impedance from port 1 to port 2.

z_{21} = open – circuit transfer impedance from port 2 to port 1.

z_{22} = open – circuit output impedance.

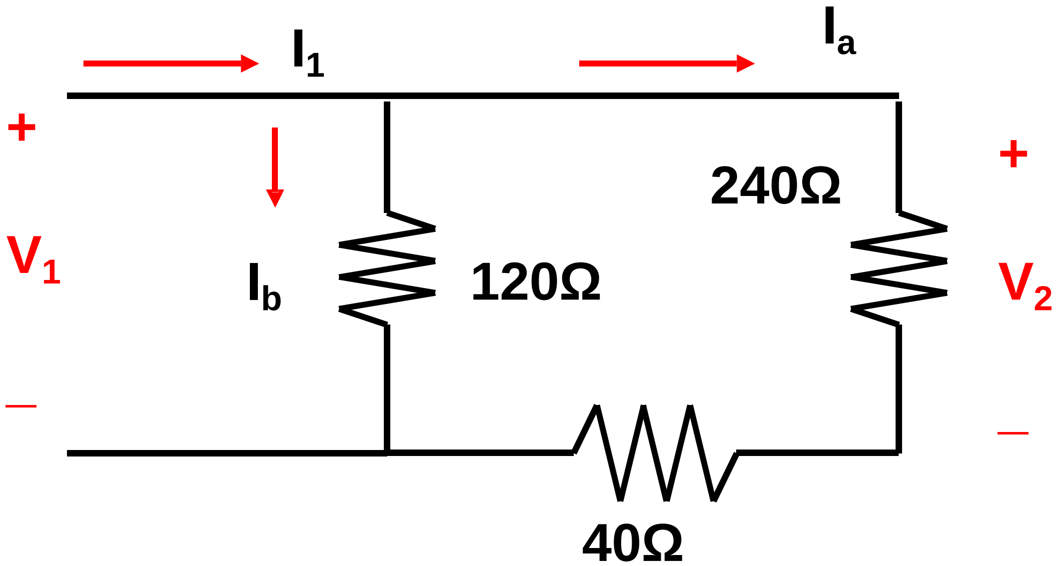
EXAMPLE

Find the Z – parameter of the circuit below.



SOLUTION

i) $I_2 = 0$(open circuit port 2). Redraw the circuit.



$$V_1 = 120 I_b \ \ldots\ldots(1)$$

$$I_b = \frac{280}{400} I_1 \ldots\ldots(2)$$

$$\text{sub}(1) \rightarrow (2)$$

$$\therefore Z_{11} = \frac{V_1}{I_1} = 84\Omega$$

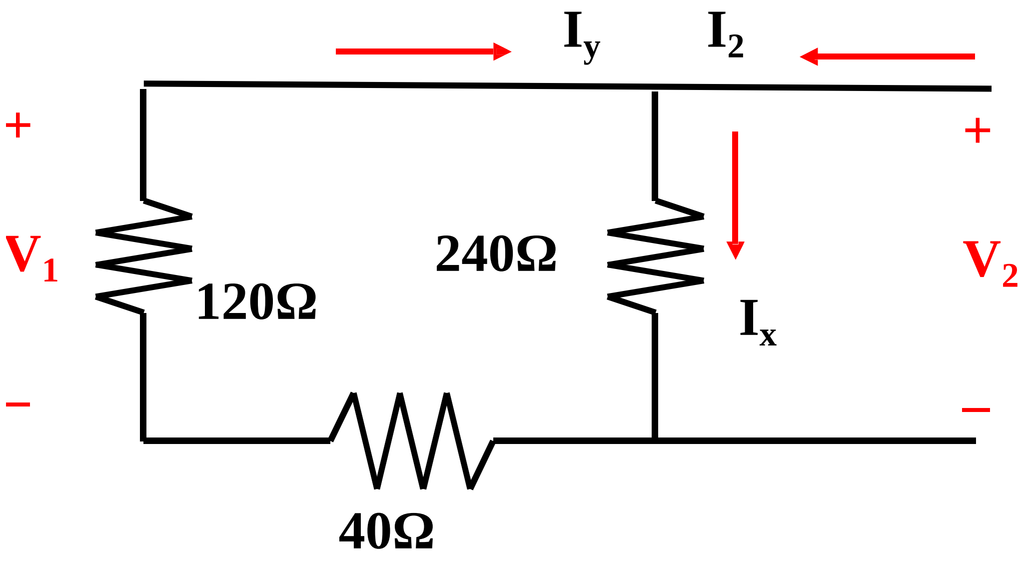
$$V_2 = 240 I_a \ldots\ldots(3)$$

$$I_a = \frac{120}{400} I_1 \ldots\ldots(4)$$

$$\text{sub}(4) \rightarrow (3)$$

$$\therefore Z_{21} = \frac{V_2}{I_1} = 72\Omega$$

ii) $I_1 = 0$ (open circuit port 1). Redraw the circuit.



$$V_2 = 240 I_x \ldots\ldots(1)$$

$$I_x = \frac{160}{400} I_2 \ldots\ldots(2)$$

$$\text{sub}(1) \rightarrow (2)$$

$$\therefore Z_{22} = \frac{V_2}{I_2} = 96\Omega$$

$$V_1 = 120 I_y \ldots\ldots(3)$$

$$I_y = \frac{240}{400} I_2 \ldots\ldots(4)$$

$$\text{sub}(4) \rightarrow (3)$$

$$\therefore Z_{12} = \frac{V_1}{I_2} = 72\Omega$$

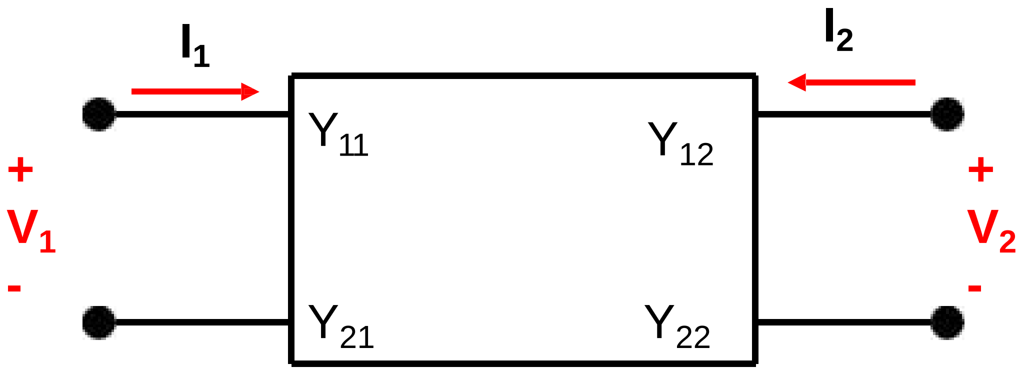
In matrix form:

$$[Z] = \begin{bmatrix} 84 & 72 \\ 72 & 96 \end{bmatrix}$$

Y - PARAMETER

Y – parameter also called admittance parameter and the units is siemens (S).

The "black box" that we want to replace with the Y-parameter is shown below.



The terminal current can be expressed in term of terminal voltage as:

$$I_1 = y_{11}V_1 + y_{12}V_2 \qquad \text{——— (1)}$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \qquad \text{——— (2)}$$

In matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

The y-parameter that we want to determine are Y_{11}, Y_{12}, Y_{21}, Y_{22}. The values of the parameters can be evaluate by setting:

i) $V_1 = 0$ (input port short – circuited).

ii) $V_2 = 0$ (output port short – circuited).

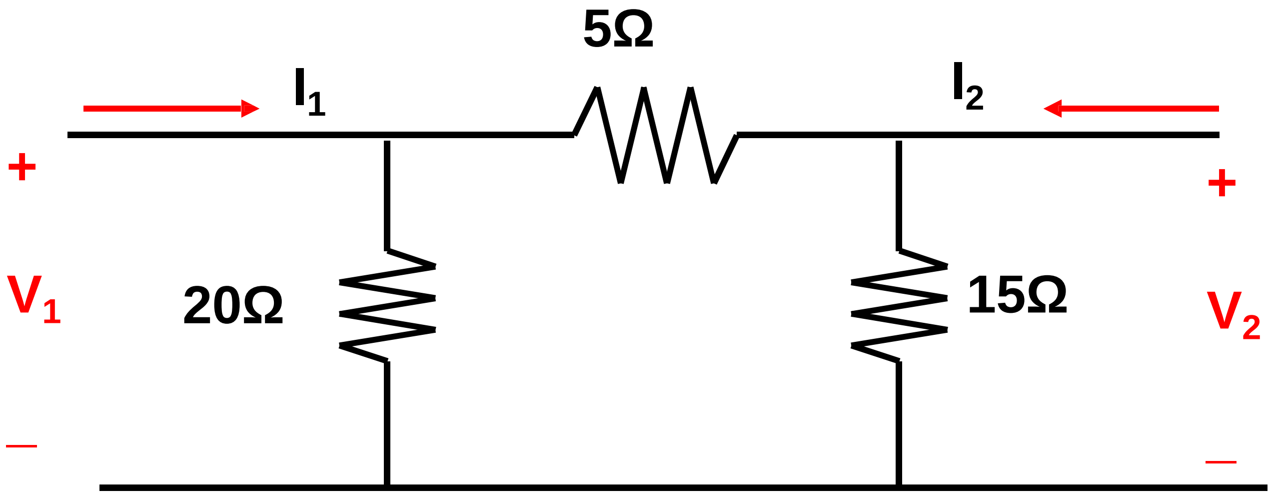
Thus;

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0}$$

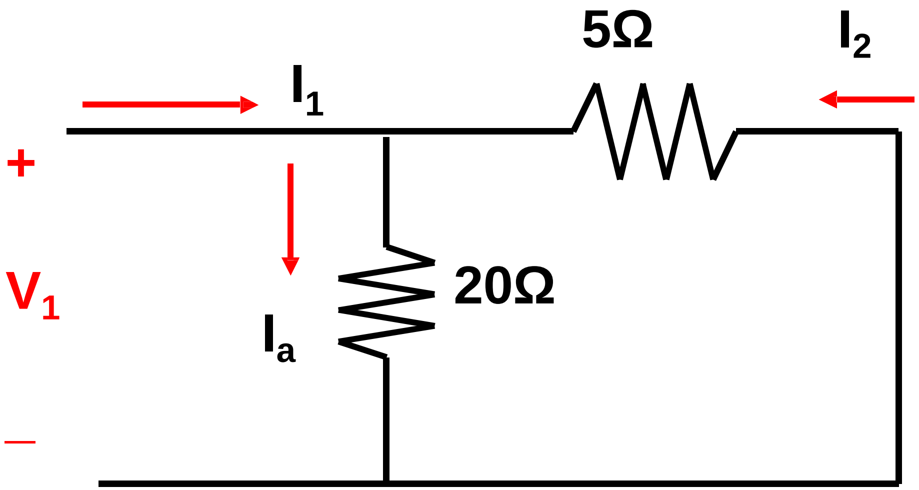
EXAMPLE

Find the Y – parameter of the circuit shown below.



SOLUTION

i) $V_2 = 0$



$$V_1 = 20 I_a \,\ldots\ldots.(1)$$

$$I_a = \frac{5}{25} I_1 \ldots\ldots.(2)$$

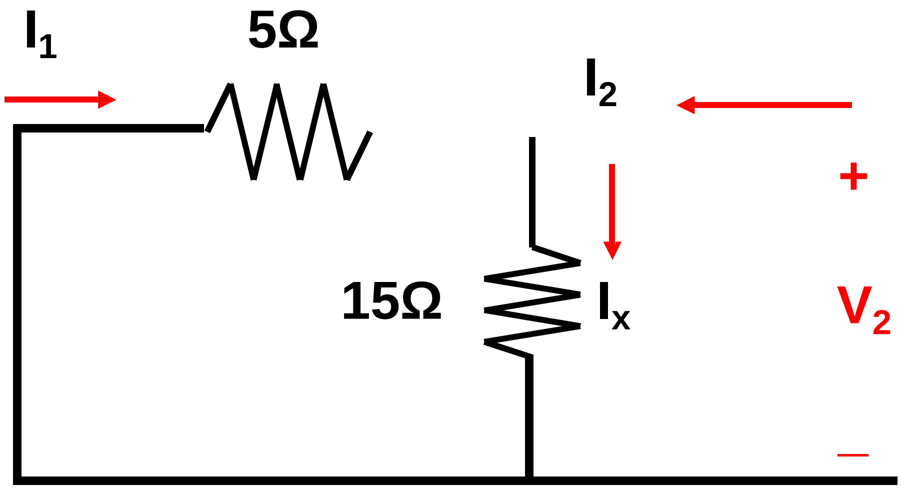
$$\text{sub}\,(1) \rightarrow (2)$$

$$\therefore Y_{11} = \frac{I_1}{V_1} = \frac{1}{4} S$$

$$V_1 = -5 I_2$$

$$\therefore Y_{21} = \frac{I_2}{V_1} = -\frac{1}{5} S$$

ii) $V_1 = 0$



$$V_2 = 15 I_x \ldots\ldots.(3)$$

$$I_x = \frac{5}{25} I_2 \ldots\ldots.(4)$$

$$\text{sub}(3) \rightarrow (4)$$

$$\therefore Y_{22} = \frac{I_2}{V_2} = \frac{4}{15} S$$

In matrix form;

$$V_2 = -5 I_1$$

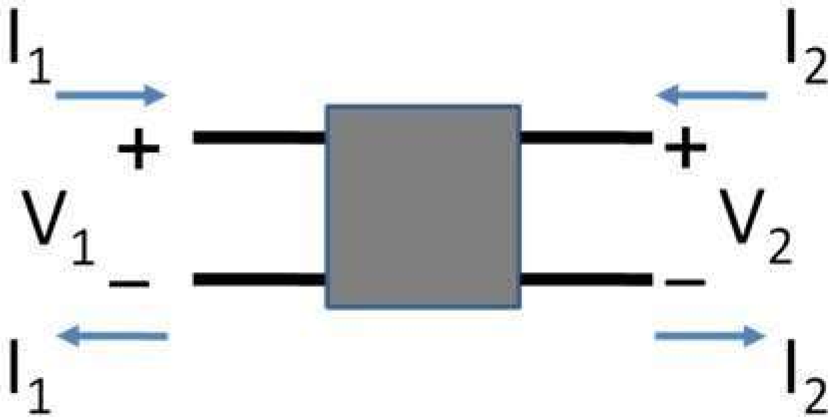
$$\therefore Y_{12} = \frac{I_1}{V_2} = -\frac{1}{5} S$$

$$\boxed{[Y] = \begin{bmatrix} \frac{1}{4} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{4}{15} \end{bmatrix} S}$$

H - PARAMETER

In these network there are four parameters called the hybrid parameters or H-parameters, one is measured in terms of ohm, one in mho and other two are dimension less. Since these parameters has mixed dimensions, so they are called as hybrid parameters.

The “black box” that we want to replace with T – parameter is as shown below.



$$\boxed{\begin{aligned} \mathbf{V}_1 &= \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \end{aligned}}$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

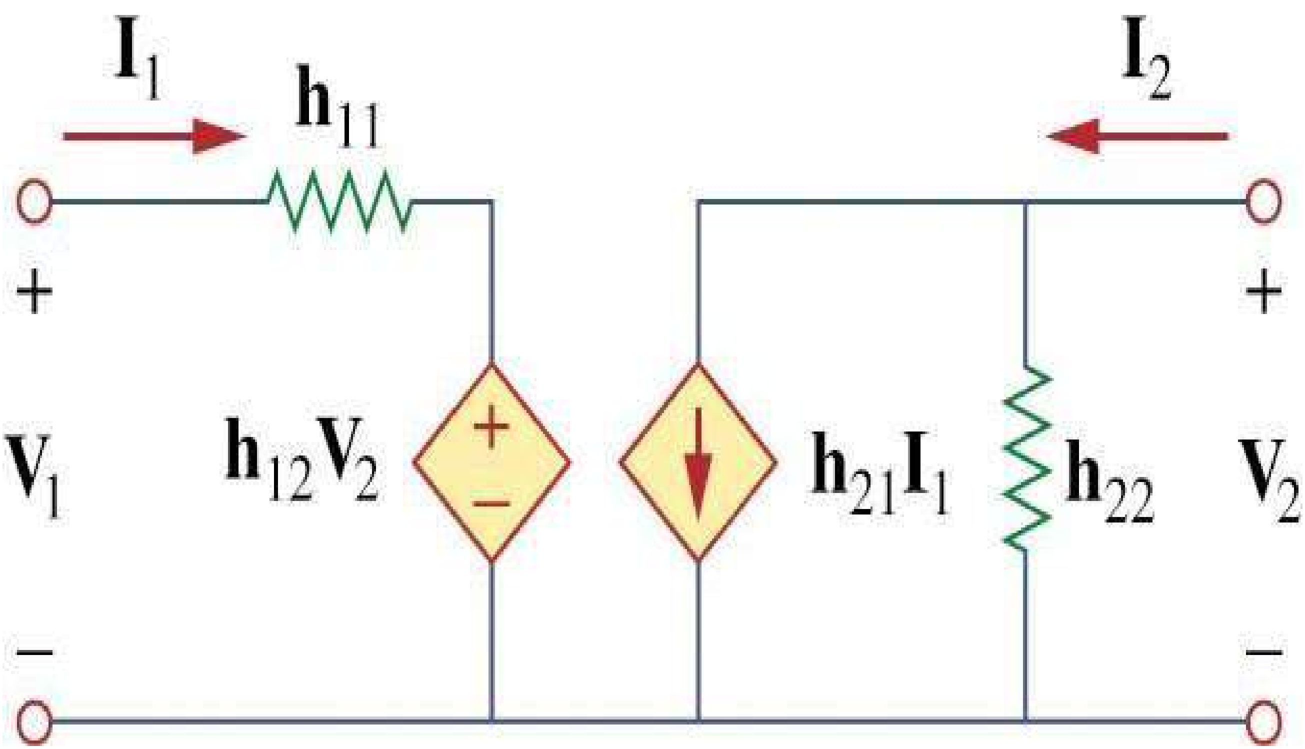
$$\boxed{\begin{aligned} \mathbf{h}_{11} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{h}_{21} &= \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} \end{aligned}}$$

h_{11} = Short-circuit input impedance

h_{12} = Open-circuit reverse voltage gain

h_{21} = Short-circuit forward current gain

h_{22} = Open-circuit output admittance



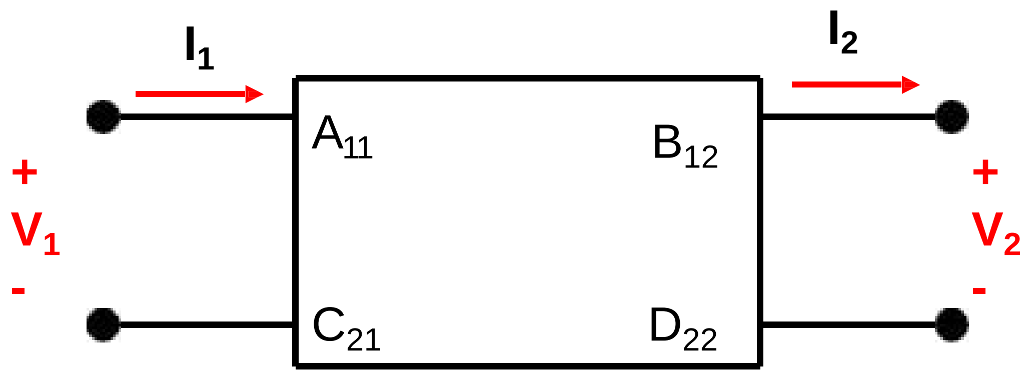
I1
h11
I2
+
V1
−
h12V2
+
−
h21I1
h22
+
V2
−

T (ABCD) PARAMETER

T – parameter or ABCD – parameter is a another set of parameters relates the variables at the input port to those at the output port.

T – parameter also called *transmission parameters* because this parameter are useful in the analysis of transmission lines because they express sending – end variables (V_1 and I_1) in terms of the receiving – end variables (V_2 and -I_2).

The “black box” that we want to replace with T – parameter is as shown below.



The equation is:

$$V_1 = AV_2 - BI_2 \ldots\ldots 1()$$

$$I_1 = CV_2 - DI_2 \ldots\ldots (2)$$

In matrix form is:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The T – parameter that we want determine are A, B, C and D where A and D are dimensionless, B is in ohm (Ω) and C is in siemens (S).

The values can be evaluated by setting

i) $I_2 = 0$ (input port open – circuit)

ii) $V_2 = 0$ (output port short circuit)

Thus;

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$

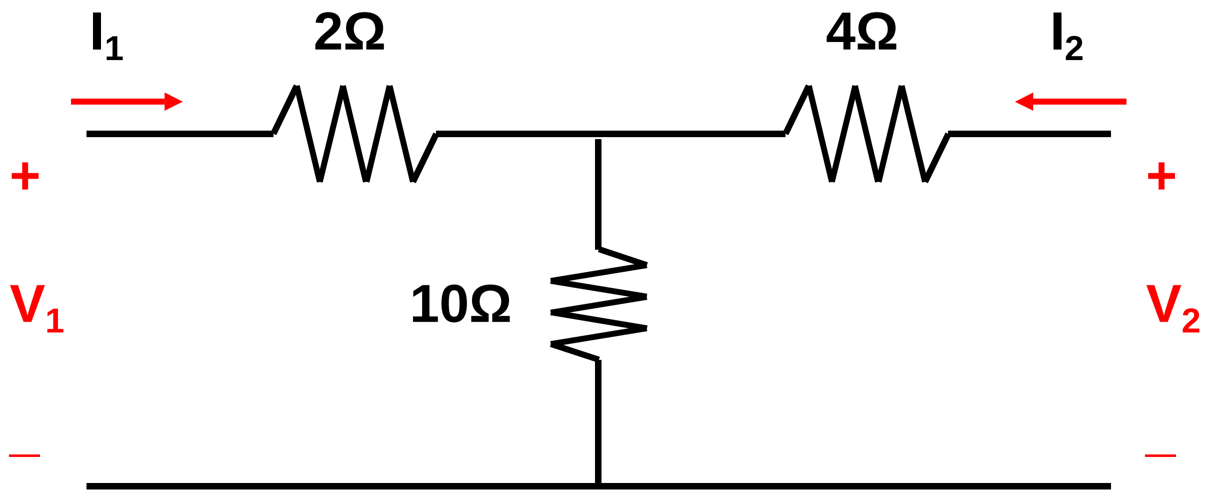
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

In term of the transmission parameter, a network is reciprocal if;

$$\boxed{AD - BC = 1}$$

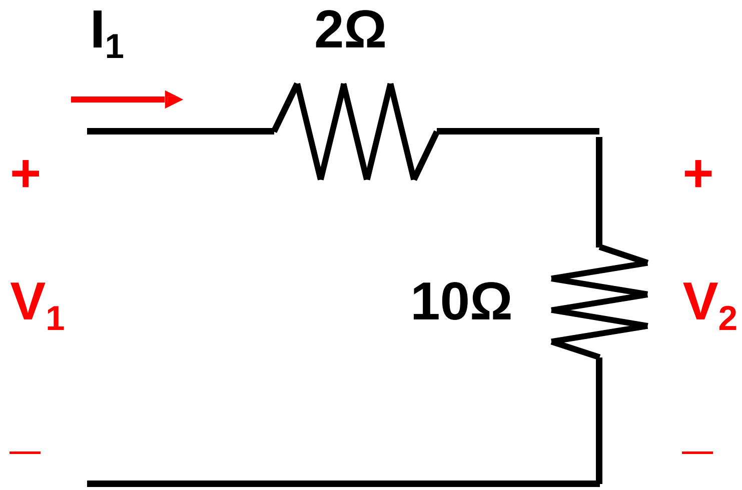
EXAMPLE

Find the ABCD – parameter of the circuit shown below.



SOLUTION

i) $I_2 = 0$,



$$V_2 = 10I_1$$

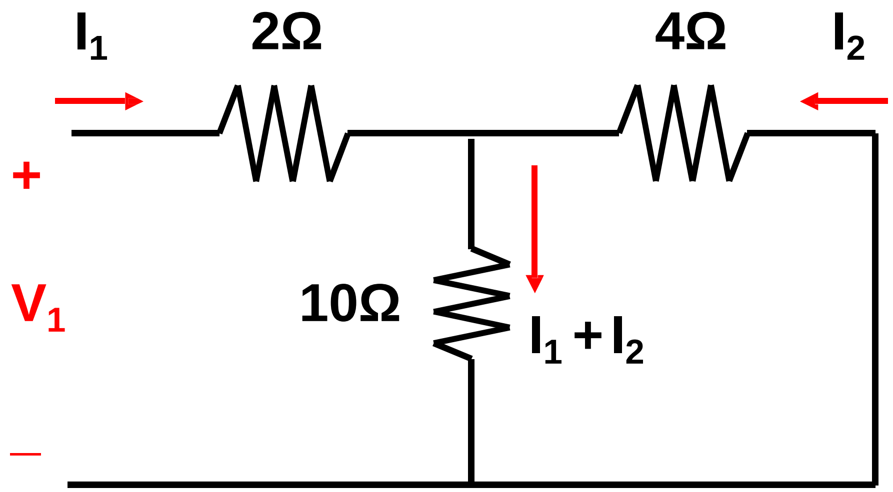
$$\therefore C = \frac{I_1}{V_2} = 0.1S$$

$$V_1 = 2I_1 + V_2$$

$$V_1 = 2\left(\frac{V_2}{10}\right) + V_2 = \frac{6}{5}V_2$$

$$\therefore A = \frac{V_1}{V_2} = 1.2$$

ii) $V_2 = 0$,



$$[T] = \begin{bmatrix} 1.2 & 6.8 \\ 0.1 & 1.4 \end{bmatrix}$$

$$I_2 = -\frac{10}{14} I_1$$

$$\therefore D = -\frac{I_1}{I_2} = 1.4$$

$$V_1 = 2I_1 + 10(I_1 + I_2)$$

$$V_1 = 12I_1 + 10I_2$$

$$V_1 = 12\left(-\frac{14}{10} I_2\right) + 10I_2$$

$$\therefore B = -\frac{V_1}{I_2} = 6.8\Omega$$

Inverse Transmission Parameter

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

$$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0} \quad B' = -\left.\frac{V_2}{I_1}\right|_{V_1=0}$$

$$C' = \left.\frac{I_2}{V_1}\right|_{I_1=0} \quad D' = -\left.\frac{I_2}{I_1}\right|_{V_1=0}$$

The h-parameter (Hybrid parameter)

H-parameter is the combination of Z and Y parameter defined by

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

or in scalar form

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

H-parameter is commonly used in transistor modeling.

The h-parameter

The h parameters can found from

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{1}{y_{11}} = z_{11} - \frac{z_{12}z_{21}}{z_{22}}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{y_{21}}{y_{11}} = -\frac{z_{21}}{z_{22}}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = y_{22} - \frac{y_{12}y_{21}}{y_{11}} = \frac{1}{z_{22}}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = -\frac{y_{12}}{y_{11}} = \frac{z_{12}}{z_{22}}$$

Inverse Hybrid Parameter (G Parameter)

g-parameter is defined by

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

or in scalar form

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

g-parameter is an alternative form of hybrid representation.

The g parameters can found from

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \frac{1}{z_{11}} = y_{11} - \frac{y_{12}y_{21}}{y_{22}} = \frac{h_{22}}{\Delta h}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{z_{21}}{z_{11}} = -\frac{y_{21}}{y_{22}} = -\frac{h_{21}}{\Delta h}$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = z_{22} - \frac{z_{12}z_{21}}{z_{11}} = \frac{1}{y_{22}} = \frac{h_{11}}{\Delta h}$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = -\frac{z_{12}}{z_{11}} = \frac{y_{12}}{y_{22}} = -\frac{h_{12}}{\Delta h}$$

where $\Delta h = h_{11}h_{22} - h_{12}h_{21}$

Condition For Reciprocity And Symmetricity:-

Parameter	Condition of Reciprocity	Condition of Symmetry
z	$z_{12} = z_{21}$	$z_{11} = z_{22}$
y	$y_{12} = y_{21}$	$y_{11} = y_{22}$
T $(ABCD)$	$(AD - BC) = 1$	$A = D$
h	$h_{12} = -h_{21}$	$(h_{11}h_{22} - h_{12}h_{21}) = 1$

Conversion of Two-port parameters

Two port parameters can be converted to any form as follows

From

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\mathbf{I} = \mathbf{YV}$$

And

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\mathbf{V} = \mathbf{ZI}$$

$$\therefore \mathbf{V} = \mathbf{ZYV} \Rightarrow \mathbf{Z} = \mathbf{Y}^{-1} \text{ and } \mathbf{Y} = \mathbf{Z}^{-1}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{z_{22}}{\Delta Z} & \dfrac{-z_{12}}{\Delta Z} \\ \dfrac{-z_{21}}{\Delta Z} & \dfrac{z_{11}}{\Delta Z} \end{bmatrix}$$

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{y_{22}}{\Delta Y} & \dfrac{-y_{12}}{\Delta Y} \\ \dfrac{-y_{21}}{\Delta Y} & \dfrac{y_{11}}{\Delta Y} \end{bmatrix}$$

where

$$\Delta Z = z_{11}z_{22} - z_{12}z_{21}$$

$$\Delta Y = y_{11}y_{22} - y_{12}y_{21}$$

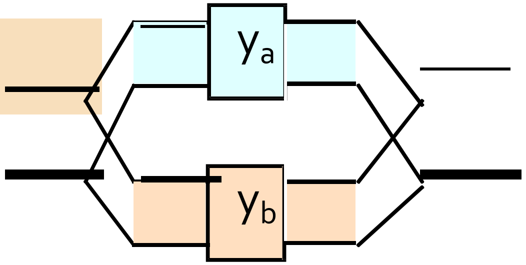
Conversion table

$_{to}\backslash^{from}$	Z	Y	H	G	T
Z	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_y} & \frac{-y_{12}}{\Delta_y} \\ \frac{-y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{g_{11}} & \frac{-g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\Delta_g}{g_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{t_{11}}{t_{21}} & \frac{\Delta_t}{t_{21}} \\ \frac{1}{t_{21}} & \frac{t_{22}}{t_{21}} \end{bmatrix}$
Y	$\begin{bmatrix} \frac{z_{22}}{\Delta_z} & \frac{-z_{12}}{\Delta_z} \\ \frac{-z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_g}{g_{22}} & \frac{g_{12}}{g_{22}} \\ \frac{-g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{t_{22}}{t_{12}} & \frac{-\Delta_t}{t_{12}} \\ \frac{-1}{t_{12}} & \frac{t_{11}}{t_{12}} \end{bmatrix}$
H	$\begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{g_{22}}{\Delta_g} & \frac{-g_{12}}{\Delta_g} \\ \frac{-g_{21}}{\Delta_g} & \frac{g_{11}}{\Delta_g} \end{bmatrix}$	$\begin{bmatrix} \frac{t_{12}}{t_{22}} & \frac{\Delta_t}{t_{22}} \\ \frac{-1}{t_{22}} & \frac{t_{21}}{t_{22}} \end{bmatrix}$
G	$\begin{bmatrix} \frac{1}{z_{11}} & \frac{-z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\Delta_z}{z_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_y}{y_{22}} & \frac{y_{12}}{y_{22}} \\ \frac{-y_{21}}{y_{22}} & \frac{1}{y_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{h_2 2}{\Delta_h} & \frac{-h_{12}}{\Delta_h} \\ \frac{-h_{21}}{\Delta_h} & \frac{h_{11}}{\Delta_h} \end{bmatrix}$	$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{t_{21}}{t_{11}} & \frac{-\Delta_t}{t_{11}} \\ \frac{1}{t_{11}} & \frac{t_{12}}{t_{11}} \end{bmatrix}$
T	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{g_{21}} & \frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\Delta_g}{g_{21}} \end{bmatrix}$	$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$

Interconnection Of Two Port Network

Three ways that two ports are interconnected:

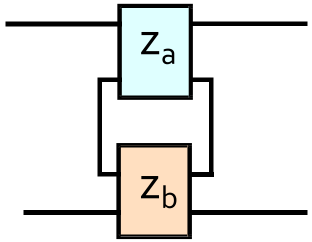
* Parallel



***Y* parameters**

$$[y] = [y_a] + [y_b]$$

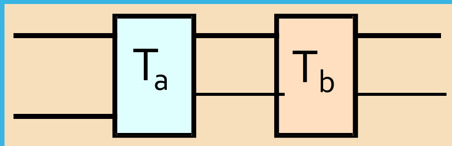
* Series



***Z* parameters**

$$[z] = [z_a] + [z_b]$$

* Cascade



***ABCD* parameters**

$$[T] = [T_a][T_b]$$

THANK YOU FOR LISTENING