Newton-Raphson Method :

1) Faster than Gauss-Seidal method of load flow
2) It's a powerful method for solving non-linear algebraic eqn.

Consider a set of n non-linear algebric equations.

$$f_i(X) = 0 \qquad i = 1, 2, \ldots n$$

→ no. of non-linear algebric eqn.

$$X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \qquad f_i(x_1, x_2, \ldots x_n) = 0$$

Assume Initial guess for unknowns

$$X^0 = \begin{Bmatrix} x_1^0 \\ x_2^0 \\ \vdots \\ x_n^0 \end{Bmatrix} \qquad \Delta X^0 = \begin{Bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{Bmatrix}$$

ΔX^0 → corresponding corrections → which are being added to initial guess

Algebric eqn. can be written as —

$$f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \ldots x_n^0 + \Delta x_n^0) = 0 \qquad i = 1, 2, \ldots n$$

So for all the eqn., using Taylor series expension we can write

$$f_i(x_1^0, x_2^0, \ldots x_n^0) + \left.\frac{\partial f_i}{\partial x_1}\right|_{X=X^0} \Delta x_1^0 + \left.\frac{\partial f_1}{\partial x_2}\right|_{X=X^0} \Delta x_2^0 + \ldots + \left.\frac{\partial f_i}{\partial x_n}\right|_{X=X^0} \Delta x_n^0 + \cancel{\begin{pmatrix}\text{Higher} \\ \text{order} \\ \text{terms}\end{pmatrix}}^{0} = 0$$

$$\begin{bmatrix} f_1(X^0) \\ f_2(X^0) \\ \vdots \\ f_n(X^0) \end{bmatrix} + \begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{X=X^0} \Delta x_1^0 & \left.\frac{\partial f}{\partial x_2}\right|_{X=X^0} \Delta x_2^0 & \cdots & \left.\frac{\partial f_1}{\partial x_n}\right|_{X=X^0} \Delta x_n^0 \\ \left.\frac{\partial f_2}{\partial x_1}\right|_{X=X^0} \Delta x_1^0 & \left.\frac{\partial f_2}{\partial x_2}\right|_{X=X^0} \Delta x_2^0 & \cdots & \left.\frac{\partial f_2}{\partial x_n}\right|_{X=X^0} \Delta x_n^0 \\ \vdots & & & \\ \left.\frac{\partial f_n}{\partial x_1}\right|_{X=X^0} \Delta x_1^0 & \left.\frac{\partial f_n}{\partial x_2}\right|_{X=X^0} \Delta x_2^0 & \cdots & \left.\frac{\partial f_n}{\partial x_n}\right|_{X=X^0} \Delta x_n^0 \end{bmatrix} = 0$$

$$\begin{bmatrix} f_1(X^0) \\ f_2(X^0) \\ \vdots \\ f_n(X^0) \end{bmatrix} + \begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{X=X^0} & \left.\frac{\partial f_1}{\partial x_2}\right|_{X=X^0} & \cdots & \left.\frac{\partial f_1}{\partial x_n}\right|_{X=X^0} \\ \left.\frac{\partial f_2}{\partial x_1}\right|_{X=X^0} & \left.\frac{\partial f_2}{\partial x_2}\right|_{X=X^0} & \cdots & \left.\frac{\partial f_2}{\partial x_n}\right|_{X=X^0} \\ \vdots & & & \\ \left.\frac{\partial f_n}{\partial x_1}\right|_{X=X^0} & \left.\frac{\partial f_n}{\partial x_2}\right|_{X=X^0} & \cdots & \left.\frac{\partial f_n}{\partial x_n}\right|_{X=X^0} \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \Delta x_3^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix}$$

$F^0 \qquad J^0 \qquad \Delta X^0$

$$\boxed{F^0 + J^0 \Delta X^0 = 0}$$

$$\boxed{F^0 = -J^0 \Delta X^0}$$

→ Jacobian Matrix evaluated by taking Partial derivative w.r.t X(vector) & evaluated at X^0

$$\Delta X^0 = \frac{-F^0}{J^0} = -[J^0]^{-1} F^0$$

After Ist iteration $\quad X^1 = X^0 + \Delta X^0$

After r^{th} iteration $\quad \boxed{X^r = X^{r-1} + \Delta X^{r-1}}$

$$\boxed{\Delta X^{r-1} = -[J^{r-1}]^{-1} F^{r-1}}$$

Corresponding X^r

$$X^r = \begin{Bmatrix} x_1^r \\ x_2^r \\ \vdots \\ x_n^r \end{Bmatrix}$$

These are solutions.

After putting values —

$\{f_1(X^r), f_2(X^r), \ldots f_n(X^r)\}$

tolerance $= \text{Max}\{f_1(X^r), f_2(X^r) \ldots f_n(X^r)\}$

If tor $\leq \varepsilon$ (specified value)
($10^{-3}, 10^{-2}$ ...)

N-R method for load flow :

Consider a power system having n-no. of Bus

1st - slack Bus

2-n - PQ Bus

$$S_i = P_i + jQ_i = V_i I_i^* = V_i \left(\sum_{k=1}^{n} \bar{Y}_{ik} \bar{V}_k \right)^*$$

$$S_i^* = P_i - jQ_i = V_i^* I = V_i^* \sum_{k=1}^{n} \bar{Y}_{ik} \bar{V}_k$$

$$\bar{V}_i = V_i \angle \delta_i$$

$$\bar{V}_k = V_k \angle \delta_k$$

$$S_i^* = P_i - jQ_i = \left[V_i \angle -\delta_i \sum_{k=1}^{n} (Y_{ik} \angle \theta_{ik})(V_k \angle \delta_k) \right]$$

$$= V_i \sum_{k=1}^{n} Y_{ik} V_k \angle (\theta_{ik} + \delta_k - \delta_i)$$

$$P_i = V_i \sum_{k=1}^{n} Y_{ik} V_k \cos(\theta_{ik} + \delta_k - \delta_i) \longrightarrow (1)$$

$$Q_i = -V_i \sum_{k=1}^{n} Y_{ik} V_k \sin(\theta_{ik} + \delta_k - \delta_i) \longrightarrow (2)$$

$\rightarrow P_i = f(\delta, V)$

$\rightarrow Q_i = f(\delta, V)$

Let us introduce two functions -

1) corresponding to P

2) corresponding to Q.

$$f_i^{\circ} = \Delta P_i^{\circ} = \underset{\substack{\downarrow \\ \text{specified} \\ \text{value}}}{P_i^{s}} - \underbrace{P_i^{\circ}(\delta_2, \delta_3 \ldots \delta_n, V_2, V_3 \ldots V_n)}_{\text{calculated value}}$$

$$g_i^{\circ} = \Delta Q_i^{\circ} = Q_i^{s} - Q_i^{\circ}(\delta_2, \delta_3 \ldots \delta_n, V_2, V_3 \ldots V_n)$$

$$F^{r-1} = -\left[J^{r-1}\right]\left[\Delta X^{r-1}\right]$$

For i^{th} PQ Bus

$$\begin{bmatrix} \Delta P_i^{\circ} \\ \Delta Q_i^{\circ} \end{bmatrix}_{2\times 1} = -\begin{bmatrix} \frac{\partial \Delta P_i^{\circ}}{\partial \delta_1} & \frac{\partial \Delta P_i^{\circ}}{\partial \delta_2} & \cdots & \frac{\partial \Delta P_i^{\circ}}{\partial \delta_n} & \frac{\partial \Delta P_i^{\circ}}{\partial V_1}, & \frac{\partial \Delta P_P}{\partial V_2} & \cdots & \frac{\partial \Delta P_i^{\circ}}{\partial V_n} \\ \frac{\partial \Delta Q_i^{\circ}}{\partial \delta_1} & \frac{\partial \Delta Q_i^{\circ}}{\partial \delta_2} & \cdots & \frac{\partial \Delta Q_i^{\circ}}{\partial \delta_n} & \frac{\partial \Delta Q_i}{\partial V_1} & \frac{\partial \Delta Q_i}{\partial V_2} & \cdots & \frac{\partial \Delta Q_i^{\circ}}{\partial V_n} \end{bmatrix}_{2\times 2(n-1)} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \vdots \\ \Delta\delta_n \\ \Delta V_2 \\ \Delta V_3 \\ \vdots \\ \Delta V_n \end{bmatrix}_{2(n-1)\times 1}$$

For All PQ Buses

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix}$$

Where

$$\Delta P = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \end{bmatrix}_{(n-1)\times 1} \qquad \Delta Q = \begin{bmatrix} \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix}_{(n-1)\times 1}$$

$$\Delta\delta = \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \vdots \\ \Delta\delta_3 \end{bmatrix}_{(n-1)\times 1} \qquad \Delta V = \begin{bmatrix} \Delta V_2 \\ \Delta V_3 \\ \vdots \\ \Delta V_n \end{bmatrix}_{(n-1)\times 1}$$

$$J_{11} = \left[\frac{\partial P}{\partial \delta}\right]$$

$$\frac{\partial \Delta P_i}{\partial \delta_j} = \frac{\partial}{\partial \delta_j}\left(\overset{\text{differenti}}{P_i^s} - P_i(\delta, V)\right)$$

$$\boxed{\frac{\partial \Delta P_i}{\partial \delta_j} = -\frac{\partial P_i(\delta,V)}{\partial \delta_j}}$$

$$\frac{\partial \Delta Q_i}{\partial \delta_j} = \frac{\partial}{\partial \delta_j}\left(Q_i^s - Q_i(\delta, V)\right)$$

$$\boxed{\frac{\partial \Delta Q_i}{\partial \delta_j} = -\frac{\partial Q_i(\delta,V)}{\partial \delta_j}}$$

similarly

$$\boxed{\frac{\partial \Delta Q_i}{\partial V_j} = -\frac{\partial Q_i}{\partial V_j}}$$

$$\boxed{\frac{\partial \Delta P_i}{\partial V_j} = -\frac{\partial P_i}{\partial V_j}}$$

$$J_{11} = \left[\frac{\partial P}{\partial \delta}\right] = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \cdots & \frac{\partial P_2}{\partial \delta_n} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \cdots & \frac{\partial P_3}{\partial \delta_n} \\ \vdots & & & \\ \frac{\partial P_n}{\partial \delta_2} & \frac{\partial P_n}{\partial \delta_3} & \cdots & \frac{\partial P_n}{\partial \delta_n} \end{bmatrix}_{(n-1)\times(n-1)}$$

$$J_{12} = \left[\frac{\partial P}{\partial V}\right] = \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_3} & \cdots & \frac{\partial P_2}{\partial V_n} \\ \frac{\partial P_3}{\partial V_2} & \frac{\partial P_3}{\partial V_3} & \cdots & \frac{\partial P_3}{\partial V_n} \\ \vdots & & & \\ \frac{\partial P_n}{\partial V_2} & \frac{\partial P_n}{\partial V_3} & \cdots & \frac{\partial P_n}{\partial V_n} \end{bmatrix}$$

$$J_{21} = \left[\frac{\partial Q}{\partial \delta}\right] = \begin{bmatrix} \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \cdots & \frac{\partial Q_2}{\partial \delta_n} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \cdots & \frac{\partial Q_3}{\partial \delta_n} \\ \vdots \\ \frac{\partial Q_n}{\partial \delta_2} & \frac{\partial Q_n}{\partial \delta_3} & \cdots & \frac{\partial Q_n}{\partial \delta_n} \end{bmatrix}_{(n-1)\times(n-1)}$$

$$J_{22} = \left[\frac{\partial Q}{\partial V}\right] = \begin{bmatrix} \frac{\partial Q_2}{\partial V_2} & \frac{\partial Q_2}{\partial V_3} & \cdots & \frac{\partial Q_2}{\partial V_n} \\ \frac{\partial Q_3}{\partial V_2} & \frac{\partial Q_3}{\partial V_3} & \cdots & \frac{\partial Q_3}{\partial V_n} \\ \vdots \\ \frac{\partial Q_n}{\partial V_2} & \frac{\partial Q_n}{\partial V_3} & \cdots & \frac{\partial Q_n}{\partial V_n} \end{bmatrix}_{(n-1)\times(n-1)}$$

$$P_i = V_i \sum_{k=1}^{n} Y_{ik} V_k \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$\left.\frac{\partial P_i}{\partial \delta_j}\right|_{i\neq j} = \frac{\partial}{\partial \delta_j}\left[V_i \sum_{k=1}^{n} Y_{ik} V_k \cos(\theta_{ik} + \delta_k - \delta_i)\right]$$

$$= \frac{\partial}{\partial \delta_j}\Big[V_i \big\{ Y_{i1} V_1 \cos(\theta_{i1} + \delta_1 - \delta_i) + Y_{i2} V_2 \cos(\theta_{i2} + \delta_2 - \delta_i) + \cdots$$

$$\cdots + Y_{ij} V_j \cos(\theta_{ij} + \delta_j - \delta_i) + \cdots + Y_{in} V_n \cos(\theta_{in} + \delta_n - \delta_i) \big\}\Big]$$

$$= V_i \left\{ 0 + 0 \cdots + \left(-Y_{ij} V_j \sin(\theta_{ij} + \delta_j - \delta_i)\right) + 0 \cdots + 0 \right\}$$

$$\boxed{\left.\frac{\partial P_i}{\partial \delta_j}\right|_{i\neq j} = -V_i Y_{ij} V_j \sin(\theta_{ij} + \delta_j - \delta_i)}$$

(31)

$$\left.\frac{\partial P_i}{\partial \delta_i}\right|_{i=j} = \frac{\partial P_i}{\partial \delta_i} = \frac{\partial}{\partial \delta_i}\left[V_i \sum_{k=1}^{n} Y_{ik} V_k \cos(\theta_{ik}+\delta_k-\delta_i)\right]$$

$$= \frac{\partial}{\partial \delta_i}\Big[V_i \Big\{ Y_{i1} V_1 \cos(\theta_{i1}+\delta_1-\delta_i) + Y_{i2} V_2 \cos(\theta_{i2}+\delta_2-\delta_i) + \cdots$$

$$\cdots + Y_{ii} V_i \cos(\theta_{ii}+\delta_i-\delta_i) + \cdots + Y_{in} V_n \cos(\theta_{in}+\delta_n-\delta_i)\Big\}\Big]$$

$$= V_i \Big\{ Y_{i1} V_1 \sin(\theta_{i1}+\delta_1-\delta_i) + Y_{i2} V_2 \sin(\theta_{i2}+\delta_2-\delta_i) + \cdots + 0 + Y_{in} V_n \sin(\theta_{in}+\delta_n-\delta_i)\Big\}$$

$$= V_i \sum_{\substack{k=1 \\ k\neq i}}^{n} Y_{ik} V_k \sin(\theta_{ik}+\delta_k-\delta_i)$$

$$= V_i \sum_{k=1}^{n} Y_{ik} V_k \sin(\theta_{ik}+\delta_k-\delta_i) - Y_{ii} V_i^2 \sin(\theta_{ii}+\delta_i-\delta_i)$$

$$= -Q_i - Y_{ii} V_i^2 \sin(\theta_{ii})$$

$$\boxed{\frac{\partial P_i}{\partial \delta_i} = -Q_i - V_i^2 B_{ii}}$$

$$\bar{Y}_{ii} = Y_{ii} \angle \theta_{ii}$$
$$= Y_{ii}\cos\theta_{ii} + j Y_{ii}\sin\theta_{ii}$$
$$= G_{ii} + j B_{ii}$$

J_{12}

$$\boxed{\left.\frac{\partial P_i}{\partial V_j}\right|_{i\neq j} = V_i Y_{ij} \cos(\theta_{ij}+\delta_j-\delta_i)}$$

$$\left.\frac{\partial P_i}{\partial V_j}\right|_{i=j} = \sum_{k=1}^{n} Y_{ik} V_k \cos(\theta_{ik}+\delta_i-\delta_i) + Y_{ii} V_i \cos\theta_{ii}$$

$$\boxed{\frac{\partial P_i}{\partial V_i} = \frac{P_i}{V_i} + V_i G_{ii}}$$

$$\boxed{\left.\frac{\partial Q_i}{\partial \delta_j}\right|_{i \neq j} = -V_i Y_{ij} V_j \cos(\theta_{ij} + \delta_j - \delta_i)}$$

$$\left.\frac{\partial Q_i}{\partial \delta_i}\right|_{i=j} = V_i \sum_{\substack{k=1 \\ i \neq j}}^{n} Y_{ik} V_k \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$= V_i \sum_{k=1}^{n} Y_{ik} V_k \cos(\theta_{ik} + \delta_k - \delta_i) - V_i^2 Y_{ii} \cos(\theta_{ii})$$

$$\boxed{\frac{\partial Q_i}{\partial \delta_i} = P_i - V_i^2 G_{ii}}$$

$$\boxed{\left.\frac{\partial Q_i}{\partial V_j}\right|_{i \neq j} = -V_i Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i)}$$

$$\left.\frac{\partial Q_i}{\partial V_j}\right|_{i=j} = -\sum_{k=1}^{n} Y_{ik} V_k \sin(\theta_{ik} + \delta_i - \delta_i) - V_i Y_{ii} \sin \theta_{ii}$$

$$\boxed{\frac{\partial Q_i}{\partial V_i} = \frac{Q_i}{V_i} - V_i B_{ii}}$$

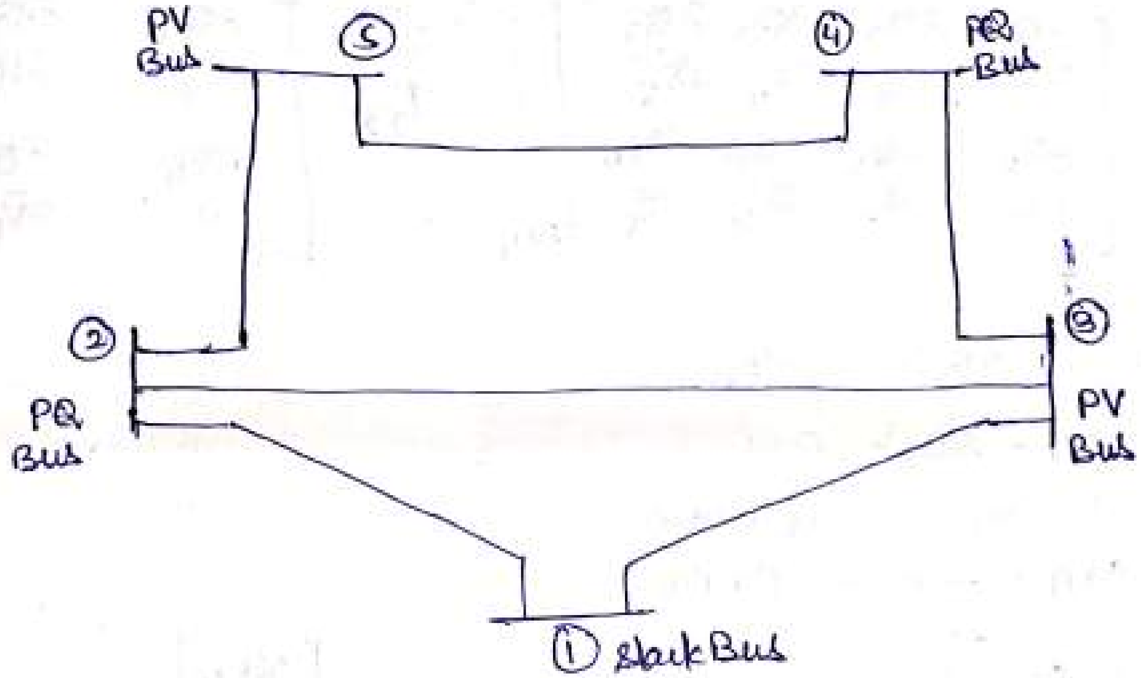
When PV Buses are available

$$\underset{\text{LHS}}{\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \underset{\text{RHS}}{\begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix}}$$

for PV Buses

i) P is specified
Q is not specified
$\Rightarrow$ ΔQ will not appear on LHS

ii) V is specified
ΔV will not appear on RHS



Bus No.	Known (specified)	Unknown (Non-specified)
2	$P_2 Q_2$	$\delta_2 V_2$
3	$P_3 V_3$	$\delta_3 Q_3$
4	$P_4\ Q_4$	$\delta_4\ V_4$
5	$P_5\ V_5$	$\delta_5\ Q_5$

$$\Delta P = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \end{bmatrix} \quad \Delta Q = \begin{bmatrix} \Delta Q_2 \\ \Delta Q_4 \end{bmatrix} \quad \Delta \delta = \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta \delta_4 \\ \Delta \delta_5 \end{bmatrix} \quad \Delta V = \begin{bmatrix} \Delta V_2 \\ \Delta V_4 \end{bmatrix}$$

$$J_{11} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial \delta_4} & \frac{\partial P_2}{\partial \delta_5} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial \delta_4} & \frac{\partial P_3}{\partial \delta_5} \\ \frac{\partial P_4}{\partial \delta_2} & \frac{\partial P_4}{\partial \delta_3} & \frac{\partial P_4}{\partial \delta_4} & \frac{\partial P_4}{\partial \delta_5} \\ \frac{\partial P_5}{\partial \delta_2} & \frac{\partial P_5}{\partial \delta_3} & \frac{\partial P_5}{\partial \delta_4} & \frac{\partial P_5}{\partial \delta_5} \end{bmatrix}_{4\times 4} \qquad J_{12} = \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_4} \\ \frac{\partial P_3}{\partial V_2} & \frac{\partial P_3}{\partial V_4} \\ \frac{\partial P_4}{\partial V_2} & \frac{\partial P_4}{\partial V_4} \\ \frac{\partial P_5}{\partial V_2} & \frac{\partial P_5}{\partial V_4} \end{bmatrix}_{4\times 2}$$

$$J_{21} = \begin{bmatrix} \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial \delta_4} & \frac{\partial Q_2}{\partial \delta_5} \\ \frac{\partial Q_4}{\partial \delta_2} & \frac{\partial Q_4}{\partial \delta_3} & \frac{\partial Q_4}{\partial \delta_4} & \frac{\partial Q_4}{\partial \delta_5} \end{bmatrix}_{2\times 4} \qquad J_{22} = \begin{bmatrix} \frac{\partial Q_2}{\partial V_2} & \frac{\partial Q_2}{\partial V_4} \\ \frac{\partial Q_4}{\partial V_2} & \frac{\partial Q_4}{\partial V_4} \end{bmatrix}_{2\times 2}$$

Generalised n-Bus System

1 - slack Bus

2 – m → PQ Buses

(m+1) – n → PV Buses

$$\Delta P = \begin{bmatrix} \left.\begin{matrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_m \end{matrix}\right\} \text{for PQ Buses} \\ \hdashline \left.\begin{matrix} \Delta P_{m+1} \\ \Delta P_{m+2} \\ \vdots \\ \Delta P_n \end{matrix}\right\} \text{for PV Buses} \end{bmatrix}_{(n-1)\times 1} \qquad \Delta Q = \begin{bmatrix} \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_m \end{bmatrix}_{(m-1)\times 1} \text{for PQ Buses}$$

$$\Delta V = \begin{bmatrix} \Delta V_2 \\ \Delta V_3 \\ \vdots \\ \Delta V_m \end{bmatrix}_{(m-1)\times 1} \qquad \Delta \delta = \begin{bmatrix} \left.\begin{matrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \vdots \\ \Delta P_m \end{matrix}\right\} \text{PQ Buses} \\ \hdashline \left.\begin{matrix} \Delta P_{m+1} \\ \Delta P_{m+2} \\ \vdots \\ \Delta P_n \end{matrix}\right\} \text{PV Buses} \end{bmatrix}_{(n-1)\times 1}$$

$$J_{11} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_m} & \frac{\partial P_2}{\partial \delta_{m+1}} & \cdots & \frac{\partial P_2}{\partial \delta_n} \\ \vdots & & & & & \\ \frac{\partial P_m}{\partial \delta_2} & \cdots & \frac{\partial P_m}{\partial \delta_m} & \frac{\partial P_m}{\partial \delta_{m+1}} & \cdots & \frac{\partial P_m}{\partial \delta_n} \\ \vdots & & & & & \\ \frac{\partial P_n}{\partial \delta_2} & \cdots & \frac{\partial P_n}{\partial \delta_m} & \frac{\partial P_n}{\partial \delta_{m+1}} & \cdots & \frac{\partial P_n}{\partial \delta_n} \end{bmatrix}_{(n-1)\times(n-1)}$$

$$J_{12} = \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \cdots & \frac{\partial P_2}{\partial V_m} \\ \vdots & & \vdots \\ \frac{\partial P_m}{\partial V_2} & \cdots & \frac{\partial P_m}{\partial V_m} \\ \vdots & & \vdots \\ \frac{\partial P_m}{\partial V_2} & \cdots & \frac{\partial P_n}{\partial V_m} \end{bmatrix}_{(n-1)\times(m-1)} \qquad J_{21} = \begin{bmatrix} \frac{\partial Q_2}{\partial \delta_2} & \cdots & \frac{\partial Q_2}{\partial \delta_n} \\ \vdots & & \vdots \\ \frac{\partial Q_m}{\partial \delta_2} & \cdots & \frac{\partial Q_m}{\partial \delta_n} \end{bmatrix}_{(m-1)\times(n-1)}$$

$$J_{22} = \begin{bmatrix} \frac{\partial Q_2}{\partial V_2} & \cdots & \frac{\partial Q_2}{\partial V_m} \\ \vdots & & \vdots \\ \frac{\partial Q_m}{\partial V_2} & \cdots & \frac{\partial Q_m}{\partial V_m} \end{bmatrix}_{(m-1)\times(m-1)}$$

$$\underset{(m-1)\times 1}{\overset{(n-1)\times 1}{\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}}} = \begin{bmatrix} \overset{(n-1)\times(n-1)}{J_{11}} & \overset{(n-1)(m-1)}{J_{12}} \\ \underset{(m-1)\times(n-1)}{J_{21}} & \underset{(m-1)\times(m-1)}{J_{22}} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} \begin{matrix} \rightarrow (n-1)\times 1 \\ \rightarrow (m-1)\times 1 \end{matrix}$$

dimension of Jacobian = $(n+m-2)\times(n+m-2)$

For PV Bus.

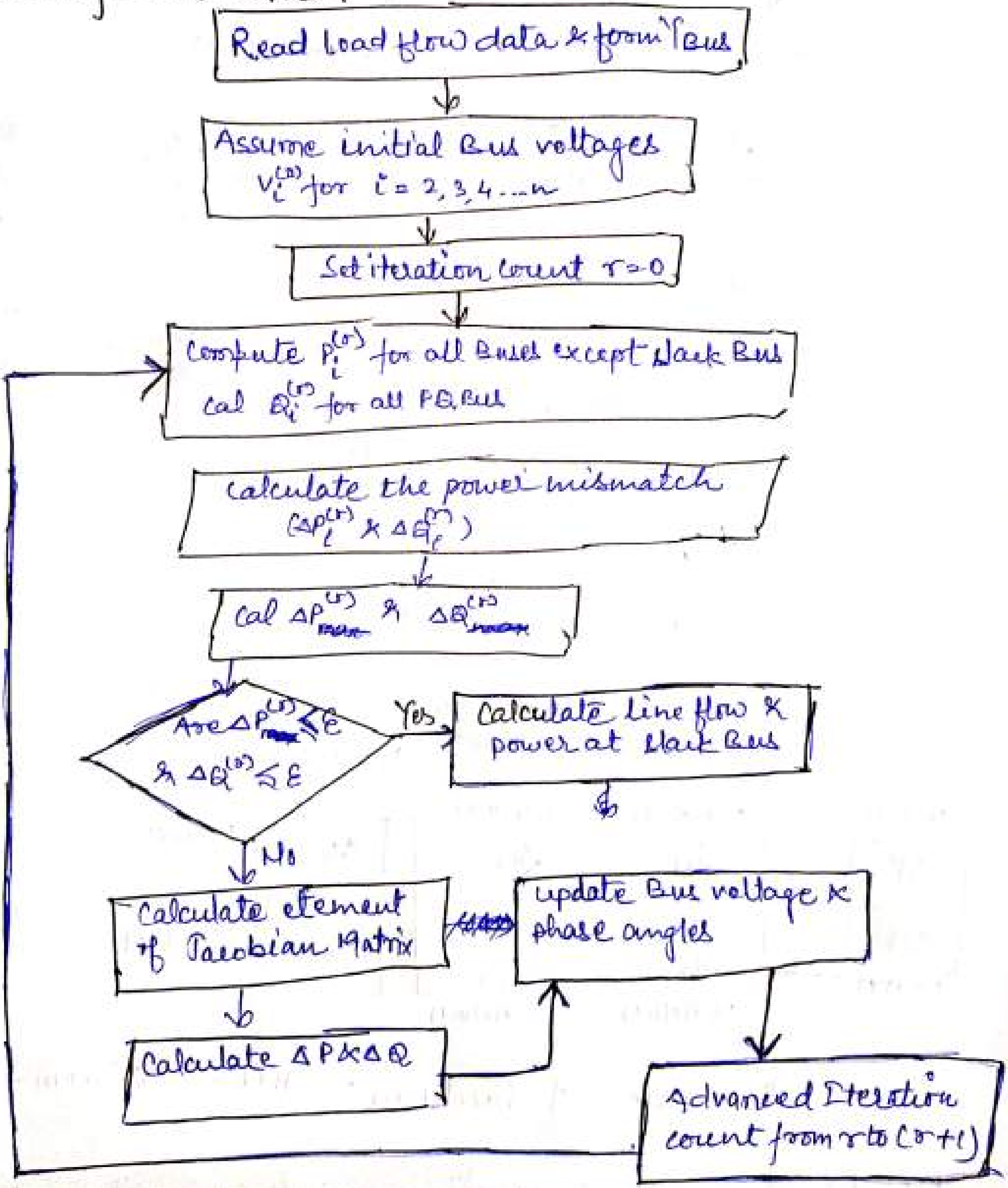
$$Q_i^r = -V_i \sum_{k=1}^{n} Y_{ik} V_k^r \sin(\theta_{ik} + \delta_k^r - \delta_i^r)$$

If $Q_{i\,min} \leq Q_i^r \leq Q_{i\,max} \longrightarrow$ PV Bus

else $\longrightarrow$ PQ Bus

$$V_i^r = V_i^{r-1} + \Delta V_i^{r-1}$$

Flow-chart for NR Method



Qus. Consider the three Bus system as shown in fig. Each of the three lines has a series impedance of $0.02+j0.08$ pu and total shunt admittance of 0.02 pu. The specified quantities at the buses are tabulated below:

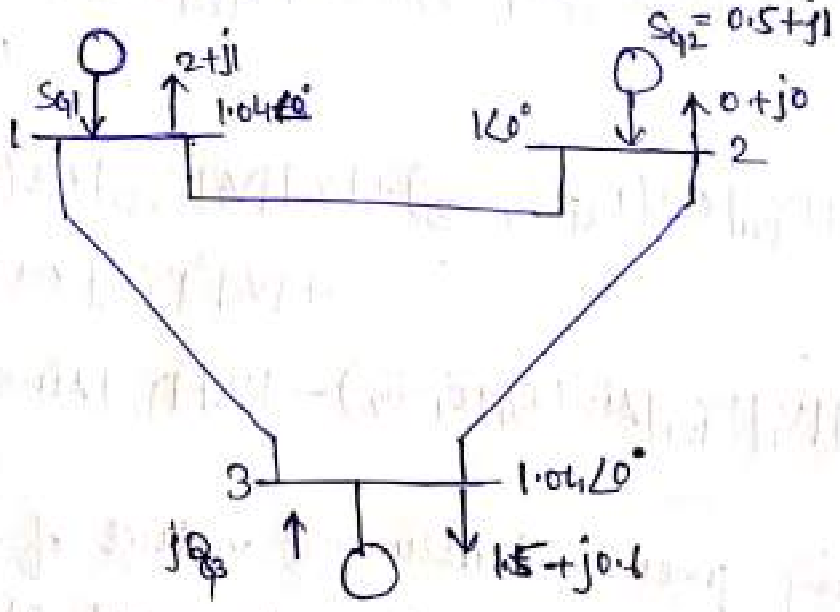
Bus	Real load Demand P_D	Reactive load Demand Q_D	Real Power Generation P_G	Reactive Power Gen. Q_G	Voltage specification
1	2.0	1.0	Unspecified	Unspecified	$V_1 = 1.04+j0$ slack Bus
2	0.0	0.0	0.5	1.0	Unspecified (PQ Bus)
3	1.5	0.6	0.0	$Q_{G3} = ?$	$\lvert V_3\rvert = 1.04$ PV Bus

Controllable reactive Power source is available at bus 3 with the constraint

$$0 \leq Q_{G3} \leq 1.5 \text{ pu.}$$

Find the load flow solution using the NR method. Use a tolerance of 0.01 for power mismatch.

Sol.



Using the nominal-π model for transmission lines, Y_{Bus} for the given system is obtained as follows:

For each line

$$y_{series} = \frac{1}{0.02+j0.08} = 2.941 - j11.764$$
$$= 12.13\angle{-75.96^\circ}$$

Each off-diagonal term $= -2.941 + j11.764$

Each self term $= 2[(2.941 - j11.764) + j0.01]$
$$= 5.882 - j23.528$$
$$= 24.23\angle{-75.95^\circ}$$

$$Y_{Bus} = \begin{bmatrix} 24.23\angle{-75.95^\circ} & 12.13\angle{104.04^\circ} & 12.13\angle{104.04^\circ} \\ 12.13\angle{104.04^\circ} & 24.23\angle{-75.95^\circ} & 12.13\angle{104.04^\circ} \\ 12.13\angle{104.04^\circ} & 12.13\angle{104.04^\circ} & 24.23\angle{-75.95^\circ} \end{bmatrix}$$

To start iteration choose $V_2^0 = 1+j0$ & $\delta_3^0 = 0$

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21}+\delta_1-\delta_2) + |V_2|^2|Y_{22}|\cos\theta_{22} + |V_2||V_3||Y_{23}|\cos(\theta_{23}+\delta_3-\delta_2)$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31}+\delta_1-\delta_3) + |V_3||V_2||Y_{32}|\cos(\theta_{32}+\delta_2-\delta_3) + |V_3|^2|Y_{33}|\cos\theta_{33}$$

$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21}+\delta_1-\delta_2) - |V_2|^2|Y_{22}|\sin\theta_{22} - |V_2||V_3||Y_{23}|\sin(\theta_{23}+\delta_2-\delta_3)$$

Substituting given and assumed values of different quantities, we get the values of powers as

$$P_2^0 = 1 \times 1.04 \times 12.13 \cos(104.04 + 0 - 0) + (1)^2 \times 24.23 \cos(-75.95)$$
$$+ 1 \times 1.04 \times 12.13 \cos(104.04)$$

$$= -3.06 + 5.88 - 3.06$$

$$= -0.24 \text{ pu.}$$

$$P_3^0 = \{1.04 \times 1.04 \times 12.13 \cos(104.04 + 0 - 0\} + \{1.04 \times 1 \times 12.13 \cos(104.04)\}$$
$$+ \{(1.04)^2 \times 24.23 \cos(-75.95)\}$$

$$= -3.18 - 3.06 + 6.36$$

$$= 0.12 \text{ pu.}$$

similarly.

$$Q_2^0 = -0.96 \text{ pu.}$$

Power Residuals

$$\Delta P_2^0 = P_2^s - P_2^0$$
$$= 0.5 - (-0.24) = 0.73$$
$$\Delta P_3^0 = -1.5 - 0.12 = -1.62$$
$$\Delta Q_2^0 = 1 - (-.96) = 1.96$$

Changes in variables at the end of the first iteration are obtained as —

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}$$

Jacobian elements can be evaluated by differentiating the expressions given above for P_2, P_3, Q_2 w.r.t. δ_2, δ_3 & $|V_2|$

$$\begin{bmatrix} \Delta\delta_2^1 \\ \Delta\delta_3^1 \\ \Delta|V_2|^1 \end{bmatrix} = \begin{bmatrix} 24.47 & -12.23 & 5.64 \\ -12.23 & 24.95 & -3.05 \\ -6.11 & 3.05 & 22.54 \end{bmatrix}^{-1} \begin{bmatrix} 0.73 \\ -1.62 \\ 1.96 \end{bmatrix} = \begin{bmatrix} -0.023 \\ -0.0654 \\ -0.089 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^1 \\ \delta_3^1 \\ |V_2^1| \end{bmatrix} = \begin{bmatrix} \delta_2^0 \\ \delta_3^0 \\ |V_2|^0 \end{bmatrix} + \begin{bmatrix} \Delta\delta_2^1 \\ \Delta\delta_3^1 \\ \Delta|V_2|^1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.023 \\ -0.0654 \\ 0.089 \end{bmatrix} = \begin{bmatrix} -0.023 \\ -0.0654 \\ 1.089 \end{bmatrix}$$

$Q_3^1 = 0.4677^{0.24}$ (By formula)

$Q_{G3}^1 = Q_3^1 + Q_{D3} = 0.4677 + 0.6 = 1.0677$

which is within limit