3.2 Microwave Tubes

- Microwave tubes are designed to overcome the principle limitations of conventional negative grid electron tubes. In microwave tubes the electron transit time is utilized for microwave oscillation or amplification. The principle uses an electron beam on which space-charge waves interact with electromagnetic fields in the microwave cavities to transfer energy to the output circuit of the cavity.
- There are basically two types of microwave tubes :

i) O-Type microwave tube

- Tubes in the O-type category are sometimes called linear or rectilinear beam tubes in recognition of the straight path taken by the electron beam. In this class of devices, both velocity and density modulation take place, creating the bunching effect. The electron bundles thus created have a period in the microwave region. Examples of O-type tubes include Klystrons and travelling-wave tubes (TWT).

ii) M-Type microwave tube

- A principle feature of such tubes is that electrons travel in a curved path. Those tubes were designated M-type. These are crossed field devices where the static magnetic field is perpendicular to the electric field. e.g. Magnetrons.
- The O-type tubes differ from M-type in that electrons travel in a straight line under the influence of parallel electric and magnetic fields.

KLYSTRONS

The two-cavity klystron is a widely used microwave amplifier operated by the principles of velocity and current modulation. All electrons injected from the cathode arrive at the first cavity with uniform velocity. Those electrons passing the first cavity gap at zeros of the gap voltage (or signal voltage) pass through with unchanged velocity; those passing through the positive half cycles of the gap voltage undergo an increase in velocity; those passing through the negative swings of the gap voltage undergo a decrease in velocity. As a result of these actions, the electrons gradually bunch together as they travel down the drift space. The variation in electron velocity in the drift space is known as *velocity modulation*. The density of the electrons in the second cavity gap varies cyclically with time. The electron beam contains an ac component and is said to be current-modulated. The maximum bunching should occur approximately midway between the second cavity grids during its retarding phase; thus the kinetic energy is transferred from the electrons to the field of the second cavity. The electrons then emerge from the second cavity with reduced velocity and finally terminate at the collector. The charateristics of a two-cavity klystron amplifier are as follows:

1. Efficiency: about 40%.
2. Power output: average power (CW power) is up to 500 kW and pulsed power is up to 30 MW at 10 GHz.
3. Power gain: about 30 dB.

Figure 9-2-1 shows the present state of the art for U.S. high-power klystrons. Figure 9-2-2 shows the schematic diagram of a two-cavity klystron amplifier. The cavity close to the cathode is known as the *buncher cavity* or input cavity, which velocity-modulates the electron beam. The other cavity is called the *catcher cavity* or output cavity; it catches energy from the bunched electron beam. The beam then passes through the catcher cavity and is terminated at the collector. The quantitative analysis of a two-cavity klystron can be described in four parts under the following assumptions:

1. The electron beam is assumed to have a uniform density in the cross section of the beam.
2. Space-charge effects are negligible.
3. The magnitude of the microwave signal input is assumed to be much smaller than the dc accelerating voltage.

9-2-1 Reentrant Cavities

At a frequency well below the microwave range, the cavity resonator can be represented by a lumped-constant resonant circuit. When the operating frequency is increased to several tens of megahertz, both the inductance and the capacitance must be reduced to a minimum in order to maintain resonance at the operating frequency. Ultimately the inductance is reduced to a minimum by short wire. Therefore the reentrant cavities are designed for use in klystrons and microwave triodes. A reentrant cavity is one in which the metallic boundaries extend into the interior of the cavity. Several types of reentrant cavities are shown in Fig. 9-2-3. One of the commonly used reentrant cavities is the coaxial cavity shown in Fig. 9-2-4.

It is clear from Fig. 9-2-4 that not only has the inductance been considerably decreased but the resistance losses are markedly reduced as well, and the shielding enclosure prevents radiation losses. It is difficult to calculate the resonant frequency of the coaxial cavity. An approximation can be made, however, using transmission-line theory. The characteristic impedance of the coaxial line is given by

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\,\ell n\frac{b}{a} \quad \text{ohms} \tag{9-2-1}$$

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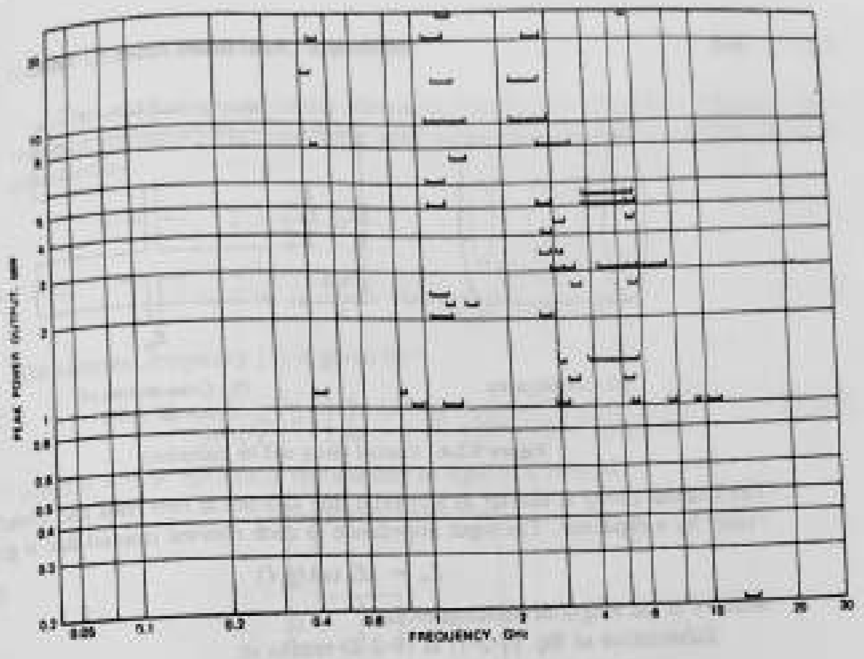


Figure 9-2-1 State of the art for U.S. high-power klystrons.

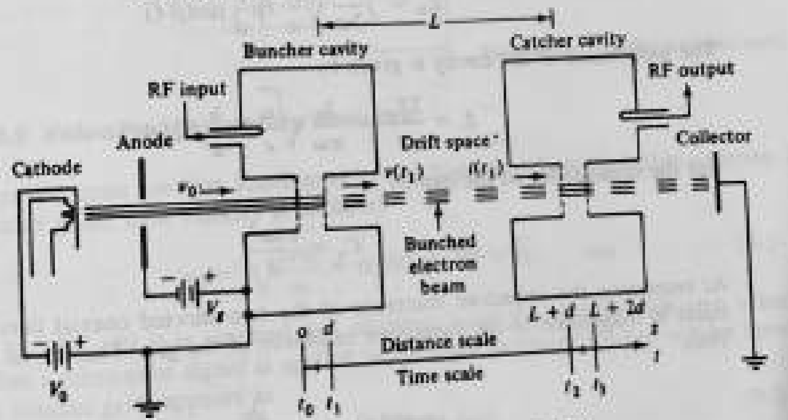


Figure 9-2-2 Two-cavity klystron amplifier.

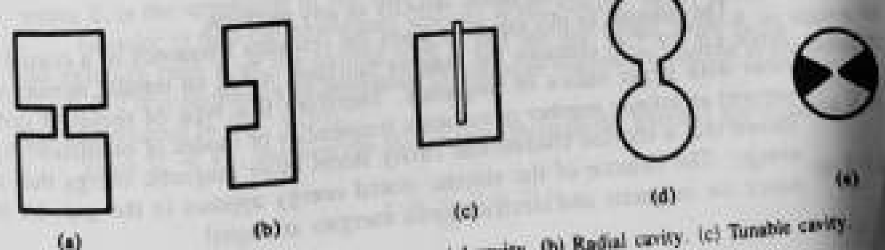
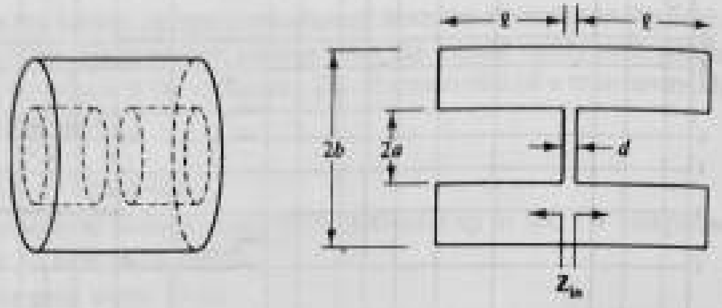


Figure 9-2-3 Reentrant cavities. (a) Coaxial cavity. (b) Radial cavity. (c) Tunable cavity. (d) Toroidal cavity. (e) Butterfly cavity.



(a) Coaxial cavity

(b) Cross-section of coaxial cavity

Figure 9-2-4 Coaxial cavity and its equivalent.

The coaxial cavity is similar to a coaxial line shorted at two ends and joined at the center by a capacitor. The input impedance to each shorted coaxial line is given by

$$Z_{in} = jZ_0 \tan(\beta \ell) \tag{9-2-2}$$

where ℓ is the length of the coaxial line.

Substitution of Eq. (9-2-1) in (9-2-2) results in

$$Z_{in} = j\frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\,\ell n\frac{b}{a}\tan(\beta \ell) \tag{9-2-3}$$

The inductance of the cavity is given by

$$L = \frac{2X_{in}}{\omega} = \frac{1}{\pi\omega}\sqrt{\frac{\mu}{\epsilon}}\,\ell n\frac{b}{a}\tan(\beta \ell) \tag{9-2-4}$$

and the capacitance of the gap by

$$C_g = \frac{\epsilon \pi a^2}{d} \tag{9-2-5}$$

At resonance the inductive reactance of the two shorted coaxial lines in series is equal in magnitude to the capacitive reactance of the gap. That is, $\omega L = 1/(\omega C_g)$. Thus

$$\tan(\beta \ell) = \frac{dv}{\omega a^2\,\ell n(b/a)} \tag{9-2-6}$$

where $v = 1/\sqrt{\mu\epsilon}$ is the phase velocity in any medium.

The solution to this equation gives the resonant frequency of a coaxial cavity. Since Eq. (9-2-6) contains the tangent function, it has an infinite number of solutions with larger values of frequency. Therefore this type of reentrant cavity can support an infinite number of resonant frequencies or modes of oscillation. It can be shown that a shorted coaxial-line cavity stores more magnetic energy than electric energy. The balance of the electric stored energy appears in the gap, for at resonance the magnetic and electric stored energies are equal.

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The radial reentrant cavity shown in Fig. 9-2-5 is another commonly used reentrant resonator. The inductance and capacitance of a radial reentrant cavity is expressed by

$$L = \frac{\mu \ell}{2\pi} \ell n \frac{b}{a} \tag{9-2-7}$$

$$C = \epsilon_0 \left[\frac{\pi a^2}{d} - 4a \, \ell n \frac{0.765}{\sqrt{\ell^2 + (b-a)^2}} \right] \tag{9-2-8}$$

The resonant frequency [1] is given by

$$f_r = \frac{c}{2\pi\sqrt{\epsilon_r}} \left\{ a\ell \left[\frac{a}{2d} - \frac{2}{\ell} \ell n \frac{0.765}{\sqrt{\ell^2 + (b-a)^2}} \right] \ell n \frac{b}{a} \right\}^{-1/2} \tag{9-2-9}$$

where $c = 3 \times 10^8$ m/s is the velocity of light in a vacuum.

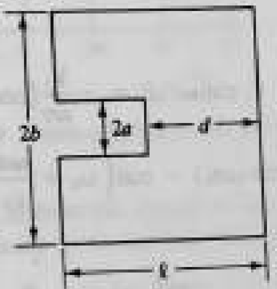

Figure 9-2-5 Radial reentrant cavity.

9-2-2 Velocity-Modulation Process

When electrons are first accelerated by the high dc voltage V_0 before entering the buncher grids, their velocity is uniform:

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0} \quad \text{m/s} \tag{9-2-10}$$

In Eq. (9-2-10) it is assumed that electrons leave the cathode with zero velocity. When a microwave signal is applied to the input terminal, the gap voltage between the buncher grids appears as

$$V_s = V_1 \sin(\omega t) \tag{9-2-11}$$

where V_1 is the amplitude of the signal and $V_1 \ll V_0$ is assumed.

In order to find the modulated velocity in the buncher cavity in terms of either the entering time t_0 or the exiting time t_1 and the gap transit angle θ_g as shown in Fig. 9-2-2 it is necessary to determine the average microwave voltage in the buncher gap as indicated in Fig. 9-2-6.

Since $V_1 \ll V_0$, the average transit time through the buncher gap distance d is

$$\tau \approx \frac{d}{v_0} = t_1 - t_0 \tag{9-2-12}$$

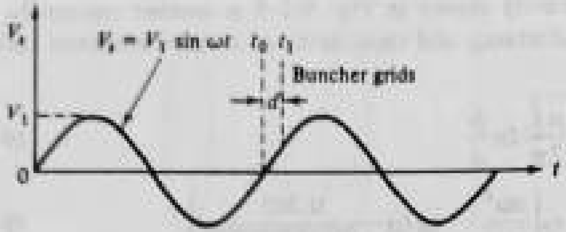

Figure 9-2-6 Signal voltage in the buncher gap.

The average gap transit angle can be expressed as

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0} \tag{9-2-13}$$

The average microwave voltage in the buncher gap can be found in the following way:

$$\begin{aligned}\langle V_s\rangle &= \frac{1}{\tau}\int_{t_0}^{t_1} V_1 \sin(\omega t)\,dt = -\frac{V_1}{\omega\tau}[\cos(\omega t_1) - \cos(\omega t_0)] \\ &= \frac{V_1}{\omega\tau}\left[\cos(\omega t_0) - \cos\left(\omega_0 + \frac{\omega d}{v_0}\right)\right]\end{aligned} \tag{9-2-14}$$

Let

$$\omega t_0 + \frac{\omega d}{2v_0} = \omega t_0 + \frac{\theta_g}{2} = A$$

and

$$\frac{\omega d}{2v_0} = \frac{\theta_g}{2} = B$$

Then using the trigonometric identity that $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$, Eq. (9-2-14) becomes

$$\langle V_s\rangle = V_1 \frac{\sin[\omega d/2v_0)]}{\omega d/(2v_0)} \sin\left(\omega t_0 + \frac{\omega d}{2v_0}\right) = V_1 \frac{\sin(\theta_g/2)}{\theta_g/2} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \tag{9-2-15}$$

It is defined as

$$\beta_i = \frac{\sin[\omega d/(2v_0)]}{\omega d/(2v_0)} = \frac{\sin(\theta_g/2)}{\theta_g/2} \tag{9-2-16}$$

Note that β_i is known as the *beam-coupling coefficient* of the input cavity gap (see Fig. 9-2-7).

It can be seen that increasing the gap transit angle θ_g decreases the coupling between the electron beam and the buncher cavity; that is, the velocity modulation

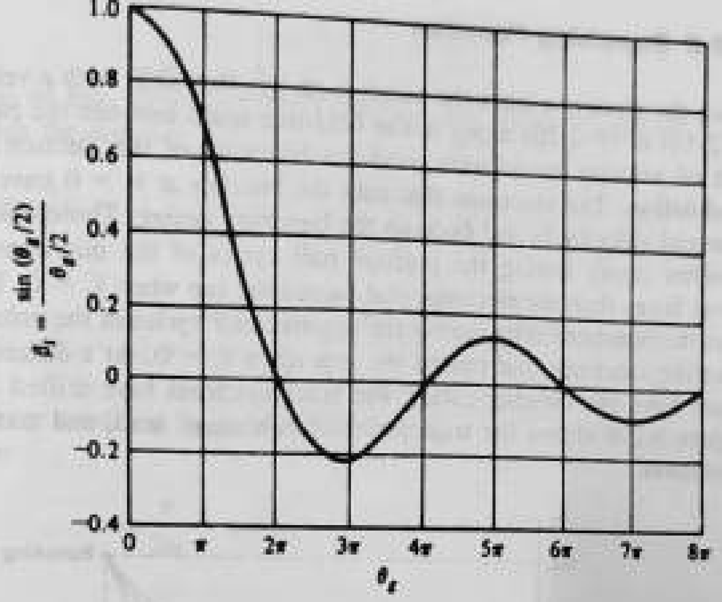

Figure 9-2-7 Beam-coupling coefficient versus gap transit angle.

of the beam for a given microwave signal is decreased. Immediately after velocity modulation, the exit velocity from the buncher gap is given by

$$v(t_1) = \sqrt{\frac{2e}{m}\left[V_0 + \beta_i V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$
$$= \sqrt{\frac{2e}{m} V_0\left[1 + \frac{\beta_i V_1}{V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]} \tag{9-2-17}$$

where the factor $\beta_i V_1/V_0$ is called the *depth of velocity modulation*.

Using binomial expansion under the assumption of

$$\beta_i V_1 \ll V_0 \tag{9-2-18}$$

Eq. (9-2-17) becomes

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \tag{9-2-19}$$

Equation (9-2-19) is the equation of velocity modulation. Alternatively, the equation of velocity modulation can be given by

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \tag{9-2-20}$$

9-2-3 Bunching Process

Once the electrons leave the buncher cavity, they drift with a velocity given by Eq. (9-2-19) or (9-2-20) along in the field-free space between the two cavities. The effect of velocity modulation produces bunching of the electron beam—or current modulation. The electrons that pass the buncher at $V_s = 0$ travel through with unchanged velocity v_0 and become the bunching center. Those electrons that pass the buncher cavity during the positive half cycles of the microwave input voltage V_s travel faster than the electrons that passed the gap when $V_s = 0$. Those electrons that pass the buncher cavity during the negative half cycles of the voltage V_s travel slower than the electrons that passed the gap when $V_s = 0$. At a distance of ΔL along the beam from the buncher cavity, the beam electrons have drifted into dense clusters. Figure 9-2-8 shows the trajectories of minimum, zero, and maximum electron acceleration.

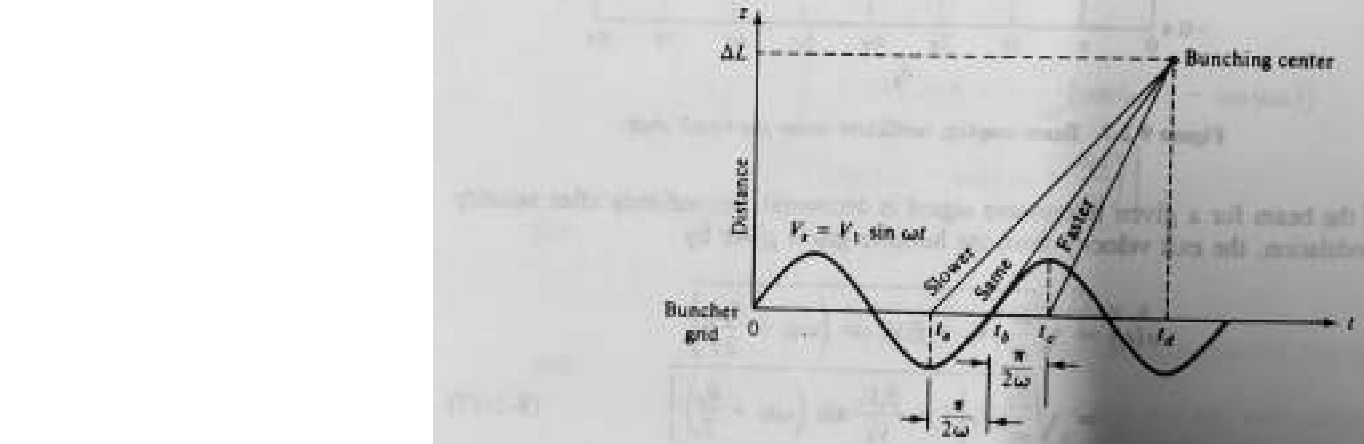

Figure 9-2-8 Bunching distance

The distance from the buncher grid to the location of dense electron bunching for the electron at t_b is

$$\Delta L = v_0(t_d - t_b) \tag{9-2-21}$$

Similarly, the distances for the electrons at t_a and t_c are

$$\Delta L = v_{min}(t_d - t_a) = v_{min}\left(t_d - t_b + \frac{\pi}{2\omega}\right) \tag{9-2-22}$$

$$\Delta L = v_{max}(t_d - t_c) = v_{max}\left(t_d - t_b - \frac{\pi}{2\omega}\right) \tag{9-2-23}$$

From Eq. (9-2-19) or (9-2-20) the minimum and maximum velocities are

$$v_{min} = v_0\left(1 - \frac{\beta_i V_1}{2V_0}\right) \tag{9-2-24}$$

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$$v_{max} = v_0\left(1 + \frac{\beta_i V_1}{2V_0}\right) \tag{9-2-25}$$

Substitution of Eqs. (9-2-24) and (9-2-25) in Eqs. (9-2-22) and (9-2-23), respectively, yields the distance

$$\Delta L = v_0(t_d - t_b) + \left[v_0\frac{\pi}{2\omega} - v_0\frac{\beta_i V_1}{2V_0}(t_d - t_b) - v_0\frac{\beta_i V_1}{2V_0}\frac{\pi}{2\omega}\right] \tag{9-2-26}$$

and

$$\Delta L = v_0(t_d - t_b) + \left[-v_0\frac{\pi}{2\omega} + v_0\frac{\beta_i V_1}{2V_0}(t_d - t_b) + v_0\frac{\beta_i V_1}{2V_0}\frac{\pi}{2\omega}\right] \tag{9-2-27}$$

The necessary condition for those electrons at t_a, t_b, and t_c to meet at the same distance ΔL is

$$v_0\frac{\pi}{2\omega} - v_0\frac{\beta_i V_1}{2V_0}(t_d - t_b) - v_0\frac{\beta_i V_1}{2V_0}\frac{\pi}{2\omega} = 0 \tag{9-2-28}$$

and

$$-v_0\frac{\pi}{2\omega} + v_0\frac{\beta_i V_1}{2V_0}(t_d - t_b) + v_0\frac{\beta_i V_1}{2V_0}\frac{\pi}{2\omega} = 0 \tag{9-2-29}$$

Consequently,

$$t_d - t_b \simeq \frac{\pi V_0}{\omega\beta_i V_1} \tag{9-2-30}$$

and

$$\Delta L = v_0\frac{\pi V_0}{\omega\beta_i V_1} \tag{9-2-31}$$

It should be noted that the mutual repulsion of the space charge is neglected, but the qualitative results are similar to the preceding representation when the effects of repulsion are included. Furthermore, the distance given by Eq. (9-2-31) is not the one for a maximum degree of bunching. Figure 9-2-9 shows the distance-time plot or Applegate diagram.

What should the spacing be between the buncher and catcher cavities in order to achieve a maximum degree of bunching? Since the drift region is field free, the transit time for an electron to travel a distance of L as shown in Fig. 9-2-2 is given by

$$T = t_2 - t_1 = \frac{L}{v(t_1)} = T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \tag{9-2-32}$$

where the binomial expansion of $(1 + x)^{-1}$ for $|x| \ll 1$ has been replaced and $T_0 = L/v_0$ is the dc transit time. In terms of radians the preceding expression can be written

$$\omega T = \omega t_2 - \omega t_1 = \theta_0 - X\sin\left(\omega t_1 - \frac{\theta_g}{2}\right) \tag{9-2-33}$$

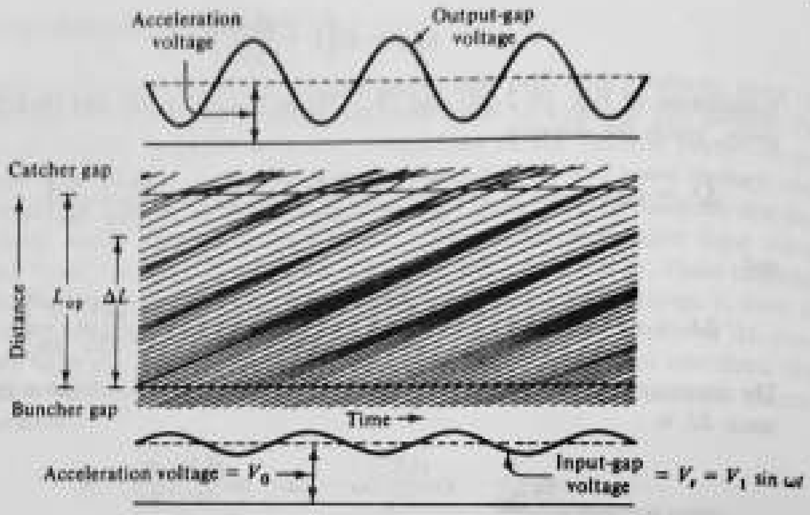

Figure 9-2-9 Applegate diagram.

where

$$\theta_0 = \frac{\omega L}{v_0} = 2\pi N \qquad (9\text{-}2\text{-}34)$$

is the dc transit angle between cavities, N is the number of electron transit cycles in the drift space, and

$$X = \frac{\beta_i V_1}{2V_0}\theta_0 \qquad (9\text{-}2\text{-}35)$$

is defined as the *bunching parameter* of a klystron.

At the buncher gap a charge dQ_0 passing through at a time interval dt_0 is given by

$$dQ_0 = I_0 dt_0 \qquad (9\text{-}2\text{-}36)$$

where I_0 is the dc current. From the principle of conservation of charges this same amount of charge dQ_0 also passes the catcher at a later time interval dt_2. Hence

$$I_0|dt_0| = i_2|dt_2| \qquad (9\text{-}2\text{-}37)$$

where the absolute value signs are necessary because a negative value of the time ratio would indicate a negative current. Current i_2 is the current at the catcher gap. Rewriting Eq. (9-2-32) in terms of Eq. (9-2-19) yields

$$t_2 = t_0 + \tau + T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \qquad (9\text{-}2\text{-}38)$$

Alternatively,

$$\omega t_2 - \left(\theta_0 + \frac{\theta_g}{2}\right) = \left(\omega t_0 + \frac{\theta_g}{2}\right) - X\sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \qquad (9\text{-}2\text{-}39)$$

where $(\omega t_0 + \theta_g/2)$ is the buncher cavity departure angle and $\omega t_2 - (\theta_0 + \theta_g/2)$ is the catcher cavity arrival angle. Figure 9-2-10 shows the curves for the catcher cav-

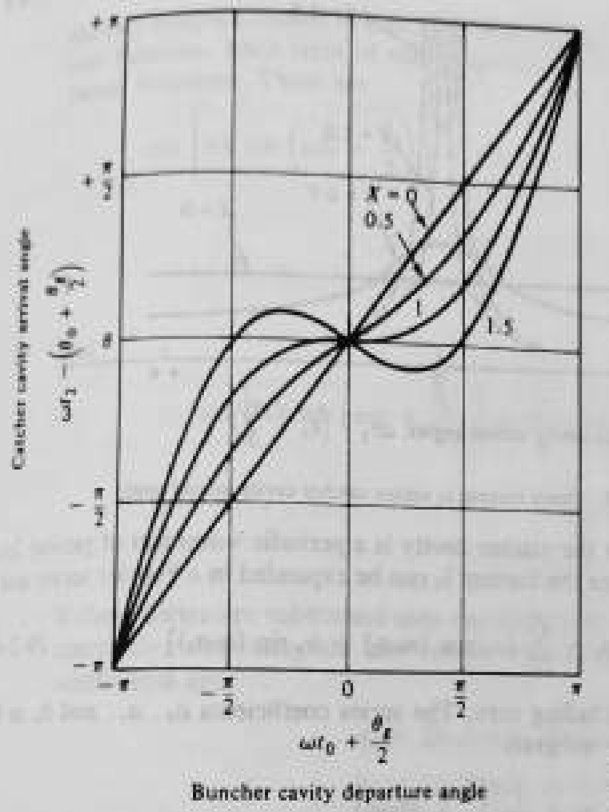

Figure 9-2-10 Catcher arrival angle versus buncher departure angle.

ity arrival angle as a function of the buncher cavity departure angle in terms of the bunching parameter X.

Differentiation of Eq. (9-2-38) with respect to t_0 results in

$$dt_2 = dt_0\left[1 - X\cos\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \tag{9-2-40}$$

The current arriving at the catcher cavity is then given as

$$i_2(t_0) = \frac{I_0}{1 - X\cos(\omega t_0 + \theta_g/2)} \tag{9-2-41}$$

In terms of t_2 the current is

$$i_2(t_2) = \frac{I_0}{1 - X\cos(\omega t_2 - \theta_0 - \theta_g/2)} \tag{9-2-42}$$

In Eq. (9-2-42) the relationsip of $t_2 = t_0 + \tau + T_0$ is used—namely, $\omega t_2 = \omega t_0 + \omega\tau + \omega T_0 = \omega t_0 + \theta_g + \theta_0$. Figure 9-2-11 shows curves of the beam current $i_2(t_2)$ as a function of the catcher arrival angle in terms of the bunching parameter X.

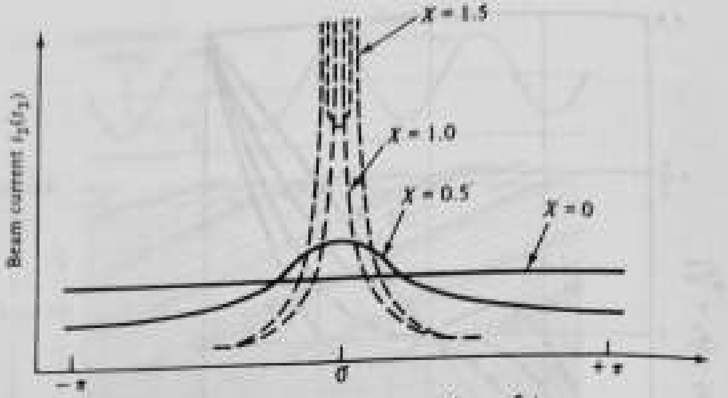

Figure 9-2-11 Beam current i_2 versus catcher cavity arrival angle.

The beam current at the catcher cavity is a periodic waveform of period $2\pi/\omega$ about dc current. Therefore the current i_2 can be expanded in a Fourier series and so

$$i_2 = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega t_2) + b_n \sin(n\omega t_2)] \tag{9-2-43}$$

where n is an integer, excluding zero. The series coefficients a_0, a_n, and b_n in Eq. (9-2-43) are given by the integrals

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} i_2\, d(\omega t_2)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos(n\omega t_2)\, d(\omega t_2) \tag{9-2-44}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} i_2 \sin(n\omega t_2)\, d(\omega t_2)$$

Substitution of Eqs. (9-2-37) and (9-2-39) in Eq. (9-2-44) yields

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} I_0\, d(\omega t_0) = I_0 \tag{9-2-45}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} I_0 \cos\left[(n\omega t_0 + n\theta_g + n\theta_0) + nX \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] d(\omega t_0) \tag{9-2-46}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} I_0 \sin\left[(n\omega t_0 + n\theta_g + n\theta_0) + nX \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] d(\omega t_0) \tag{9-2-47}$$

By using the trigonometric functions

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

and

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

the two integrals shown in Eqs. (9-2-40) and (9-2-41) involve cosines and sines of a sine function. Each term of the integrand contains an infinite number of terms of Bessel functions. These are

$$\cos\left[nX\sin\left(\omega t_0+\frac{\theta_g}{2}\right)\right]=2J_0(nX)+2\left[J_2(nX)\cos 2\left(\omega t_0+\frac{\theta_g}{2}\right)\right]+2\left[J_4(nX)\cos 4\left(\omega t_0+\frac{\theta_g}{2}\right)\right]+\cdots \tag{9-2-48}$$

and

$$\sin\left[nX\sin\left(\omega t_0+\frac{\theta_g}{2}\right)\right]=2\left[J_1(nX)\sin\left(\omega t_0+\frac{\theta_g}{2}\right)\right]+2\left[J_3(nX)\sin 3\left(\omega t_0+\frac{\theta_g}{2}\right)\right]+\cdots \tag{9-2-49}$$

If these series are substituted into the integrands of Eqs. (9-2-46) and (9-2-47), respectively, the integrals are readily evaluated term by term and the series coefficients are

$$a_n=2I_0J_n(nX)\cos(n\theta_g+n\theta_0) \tag{9-2-50}$$

$$b_n=2I_0J_n(nX)\sin(n\theta_g+n\theta_0) \tag{9-2-51}$$

where $J_n(nX)$ is the nth-order Bessel function of the first kind (see Fig. 9-2-12).

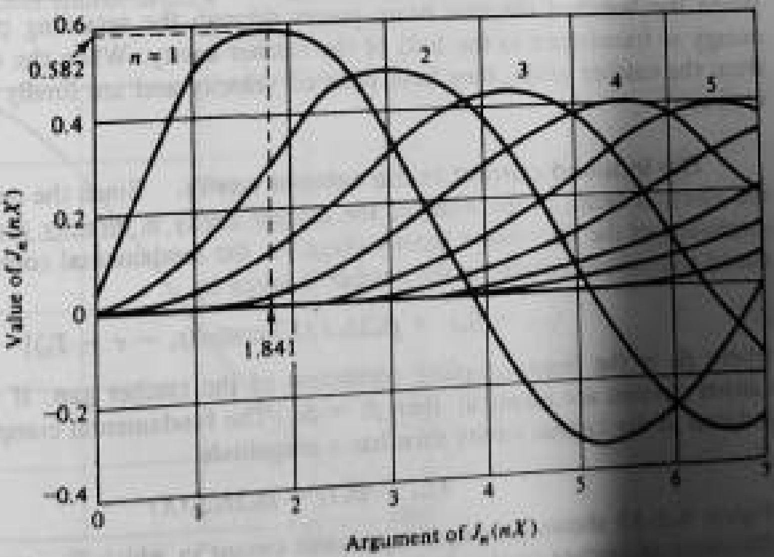

Figure 9-2-12 Bessel functions $J_n(nX)$

Substitution of Eqs. (9-2-45), (9-2-50), and (9-2-51) in Eq. (9-2-43) yields the beam current i_2 as

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2I_0 J_n(nX) \cos [n\omega(t_2 - \tau - T_0)] \tag{9-2-52}$$

The fundamental component of the beam current at the catcher cavity has a magnitude

$$I_f = 2I_0 J_1(X) \tag{9-2-53}$$

This fundamental component I_f has its maximum amplitude at

$$X = 1.841 \tag{9-2-54}$$

The optimum distance L at which the maximum fundamental component of current occurs is computed from Eqs. (9-2-34), (9-2-35), and (9-2-54) as

$$L_{\text{optimum}} = \frac{3.682 v_0 V_0}{\omega \beta_i V_1} \tag{9-2-55}$$

It is interesting to note that the distance given by Eq. (9-2-31) is approximately 15% less than the result of Eq. (9-2-55). The discrepancy is due in part to the approximations made in deriving Eq. (9-2-31) and to the fact that the maximum fundamental component of current will not coincide with the maximun electron density along the beam because the harmonic components exist in the beam.

9-2-4 Output Power and Beam Loading

The maximum bunching should occur approximately midway between the catcher grids. The phase of the catcher gap voltage must be maintained in such a way that the bunched electrons, as they pass through the grids, encounter a retarding phase. When the bunched electron beam passes through the retarding phase, its kinetic energy is transferred to the field of the catcher cavity. When the electrons emerge from the catcher grids, they have reduced velocity and are finally collected by the collector.

The induced current in the catcher cavity. Since the current induced by the electron beam in the walls of the catcher cavity is directly proportional to the amplitude of the microwave input voltage V_1, the fundamental component of the induced microwave current in the catcher is given by

$$i_{2\text{ind}} = \beta_0 i_2 = \beta_0 2I_0 J_1(X) \cos[\omega(t_2 - \tau - T_0)] \tag{9-2-56}$$

where β_0 is the beam coupling coefficient of the catcher gap. If the buncher and catcher cavities are identical, then $\beta_i = \beta_0$. The fundamental component of current induced in the catcher cavity then has a magnitude

$$I_{2\text{ind}} = \beta_0 I_2 = \beta_0 2I_0 J_1(X) \tag{9-2-57}$$

Figure 9-2-13 shows an output equivalent circuit in which R_{sho} represents the wall resistance of catcher cavity, R_B the beam loading resistance, R_L the external load resistance, and R_{sh} the effective shunt resistance.

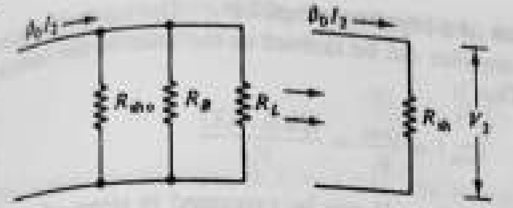

Figure 9-2-13 Output equivalent circuit.

The output power delivered to the catcher cavity and the load is given as

$$P_{out} = \frac{(\beta_0 I_2)^2}{2} R_{sh} = \frac{\beta_0 I_2 V_2}{2} \tag{9-2-58}$$

where R_{sh} is the total equivalent shunt resistance of the catcher circuit, including the load, and V_2 is the fundamental component of the catcher gap voltage.

Efficiency of klystron. The electronic efficiency of the klystron amplifier is defined as the ratio of the output power to the input power:

$$\text{Efficiency} = \frac{P_{out}}{P_{in}} = \frac{\beta_0 I_2 V_2}{2 I_0 V_0} \tag{9-2-59}$$

in which the power losses to the beam loading and cavity walls are included.

If the coupling is perfect, $\beta_0 = 1$, the maximum beam current approaches $I_{2max} = 2I_0(0.582)$, and the voltage V_2 is equal to V_0. Then the maximum electronic efficiency is about 58%. In practice, the electronic efficiency of a klystron amplifier is in the range of 15 to 30%. Since the efficiency is a function of the catcher gap transit angle θ_g, Fig. 9-2-14 shows the maximum efficiency of a klystron as a function of catcher transit angle.

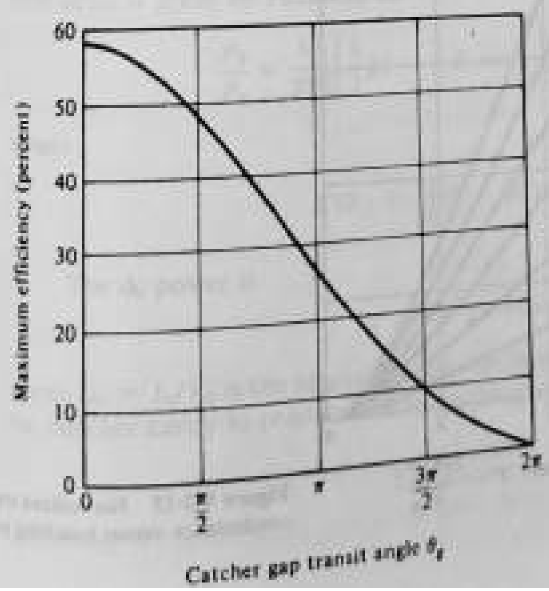

Figure 9-2-14 Maximum efficiency of klystron versus transit angle.

Mutual conductance of a klystron amplifier. The equivalent mutual conductance of the klystron amplifier can be defined as the ratio of the induced output current to input voltage. That is

$$|G_m| = \frac{i_{2\,ind}}{V_1} = \frac{2\beta_0 I_0 J_1(X)}{V_1} \tag{9-2-60}$$

From Eq. (9-2-35) the input voltage V_1 can be expressed in terms of the bunching parameter X as

$$V_1 = \frac{2V_0}{\beta_0 \theta_0} X \tag{9-2-61}$$

In Eq. (9-2-61) it is assumed that $\beta_0 = \beta_i$. Substitution of Eq. (9-2-61) in Eq. (9-2-60) yields the normalized mutual conductance as

$$\frac{|G_m|}{G_0} = \beta_0^2 \theta_0 \frac{J_1(X)}{X} \tag{9-2-62}$$

where $G_0 = I_0/V_0$ is the dc beam conductance. The mutual conductance is not a constant but decreases as the bunching parameter X increases. Figure 9-2-15 shows the curves of normalized transconductance as a function of X.

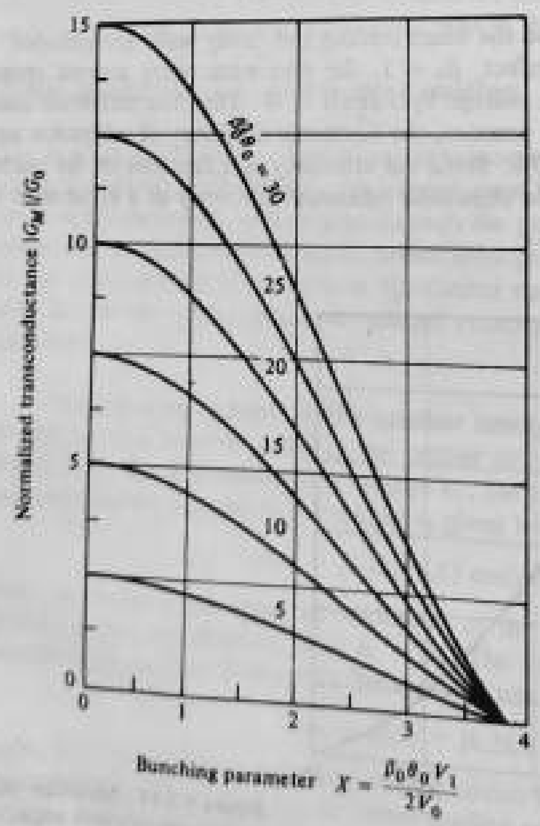

Figure 9-2-15 Normalized transconductance versus bunching parameter.

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It can be seen from the curves that, for a small signal, the normalized transconductance is maximum. That is,

$$\frac{|G_m|}{G_0} = \frac{\beta_0^2 U_0}{2} \tag{9-2-63}$$

For a maximum output at $X = 1.841$, the normalized mutual conductance is

$$\frac{|G_m|}{G_0} = 0.316\beta_0^2\theta_0 \tag{9-2-64}$$

The voltage gain of a klystron amplifier is defined as

$$A_v = \frac{|V_2|}{|V_1|} = \frac{\beta_0 I_2 R_{sh}}{V_1} = \frac{\beta_0^2\theta_0}{R_0}\frac{J_1(X)}{X}R_{sh} \tag{9-2-65}$$

where $R_0 = V_0/I_0$ is the dc beam resistance. Substitution of Eqs. (9-2-57) and (9-2-61) in Eq. (9-2-65) results in

$$A_v = G_m R_{sh} \tag{9-2-66}$$

Power required to bunch the electron beam. As described earlier, the bunching action takes place in the buncher cavity. When the buncher gap transit angle is small, the average energy of electrons leaving the buncher cavity over a cycle is nearly equal to the energy with which they enter. When the buncher gap transit angle is large, however, the electrons that leave the buncher gap have greater average energy than when they enter. The difference between the average exit energy and the entrance energy must be supplied by the buncher cavity to bunch the electron beam. It is difficult to calculate the power required to produce the bunching action. Feenberg did some extensive work on beam loading [6]. The ratio of the power required to produce bunching action over the dc power required to perform the electron beam is given by Feenberg as

$$\frac{P_B}{P_0} = \frac{V_1^2}{2V_0^2}\left[\frac{1}{2}\beta_i^2 - \frac{1}{2}\beta_i \cos\left(\frac{\theta_g}{2}\right)\right] = \frac{V_1^2}{2V_0^2}F(\theta_g) \tag{9-2-67}$$

where

$$F(\theta_g) = \frac{1}{2}\left[\beta_i^2 - \beta_i \cos\left(\frac{\theta_g}{2}\right)\right]$$

The dc power is

$$P_0 = V_0^2 G_0 \tag{9-2-68}$$

where $G_0 = I_0/V_0$ is the equivalent electron beam conductance. The power given by the buncher cavity to produce beam bunching is

$$P_B = \frac{V_1^2}{2}G_B \tag{9-2-69}$$

where G_B is the equivalent bunching conductance. Substitution of Eqs. (9-2-68) and (9-2-69) in Eq. (9-2-67) yields the normalized electronic conductance as

$$\frac{G_B}{G_0} = \frac{R_0}{R_B} = F(\theta_g) \tag{9-2-70}$$

Figure 9-2-16 shows the normalized electronic conductance as a function of the buncher gap transit angle. It can be seen that there is a critical buncher gap transit angle for a minimum equivalent bunching resistance. When the transit angle θ_g is 3.5 rad, the equivalent bunching resistance is about five times the electron beam resistance. The power delivered by the electron beam to the catcher cavity can be expressed as

$$\frac{V_2^2}{2R_{sh}} = \frac{V_2^2}{2R_{sho}} + \frac{V_2^2}{2R_B} + \frac{V_2^2}{2R_L} \tag{9-2-71}$$

As a result, the effective impedance of the catcher cavity is

$$\frac{1}{R_{sh}} = \frac{1}{R_{sho}} + \frac{1}{R_B} + \frac{1}{R_L} \tag{9-2-72}$$

Finally, the loaded quality factor of the catcher cavity circuit at the resonant frequency can be written

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_B} + \frac{1}{Q_{ext}} \tag{9-2-73}$$

where Q_L = loaded quality factor of the whole catcher circuit
Q_0 = quality factor of the catcher walls
Q_B = quality factor of the beam loading
Q_{ext} = quality factor of the external load

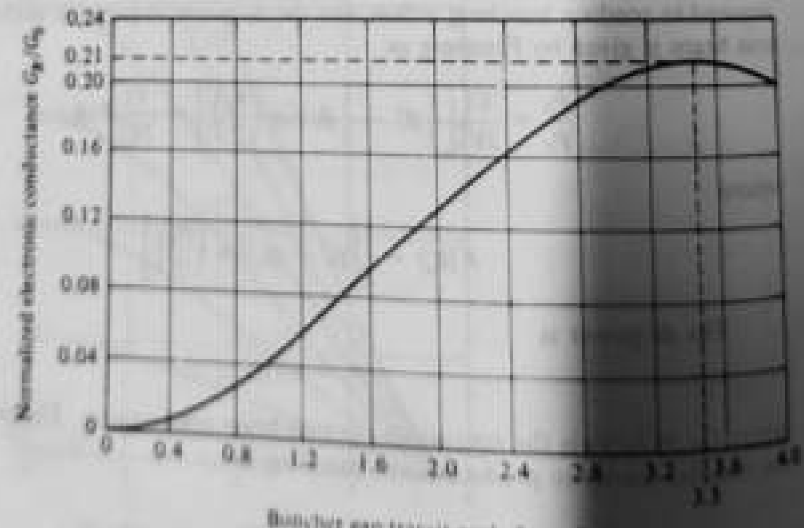

Figure 9-2-16 Normalized electronic conductance versus buncher gap transit angle.

9-3-2 Output Current and Output Power of Two-Cavity Klystron

If the two cavities of a two-cavity klystron amplifier are assumed to be identical and are placed at the point where the RF current modulation is a maximum, the magnitude of the RF convection current at the output cavity for a two-cavity klystron can be written from Eq. (9-3-17) as

$$|i_2| = \frac{1}{2}\frac{I_0\omega}{V_0\omega_q}\beta_i|V_1| \tag{9-3-18}$$

where V_1 = magnitude of the input signal voltage. Then the magnitudes of the induced current and voltage in the output cavity are equal to

$$|I_2| = \beta_0|i_2| = \frac{1}{2}\frac{I_0\omega}{V_0\omega_q}\beta_0^2|V_1| \tag{9-3-19}$$

and

$$|V_2| = |I_2|R_{shl} = \frac{1}{2}\frac{I_0\omega}{V_0\omega_q}\beta_0^2|V_1|R_{shl} \tag{9-3-20}$$

where $\beta_0 = \beta_i$ is the beam coupling coefficient
R_{shl} = total shunt resistance of the output cavity in a two-cavity klystron amplifier including the external load

The output power delivered to the load in a two-cavity klystron amplifier is given by

$$P_{out} \doteq |I_2|^2 R_{shl} = \frac{1}{4}\left(\frac{I_0 \omega}{V_0 \omega_q}\right)^2 \beta_0^4 |V_1|^2 R_{shl} \tag{9-3-21}$$

The power gain of a two-cavity klystron amplifier is then expressed by

$$\text{Power gain} = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{|V_1|^2/R_{sh}} = \frac{1}{4}\left(\frac{I_0 \omega}{V_0 \omega_q}\right)^2 \beta_0^4 R_{sh} \cdot R_{shl} \tag{9-3-22}$$

where R_{sh} = total shunt resistance of the input cavity. The electronic efficiency of a two-cavity klystron amplifier is

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{I_0 V_0} = \frac{1}{4}\left(\frac{I_0}{V_0}\right)\left(\frac{|V_1|\omega}{V_0 \omega_q}\right)^2 \beta_0^4 R_{shl} \tag{9-3-23}$$