For the students of

M. Com. (Applied Economics) Sem. IV

Paper: Research Methodology (Unit IV)

Note: Study material may be useful for the courses wherever Research Methodology paper is being taught.

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Topic: Chi Square- Test

The χ^2 test (pronounced as chi-square test) is an important and popular test of hypothesis which fall is categorized in non-parametric test. This test was first introduced by Karl Pearson in the year 1900.

It is used to find out whether there is any significant difference between observed frequencies and expected frequencies pertaining to any particular phenomenon. Here frequencies are shown in the different cells (categories) of a so-called contingency table. It is noteworthy that we take the observations in categorical form or rank order, but not in continuation or normal distribution.

The test is applied to assess how likely the observed frequencies would be assuming the null hypothesis is true.

This test is also useful in ascertaining the independence of two random variables based on observations of these variables.

This is a non parametric test which is being extensively used for the following reasons:

1. This test is a Distribution free method, which does not rely on assumptions that the data are drawn from a given parametric family of probability distributions.

2. This is easier to compute and simple enough to understand as compared to parametric test.

3. This test can be used in the situations where parametric test are not appropriate or measurements prohibit the use of parametric tests.

It is defined as:

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Where O refers to the observed frequencies and E refers to the expected frequencies.

Uses of Chi-Square Test

Chi Square test has a large number of applications where paremertic tests can not be applied. Their uses can be summarized as under along with examples:

(a) A test of independence.

This test is helpful in detecting the association between two or more attributes. Suppose we have N observations classified according to two attributes. By applying this test on the given observations (data) we try to find out whether the attributes have some association or they are independent. This association may be positive, negative or absence of association. For example we can find out whether there is any association between regularity in class and division of passing of the students, similarly we can find out whether quinine is effective in controlling fever or not. In order to test whether or not the attributes are associated we take the null hypothesis that there is no association in the attributes under study. In other words, the two attributes are independent.

After computing the value of chi square, we compare the calculated value with its corresponding critical value for the given degree of freedom at a certain level of significance. If calculate value of χ^2 is less than critical or table value, null hypothesis is said to be accepted and it is concluded that two attributes have no association that means they are independent. On the other hand, if the calculated value is greater than the table value, it means that the results of the experiment do not support the hypothesis and hypothesis is rejected, and it is concluded that the attributes are associated.

Illustration 1: From the data given in the following table, find out whether there is any relationship between gender and the preference of colour.

Colour	Male	Female	Total
Red	25	45	70
Blue	45	25	70
Green	50	10	60
Total	120	80	200

(Given : For ν =2, $\chi^2_{\ 0.05}$ = 5.991)

Solution: Let us take the following hypothesis:

Null Hypothesis H_0: There is no relationship between gender and preference of colour.

Alternative Hypothesis H_a: There is relationship between gender and preference of colour.

We have to first calculate the expected value for the observed frequencies. These are shown below along with the observed frequencies:

Colour	Gender	O	E	O-E	$(O-E)^2$	$(O-E)^2/E$
Red	M F	25 45	42 28	-17 17	289 289	6.88 10.32
Blue	M F	45 25	42 28	3 -3	9 9	0.21 0.32
Green	M F	50 10	36 24	14 -14	196 196	5.44 8.16
						χ^2= 31.33

The degree of freedom are (r-1) (c-1) = (3-1) (2-1) = 2.

The critical value of χ^2 for 2 degrees of freedom at 5% level of significance is 5.991.

Since the calculated χ^2 =31.33 exceeds the critical value of χ^2 , the null hypothesis is rejected. Hence, the conclusion is that there is a definite relationship between gender and preference of colour.

(B) A test of goodness of fit

It is the most important utility of the Chi Square test. This method is mainly used for testing of goodness of fit. It attempts to set up whether an observed frequency distribution differs from an estimated frequency distribution. When an ideal frequency curve whether normal or some other type is fitted to the data, we are interested in finding out how well this curve fits with the observed facts.

The following steps are followed for the above said purpose:

i. A null and alternative hypothesis pertaining to the enquiry are established,

ii. A level of significance is chosen for rejection of the null hypothesis.

iii. A random sample of observations is drawn from a relevant statistical population.

iv. On the basis of given actual observations, expected or theoretical frequencies are derived through probability. This generally takes the form of assuming that a particular probability distribution is applicable to the statistical population under consideration.

v. The observed frequencies are compared with the expected or theoretical frequencies.

vi. If the calculated value of χ^2 is less than the table value at a certain level of significance (generally 5% level) and for certain degrees of freedom the, fit is considered to be good. i.e.. the divergence between the actual and expected frequencies is attributed to fluctuations of simple sampling. On the other hand, if the calculated value of χ^2 is greater than the table value, the fit is considered to be poor i.e. it cannot be attributed to the fluctuations of simple sampling rather it is due to the inadequacy of the theory to fit the observed facts.

Illustration 2:

In an anti malaria campaign in a certain area, quinine was administered to 812 persons out of a total population of 3248. The number of fever cases is shown below:

Treatment	Fever (A)	No fever (a)	Total
Quinine (B)	140(AB)	30 (aB)	170 (B)
No Quinine (b)	60(Ab)	20 (ab)	80 (b)
Total	200(A)	50 (a)	250 (N)

Discuss the usefulness of quinine in checking malaria.

(Given: For ν =1, $\chi^2_{\ 0.05}$ = 3.84)

Solution: Let us take the following hypotheses:

Null Hypothesis H_O: *Quinine is not effective in checking malaria.*

Alternative Hypothesis H_a: *Quinine is effective in checking malaria.*

Applying χ^2 test:

Expectated frequency of say AB $= \frac{(A)X(B)}{N} = \frac{200X\ 170}{250} = 136$

Or E_1, i.e., expected frequency corresponding to first row and first column is 60.

The table of expected frequencies shall be :

Treatment	Fever	No Fever	Total
Quinine	136	34	170
No quinine	64	16	80
Total	200	50	250 (N)

<u>Computation of Chi Square value</u>

O	E	$(O-E)^2$	$(O-E)^2/E$
140	136	16	0.118
60	64	16	0.250
30	34	16	0.471
20	16	16	1.000
			$\sum \frac{(O-E)^2}{E} = 1.839$

$\chi^2 = \sum \frac{(O-E)^2}{E} = 1.839$

Degree of freedom ν = (r-1) (c-1) = (2-1) (2-1) = 1

Table Value: For ν =1, $\chi^2_{\ 0.05} = 3.84$

The calculated value of χ^2 i.e. 1.839 is less than the table value i.e. 3.84, the null hypothesis is accepted. Hence quinine is not useful in checking malaria.

(C) A test of homogeneity

The χ^2 test of homogeneity is an extension of the χ^2 test of independence. Such tests indicate whether two or more independent samples are drawn from the same population or from different populations. Instead of one sample as we use in the independence problem, we shall now have two or more samples. Supposes a test is given to students in two different higher secondary schools. The sample size in both the cases is the same. The question we have to ask: is there any difference between the two higher secondary schools? In order to find the answer, we have to set up the null hypothesis that the two samples came from the same population. The word 'homogeneous' is used frequently in Statistics to indicate 'the same' or 'equal'. Accordingly, we can say that we want to test in our example whether the two samples are homogeneous. Thus, the test is called a test of homogeneity.

Illustration 3: Two hundred bolts were selected at random from the output of each of the five machines. The number of defective bolts found were 5, 9, 13, 7 and 6 . Is there a significant difference among the machines? Use 5% level of significance.

(Given: For ν =4, $\chi^2{}_{0.05}$ = 9.488)

Solution: Let us take the following hypothesis:

H_O*: There is no significant difference among the machines.*

H_a*: There is significant difference among the machines.*

As there are five machines, the total number of defective bolts should be equally distributed among these machines. That is how we can get expected frequencies as under:

Here expected no. of defective bolts for each machine (E) =

$\frac{Sum\ of\ defective\ bolts}{No.\ of\ machines\ producing\ these\ defective\ bolts} = \frac{40}{5} = 8.$

Computation of Chi Square test

Machine	O	E	O-E	$(O-E)^2$	$(O-E)^2/E$
1	5	8	-3	9	1.125
2	9	8	1	1	0.125
3	13	8	5	25	3.125
4	7	8	-1	1	0.125
5	6	8	-2	4	0.5
					$\sum(O-E)^2/E = 5.00$

Decision: The critical value of χ^2 at 0.05 level of significance for 4 degrees of freedom is 9.488. As the calculated value of $\chi^2 = 5$ is less than the critical value, H_O is accepted. In other words, the difference among the five machines in respect of defective bolts is not significant.