UNIT-III

Load Flow Analysis :- is a numerical analysis of the flow of electric power in an interconnected system. A power flow analysis usually uses simplified notations such as a one-line diagram and per unit system, and focuses on various aspects of AC power parameters, such as voltages, voltage angles, real power and reactive power. It analyses the power systems in normal steady state operation.

OR

It is a computational procedure (numerical algorithms) required to determine the steady state operating characteristics of a power system network from the given line data and Bus data

Power flow or load flow studies are important for planning future expansion of power systems as well as determining the best operating of existing systems. The principal information obtained from the power-flow study is the magnitude and phase angle of the voltage at each bus, and the real and reactive power flowing in each line.

Bus – A Bus in a power system is defined as the ~~vertile~~ vertical line at which the several components of the power system like generator, loads, and feeders, etc. are connected. The buses in a power system are associated with four quantities. These quantities are the magnitude of the voltage, the phase angle of the voltage, active or true power and the reactive power.

OR

A bus is a node where a line or several lines are connected and may also include several components such as loads and generators in a power system. Each Bus or node is correlated with one of four quantities (1) magnitude of voltage, (2) phase angle of voltage (3) active power or ~~tute~~ true power, and (4) reactive power.

In a specific load flow, two out of four quantities have a definite value, while the other two will need to be determind by calculating using power flow equations. The unknown and known variables are not fixed and vary depending on the bus type.

Depending upon which quantities have been specified, the buses are classified in the following three categories-

Generation Bus : The generation bus, also known as the PV Bus voltage-controlled bus, or generator bus, represents the generator satations found in a power system. The quantities specified for this type of bus are voltage magnitude and the ~~real~~ real power. It means that the unknown variables for the generation bus are the phase, ~~and~~ angle of voltage and reactive power

Generator in the power system are connected to this type of bus. Therefore, the bus voltage corresponds to the generator's voltage, and the generation of active power is correlated to the generator rating specific to the Bus.

The voltage magnitude of the generator bus is kept steady by adjusting the synchronous generator's field current. Real power generation for every generator is assigned concerning economic dispatch.

Load Bus : At this bus the real and reactive components or power are specified. It is desired to find out the voltage magnitude and phase angle through the load flow solution.

- It is required to specify and P_D & Q_D at such a bus as at a load Bus voltage can be allowed to vary with in permissible values e.g. 5%. Also phase angle of the voltage is not very important for the load.

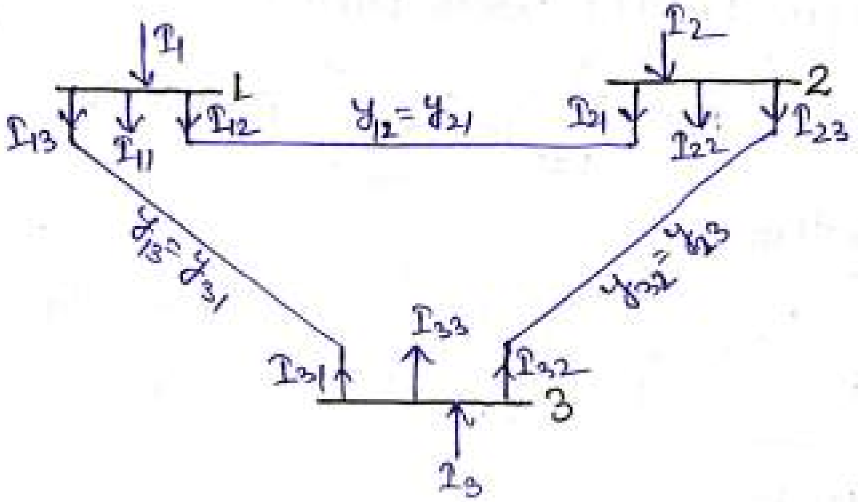
No generator is connected to the load Bus. ~~Voltage in this type of~~ The load bus is the most numerous bus type typically found in the power system.

Slack Bus :- In load flow analysis losses remain unknown until the load flow solution is complete. It is for this reason that generally one of the generator buses is made to take the additional real and reactive power to supply transmission losses. That is why this type of bus is also known as the slack or swing bus. At this Bus, the voltage magnitude V and phase angle δ are specified. whereas real and reactive powers P_g & Q_g are obtained through the load flow solution.

Bus type	Quantities specified	Quantities to be obtained
Load Bus	P, Q	|V|, δ
Generator Bus	P, |V|	Q, δ
Slack Bus	|V|, δ	P, Q

Nodal Admittance Matrix :

The load flow equations, using nodal admittance formulation for a three-bus system are developed first and then they are generalized for an n-bus system



Three-bus System

At node 1

$$I_1 = I_{11} + I_{12} + I_{13}$$
$$= V_1 y_{11} + (V_1 - V_2) y_{12} + (V_1 - V_3) y_{13}$$
$$= V_1 y_{11} + V_1 y_{12} - V_2 y_{12} + V_3 y_{13} - V_3 y_{13}$$
$$= V_1 (y_{11} + y_{12} + y_{13}) - V_2 y_{12} - V_3 y_{13}$$
$$I_1 = V_1 Y_{11} + V_2 Y_{12} + V_3 Y_{13} \longrightarrow (1)$$

Here y_{11} is the shunt charging admittance at bus 1 and ground.

$$Y_{11} = y_{11} + y_{12} + y_{13}$$
$$Y_{12} = -y_{12}, \quad Y_{13} = -y_{13}$$

Similarly nodal current equations for the other nodes can be written as follows —

$$I_2 = V_1 Y_{21} + V_2 Y_{22} + V_3 Y_{23} \longrightarrow (2)$$

$$I_3 = V_1 Y_{31} + V_2 Y_{32} + V_3 Y_{33} \longrightarrow (3)$$

These equations canbe written in a matrix form asfollows -

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \longrightarrow (4)$$

$Y_{12} = y_{12}$
$Y_{31} = Y_{13} = -y_{13}$
$Y_{21} = -y_{12}$
$Y_{32} = Y_{23} = -y_{23}$

Generalise equ. for the node current

$$\boxed{I_p = \sum_{q=1}^{3} Y_{pq} V_q ,} \quad p = 1 \text{ to } 3 \longrightarrow (5)$$

For n-Bus system

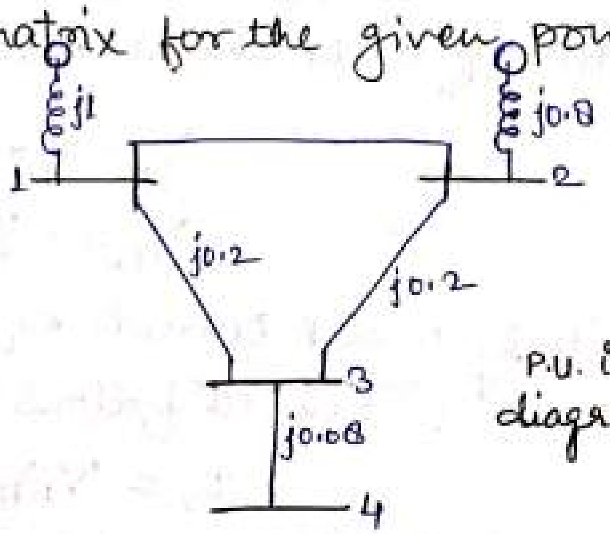
$$\boxed{I_p = \sum_{q=1}^{n} Y_{pq} V_q} \quad p = 1, 2 \ldots n$$

In matrix form

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

It can be shown that the nodal admittance matrix is a sparse matrix (a few no. of elements are non-zero) for an actual power system.

Find the admittance matrix for the given power system network.

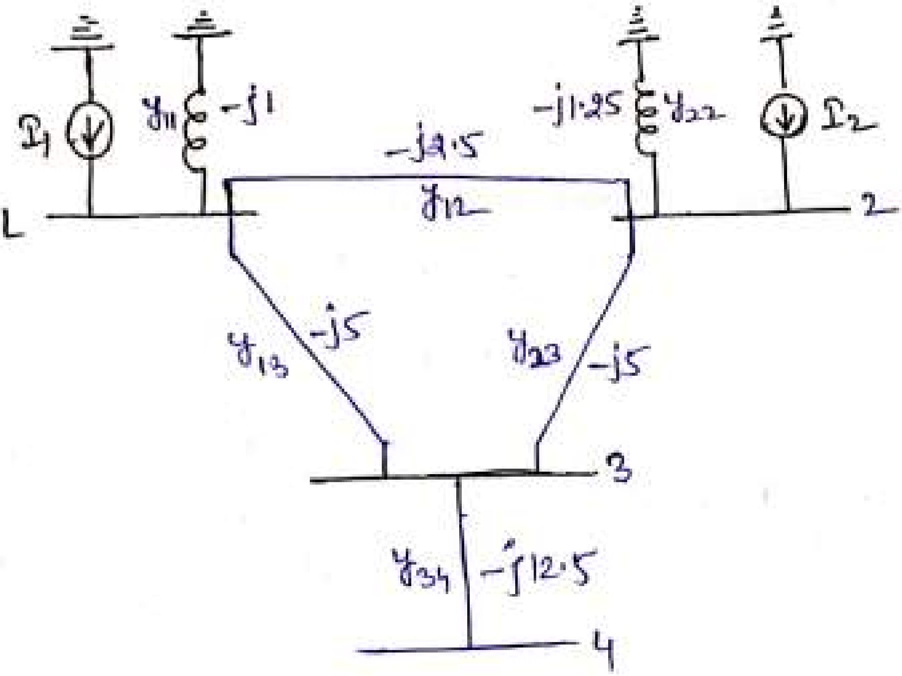


P.U. impedance diagram of P.S. N/W

$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

Since the nodal solution is based upon Kirchhoff's current law, impedance are converted to admittance i.e.

$$y_{ij} = \frac{1}{z_{ij}} = \frac{1}{r_{ij} + jx_{ij}}$$



$Y_{11} = y_{11} + y_{12} + y_{13} + y_{14}$
$= -j1 + (-j2.5) - j5 + 0$
$= -j8.5$

$Y_{22} = y_{21} + y_{22} + y_{23} + y_{24}$
$= -j2.5 - j1.25 - j5 + 0$
$= -j8.75$

$Y_{33} = y_{31} + y_{32} + y_{33} + y_{34}$
$= -j5 - j5 + 0 - j12.5$
$= -j22.5$

$Y_{44} = y_{41} + y_{42} + y_{43} + y_{44}$
$= 0 + 0 - j12.5 + 0$
$= -j12.5$

$Y_{12} = +j2.5 \quad Y_{13} = +j5, \; Y_{14} = 0$

$Y_{21} = +j2.5 \quad Y_{23} = +j5, \; Y_{24} = 0$

$Y_{31} = +j5 \quad Y_{32} = +j5, \; Y_{34} = +j12.5$

$Y_{41} = 0 \quad Y_{42} = 0, \; Y_{43} = +j12.5.$

$$Y_{Bus} = \begin{bmatrix} -j8.5 & +j2.5 & +j5 & 0 \\ +j2.5 & -j8.75 & +j5 & 0 \\ +j5 & +j5 & -j22.5 & +j12.5 \\ 0 & 0 & +j12.5 & -j12.5 \end{bmatrix}$$

LOAD FLOW PROBLEM FORMULATION —

The complex power injected by the source into the i^{th} bus of a power system is —

$$S_i = P_i + jQ_i = V_i I_i^* \qquad i = 1, 2 \ldots n \longrightarrow (1)$$

Where $V_i \rightarrow$ voltage at i^{th} Bus w.r.t. ground

$I_i \rightarrow$ source current injected into the Bus.

The load flow problem is handled more conveniently by use of I_i rather than I_i^*. Therefore, taking the complex conjugate of equ. (1)

$$P_i - jQ_i = V_i^* I_i \longrightarrow (2)$$

As we know that

$$I_i = \sum_{k=1}^{n} Y_{ik} V_k \longrightarrow (3)$$

Use equ. no. (3) in (2)

$$P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k \qquad i = 1, 2 \ldots n \longrightarrow (4)$$

Equating real and imaginary parts

$$P_i = \mathrm{Re}\left\{ V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right\} \longrightarrow (5)$$

$$Q_i = -\mathrm{Im}\left\{ V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right\} \longrightarrow (6)$$

In polarform

$$V_i = |V_i| e^{j\delta_i}$$

$$Y_{ik} = |Y_{ik}| e^{j\theta_{ik}}$$

Now Real & Reactive powers can be expressed as —

$$P_i = |V_i| \sum_{k=1}^{n} |V_k||Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i) \longrightarrow (7) \quad i = 1, 2 \ldots n$$

$$Q_i = -|V_i| \sum_{k=1}^{n} |V_k||Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i) \longrightarrow (8) \quad i = 1, 2 \ldots n$$

Equ. (7) & (8) represent 2n power flow equations at 'n' buses of a power system ($n \rightarrow$ real power flow equ + $n \rightarrow$ reactive powerflow equ.). Each bus is characterized by four variable; P_i, Q_i, $|V_i|$ & δ_i resulting in a total of 4n variables.

Equ. (7) & (8) can be solved for 2n variables if remaining 2n variables are speified.

Practical considerations allow a power system analyst to fix a priori two variables at each bus.

The sol.n of remaining 2n bus variables is rendered difficult by fact that equ. (7) & (8) are non-linear algebric equations (bus voltages are involved in product form and sine & cosine Terms are present and therefore, explicit sol.n is not possible. Sol.n can be obtained by iterative numerical techniques.

An approximate load flow Solution —

Following assumptions and approximations in the load flow equ. (7) & (8) have been made —

i) Line resistance — very small & neglected (shunt conductance of overhead line is always negligible) so, P_L - the active power loss of the system is zero. Thus. $\theta_{ik} = 90°$ & $\theta_{ii} = -90°$

ii) $(\delta_i - \delta_k)$ is small ($< \pi/6$) so that $\sin(\delta_i - \delta_k) \simeq (\delta_i - \delta_k)$

iii) All buses other than the slack bus (numbered as bus 1) are PV buses, i.e. voltage magnitudes at all the buses including the slack bus are specified.

Now equ. (7) & (8) becomes —

$$P_i = |V_i| \sum_{k=1}^{n} |V_k||Y_{ik}|(\delta_i - \delta_k); \quad i = 2, 3, \ldots n \longrightarrow (9)$$

$$Q_i = -|V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k||Y_{ik}| \cos(\delta_i - \delta_k) + |V_i|^2 |Y_{ii}| \quad i = 1, 2, \ldots n \longrightarrow (10)$$

Gauss-Seidel Method :-

Let it be assumed that all buses other than the slack bus are PQ buses to explain how the GS method is applied to obtain the load flow solution,

Later we will include PV buses as well.

The slack bus voltage being specified, there are (n-1) bus voltages starting values of whose magnitudes and angles are assumed. These values are then ~~upd~~ updated through an iterative process.

from eqn. 2

$$I_i = \frac{(P_i - jQ_i)}{V_i^*} \longrightarrow (11)$$

from eqn. 4

$$P_i - jQ_i = V_i^* \left[\sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k + Y_{ii} V_i \right]$$

$$V_i Y_{ii} = \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad i = 2, 3, \ldots n$$

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad i = 2, 3, \ldots n \longrightarrow (12)$$

During each iteration voltages at buses $i = 2, 3 \ldots n$ are sequentially updated through eqn. (12). Slack bus voltage being fixed & will not update (V_1)

Iteration are repeated till no bus voltage magnitude changes by more than a prescribed value during an iteration. ~~The comput process~~ Computation process is then said to converge to a solution.

If instead of updating voltage at every step of an iteration, updating is carried out at the end of a complete iteration, the process is known as the Gauss iterative method. It is much slower to converge and may sometimes fail to do so.

Algorithms for Load Flow Solution :

All buses are PQ bus other than slack Bus

1. Load ~~profile~~ profile is known at each bus (P_{Di} & Q_{Di} known), allocate P_{Gi} & Q_{Gi} at all generating stations.

2. Assembly of bus admittance matrix Y_{BUS}

3. Iterative computation of bus voltages (V_i ; $i = 2, 3, \ldots n$) — A set of initial voltage values is assumed. Initially all voltages are set equal to $(1+j0)$ except the voltage of the slack bus which is fixed.

No. of equ. $(n-1)$ in complex no. to find $(n-1)$ complex voltages $V_2, V_3 \ldots V_n$

$$A_i = \frac{P_i - jQ_i}{Y_{ii}} \qquad i = 2, 3, \ldots n$$

$$B_{ik} = \frac{Y_{ik}}{Y_{ii}} \qquad i = 2, 3, \dots n$$

$$k = 1, 2, \dots n$$

$$k \neq i$$

Now for the $(r+1)^{th}$ iteration, the voltage becomes

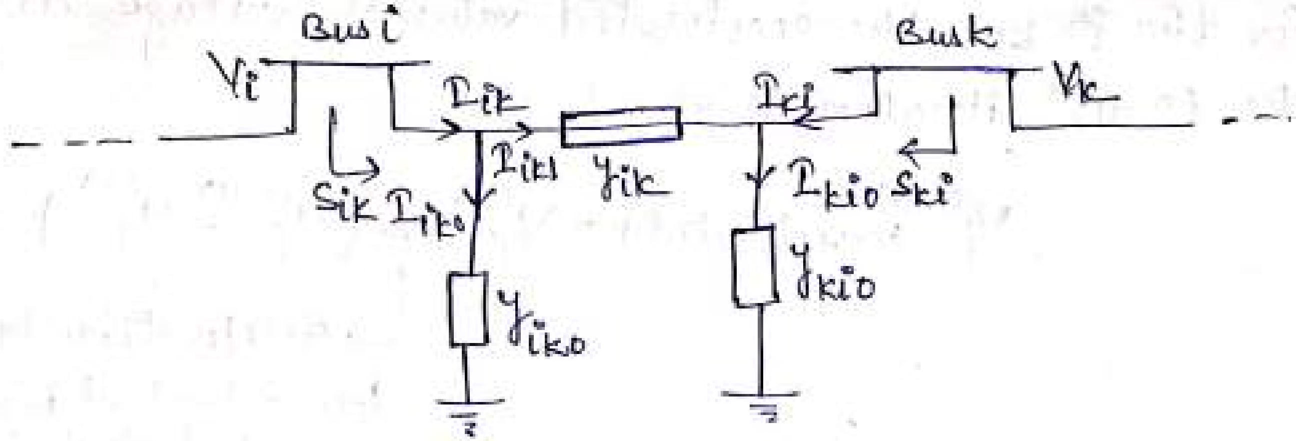
$$V_i^{(r+1)} = \frac{A_i}{\left(V_i^{(r)*}\right)} - \sum_{k=1}^{i-1} B_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} B_{ik} V_k^{(r)} \quad i = 2, 3, \dots n \qquad (13)$$

The iterative process is continued till the change in magnitude of bus voltage $|\Delta V_i^{(r+1)}|$, between two consecutive iterations is less than a certain tolerance for all bus voltages i.e.

$$|\Delta V_i^{(r+1)}| = |V_i^{(r+1)} - V_i^{(r)}| < \epsilon \; ; \; i = 2, 3, \dots n$$

4. Computation of slack bus Power — substitute all bus voltages computed in step 3 along with V_1 in eqn. (4) yields $S_1^* = P_1 - jQ_1$

5. Computation of line flows — Power flows on the various lines of the network are computed. Consider a line connecting buses i & k. The line and ~~transformes~~ transfomers at each end can be represented by a circuit with series admittance y_{ik} and two shunt admittances y_{ik0} & y_{ki0} as shown in fig.



π- representation of a line and transformers connected b/w two buses

The current fed by bus i into the line can be expressed as

$$I_{ik} = I_{ik1} + I_{ik0} = (V_i - V_k) y_{ik} + V_i y_{ik0} \longrightarrow (14)$$

The power fed into the line from bus i is—

$$S_{ik} = P_{ik} + jQ_{ik}$$
$$= V_i I_{ik}^*$$
$$= V_i (V_i^* - V_k^*) y_{ik}^* + V_i V_i^* y_{ik0}^* \longrightarrow (15)$$

Similarly, the power fed into the line from bus 'k' is

$$S_{ki} = V_k (V_k^* - V_i^*) y_{ik}^* + V_k V_k^* y_{ki0} \longrightarrow (16)$$

The power loss in the $(i\text{-}k)^{th}$ Line is the sum of the power flows determined from (15) & (16). Total transmission loss can be computed by summing all the Line flows (i.e. $S_{ik} + S_{ki}$ for all i, k).

<u>Acceleration of Convergence</u> :- To speed up the convergence we use acceleration factor.

for the i^{th} bus, the accelerated value of voltage at the $(r+1)^{th}$ iteration is give by

$$V_i^{(r+1)}(\text{accelerated}) = V_i^{(r)} + \alpha\left(V_i^{(r+1)} - V_i^{(r)}\right) \rightarrow (17)$$

$\hookrightarrow$ Acceleration factor found by trial ~~towal~~ load flow studies.

- Generally $\alpha = 1.6$
- Wrong value of α may indeed slow down convergence.

Algorithm Modification when PV Buses are also Present :

At the PV buses, P & |V| are specified and Q & δ are the unknowns to be determined. Therefore, the values of Q and δ are to be updated in every GS iteration through appropriate bus equations.

1. From eqn. (6)

$$Q_i = -\text{Im}\left\{V_i^* \sum_{k=1}^{n} Y_{ik} V_k\right\}$$

revised value of Q_i is obtained by above eqn. by substituting most updated values of voltages on righ hand side.

for $(r+1)^{th}$ iteration

$$Q_i^{(r+1)} = -\text{Im}\left\{(V_i^{(r)})^* \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} + (V_i^{(r)})^* \sum_{k=i}^{n} Y_{ik} V_k^{(r)}\right\} \rightarrow (18)$$

2. The revised value of δ_i is obtained from equ. 13

$$\delta_i^{(r+1)} = \angle V_i^{(r+1)}$$

$$= \text{Angle of} \left[\frac{A_i^{(r+1)}}{V_i^{(r)*}} - \sum_{k=1}^{i-1} B_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} B_{ik} V_k^{(r)} \right] \longrightarrow (19)$$

where

$$A_i^{(r+1)} = \frac{P_i - jQ_i^{(r+1)}}{Y_{ii}} \longrightarrow (20)$$

The algorithm for PQ buses remains unchanged.

Flow Chart :-

Read 1. Primitive Y matrix
2. Bus Incidence Matrix A
3. Slack Bus voltage ($|V_1|, \delta_1$)
4. Real Bus Powers P_i for $i = 2, 3, 4 \ldots n$
5. Reactive Bus Powers Q_i for $i = m+1, \ldots n$ (PQ Buses)
6. Voltage magnitude $|V_i^s|$ for $i = 2, \ldots m$ (PV Buses)
7. Voltage magnitude Limits $|V_i|_{min}$ & $|V_i|_{max}$ for PQ Buses
8. Reactive Power limits $Q_{i\,min}$ & $Q_{i\,max}$ for PV buses.

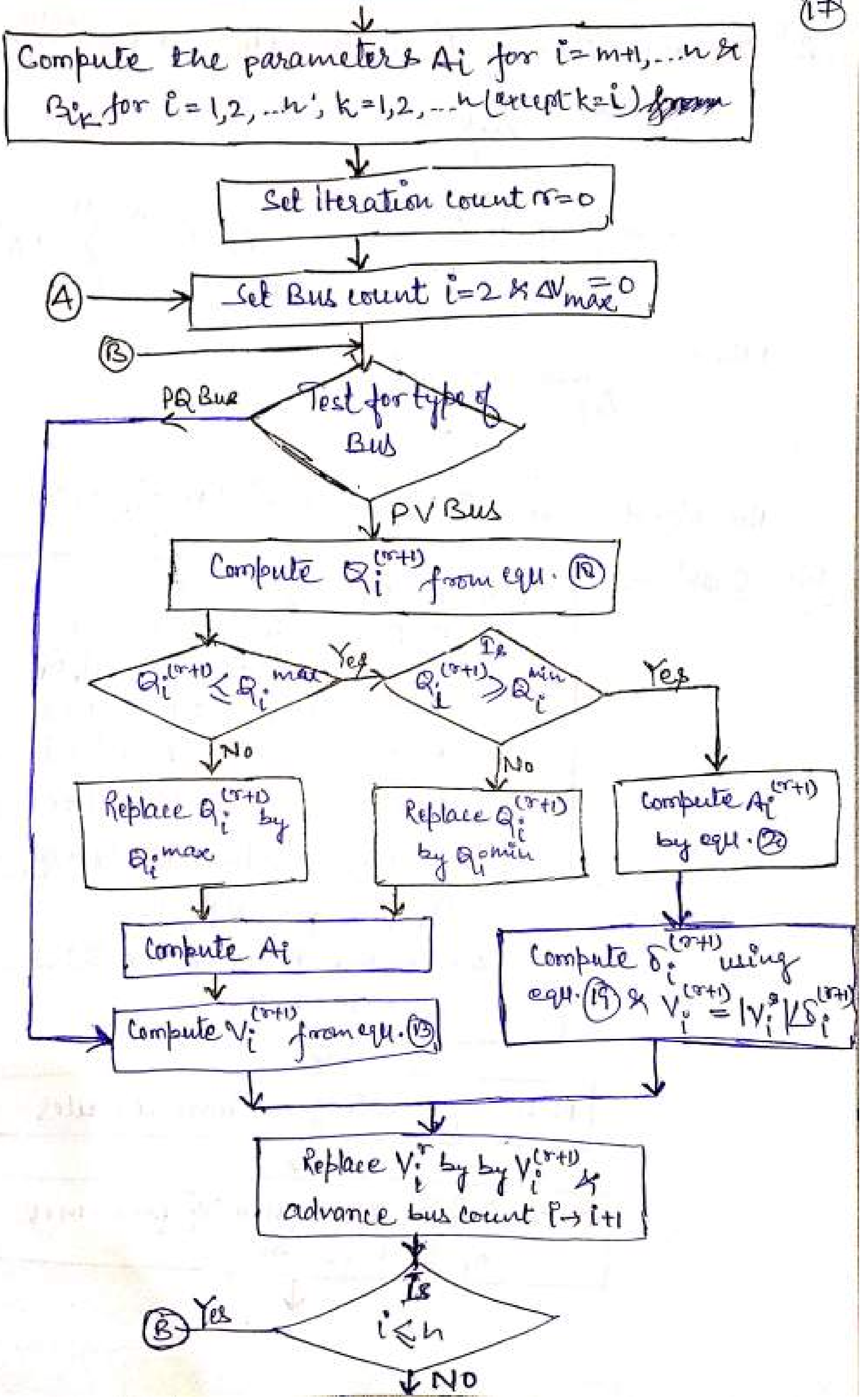
↓

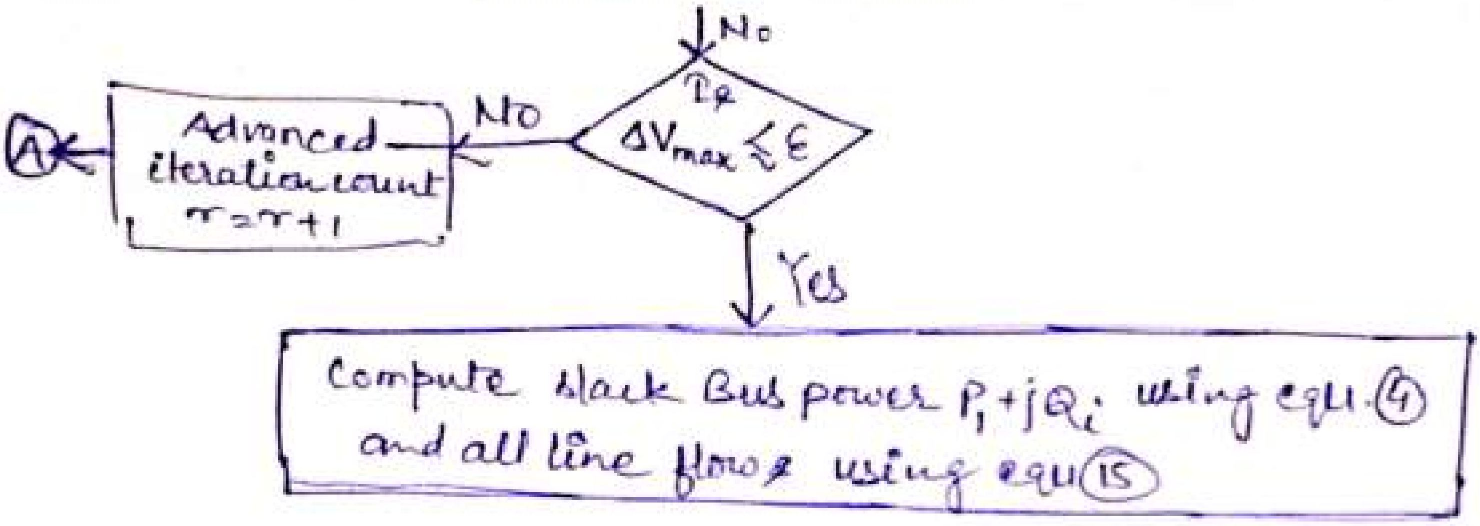
Form Y_{Bus} using relevant rules

↓

Make initial assumption V_i^0 for $i = m+1, \ldots n$ & δ_i^0 for $i = 2, \ldots m$

↓


Compute the parameters Ai for i=m+1,...n & Bik for i=1,2,...n; k=1,2,...n (except k=i) from
Set iteration count r=0
A
Set Bus count i=2 & ΔVmax=0
B
Test for type of Bus
PQ Bus
PV Bus
Compute Qi(r+1) from eqn. 18
Qi(r+1) ≤ Qi max
Yes
Is Qi(r+1) ≥ Qi min
Yes
No
No
Replace Qi(r+1) by Qi max
Replace Qi(r+1) by Qi min
Compute Ai(r+1) by eqn. 20
Compute Ai
Compute δi(r+1) using eqn. 19 & Vi(r+1) = |Vi|∠δi(r+1)
Compute Vi(r+1) from eqn. 13
Replace Vi r by by Vi(r+1) & advance bus count i→i+1
Is i≤n
B
Yes
NO


No
Is $\Delta V_{max} \le \epsilon$
No
Advanced iteration count $r = r+1$
A
Yes
Compute slack Bus power $P_1 + jQ_1$ using equ. 4 and all line flows using equ 15