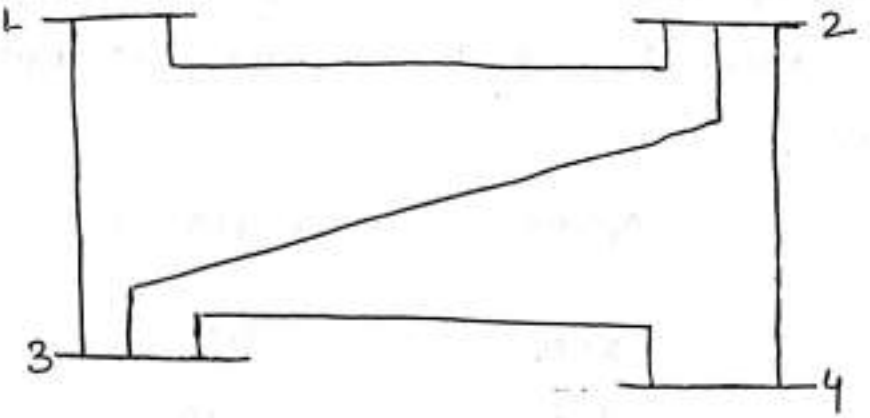
Qu



Line Bus to Bus	R (PU)	X (PU)
1–2	0.05	0.5
1–3	0.10	0.30
2–3	0.15	0.45
2–4	0.10	0.30
3–4	0.05	0.15

For the sample system shown in figure, the generators are connected at all the four buses, while loads are at buses 2 and 3. Values of real and reactive power are listed in table. All buses other than the slack are PQ type. Assuming a flat voltage start, find the voltages and bus angles at the three buses at the end of the first GS iteration.

Input data

Bus	P_i (PU)	Q_i (PU)	V_i (PU)	Remarks
1.	—	—	$1.04\angle 0^\circ$	Slack Bus
2.	0.5	−0.2	—	PQ Bus
3.	−1.0	0.5	—	PQ Bus
4.	0.3	−0.1	—	PQ Bus.

Sol. Calculation of Y_{BUS}.

Change the value of impedances b/w each buses in admittance.

Line Bus to Bus	G(PU)	B(PU)
1-2	2.0	-6.0
1-3	1.0	-3.0
2-3	0.666	-2.0
2-4	1.0	-3.0
3-4	2.0	-6.0

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} y_{12}+y_{13} & -y_{12} & -y_{13} & 0 \\ -y_{21} & y_{21}+y_{23}+y_{24} & -y_{23} & y_{24} \\ -y_{31} & -y_{32} & y_{31}+y_{32}+y_{34} & -y_{34} \\ 0 & -y_{42} & -y_{43} & y_{42}+y_{43} \end{bmatrix}$$

$$= \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

Bus voltages at the end of the first iteration are calculated using equ. (13)

$$V_2^1 = \frac{1}{Y_{22}} \left\{ \frac{P_2 - jQ_2}{(1-j0)} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0 \right\}$$

$\hookrightarrow (V_2^0)^*$

$$= \frac{1}{Y_{22}} \left\{ \frac{0.5+j0.2}{1-j0} - 1.04(-2+j6) - (-0.666+j2)(1+j0) - \frac{(-1+j3)}{(1+j0)} \right\}$$

$$= \frac{1}{(3.666-j11)}\left\{\frac{0.5+j0.2}{1} - 1.04(-2+j6) + 0.666-j2+1-j3\right\}$$

$$= \frac{1}{(3.666-j11)}\left\{0.5+j0.2+2.08-j6.24+1.666-j5\right\}$$

$$= \frac{1}{(3.666-j11)}(4.246-j11.04)$$

$$= 1.019+j0.046 \text{ P.U.}$$

$$V_3' = \frac{1}{Y_{33}}\left\{\frac{P_3-jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_2' - Y_{34}V_4^0\right\}$$

→ Always same due to slack Bus

$$= \frac{1}{Y_{33}}\left\{\frac{-1-j0.5}{1-j0} - 1.04(-1+j3) - (-0.666+j2)(1.019+j0.046) - (-2+j6)(1+j0)\right\}$$

$$= \frac{2.01-j11.627}{3.666-j11} = (1.028-j0.087) \text{ P.U.}$$

$$V_4' = \frac{1}{Y_{44}}\left\{\frac{P_4-jQ_4}{(V_4^0)^*} - Y_{41}V_1^0 - Y_{42}V_2' - Y_{43}V_3'\right\}$$

$$= \frac{1}{Y_{44}}\left\{\frac{0.3+j0.1}{1-j0} - (-1+j3)(1.019+j0.046) - (-2+j6)(1.028-j0.087)\right\}$$

$$= \frac{2.991-j9.253}{3-j9} = 1.025-j0.0053 \text{ P.U.}$$

Ques- In above example, let Bus 2 be PV bus now with $|V_2|=1.04$ p.u. Once again assuming a flat voltage start, find Q_2, δ_2, V_3, V_4 at the end of the first GS iteration. (Given: $0.2 \le Q_2 \le 1$)

Note $\delta_2^\circ = 0$ i.e. $V_2^\circ = 1.04 + j0$

from equ (18)

$$Q_2' = -Im\{(V_2^\circ)^* Y_{21} V_1' + (V_2^\circ)^* [Y_{22} V_2^\circ + Y_{23} V_3^\circ + Y_{24} V_4^\circ]\}$$

$$= -Im\{1.04(-2+j6)1.04 + 1.04[(3.666-j11)1.04 + (-0.666+j2) + (-1+j3)]\}$$

$$= -Im\{-0.0693 - j0.2079\}$$

$$Q_2' = 0.2079 \text{ pu}$$

from equ (19)

$$\delta_2' = \angle\left\{\frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2'}{(V_2^\circ)^*} - Y_{21}V_1' - Y_{23}V_3^\circ - Y_{24}V_4^\circ\right]\right\}$$

$$= \angle\left\{\frac{1}{3.666-j11}\left[\frac{0.5-j0.2079}{1.04-j0} - (-2+j6)(1.04+j0) - (-0.666+j2)(1+j0) - (-1+j3)(1+j0)\right]\right\}$$

$$= \angle\left(\frac{4.2267 - j11.439}{3.66-j11}\right) = \angle(1.0512 + j0.0339)$$

or $\delta_2' = 1.8465^\circ = 0.032$ rad.

$$V_2' = 1.04(\cos\delta_2' + j\sin\delta_2')$$

$$= 1.04(0.99948 + j0.0322)$$

$$V_3' = \frac{1}{Y_{33}}\left\{\frac{P_3 - jQ_3}{(V_3^\circ)^*} - Y_{31}V_1' - Y_{32}V_2' - Y_{34}V_4^\circ\right\}$$

$$= \frac{1}{3.666 - j11}\left[\frac{-1-j0.5}{1-j0} - (-1+j3)1.04 - (-0.666+j2)(1.0394 + j0.0335) - (-2+j6)\right]$$

$$= \frac{2.7992 - j11.6766}{3.666 - j11} = 1.0317 - j0.08937$$

$$V_4^1 = \frac{1}{Y_{44}}\left\{\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 - Y_{42}V_2^1 - Y_{43}V_3^1\right\}$$

$$= \frac{1}{3-j9}\left[\frac{0.3+j0.1}{1-j0} - (-1+j3)(1.0394 + j0.0335) - (-2+j6)(1.0317 - j0.08937)\right]$$

$$= \frac{2.9671 - j8.9962}{3-j9} = 0.9985 - j0.0031$$

When Q is not in range

Now suppose the permissible limits on Q_2 (reactive power injection) are revised as follows:

$$0.25 \leq Q_2 \leq 1.0 \text{ pu.}$$

It is clear, that other data remaining the same, the calculated Q_2 (= 0.2079) is now less than the $Q_{2,min}$. Hence Q_2 is set equal to $Q_{2,min}$ i.e.

$$Q_2 = 0.25 \text{ pu.}$$

Bus 2, therefore, becomes a PQ bus from a PV Bus. Therefore, $|V_2|$ can no longer remain fixed at 1.04 pu. The value of V_2 at the end of the first iteration is calculated as follows- (Note $V_2^0 = 1+j0$ by virtue of flat start)

$$V_2^1 = \frac{1}{Y_{22}}\left(\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right)$$

$$= \frac{1}{3.666 - j11}\left[\frac{0.5 - j0.25}{1 - j0} - (-2+j6)1.04 - (-0.666+j2) - (-1+j3)\right]$$

$$= \frac{4.246 - j11.49}{3.666 - j11} = 1.0559 + j0.0341$$

$$V_3^1 = \frac{1}{Y_{33}}\left(\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0\right)$$

$$= \frac{1}{3.666 - j11}\left[\frac{-1 - j0.5}{1 - j0} - (-1+j3)1.04 - (-0.666+j2)(1.0559 + j0.0341) - (-2+j6)\right]$$

$$= \frac{2.8112 - j11.709}{3.666 - j11} = 1.0347 - j0.0893 \text{ pu.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left(\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 - Y_{42}V_2^1 - Y_{43}V_3^1\right)$$

$$= \frac{1}{3 - j9}\left[\frac{0.3 + j0.1}{1 - j0} - (-1+j3)(1.0509 + j0.0341) - (-2+j6)(1.0347 - j0.0893)\right]$$

$$= \frac{4.0630 - j9.4204}{3 - j9} = 1.0775 + j0.0923 \text{ pu.}$$