In polar form

$$V_i = |V_i| e^{j\delta_i}$$

$$Y_{ik} = |Y_{ik}| e^{j\theta_{ik}}$$

Now Real & Reactive powers can be expressed as —

$$P_i = |V_i| \sum_{k=1}^{n} |V_k||Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i) \longrightarrow (7) \quad i = 1, 2 \ldots n$$

$$Q_i = -|V_i| \sum_{k=1}^{n} |V_k||Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i) \longrightarrow (8) \quad i = 1, 2 \ldots n$$

Equ. (7) & (8) represent 2n power flow equations at 'n' buses of a power system ($n \rightarrow$ real power flow equ + $n \rightarrow$ reactive powerflow equ.). Each bus is characterized by four variable; P_i, Q_i, $|V_i|$ & δ_i resulting in a total of 4n variables.

Equ. (7) & (8) can be solved for 2n variables if remaining 2n variables are speified.

Practical considerations allow a power system analyst to fix a priori two variables at each bus.

The sol.n of remaining 2n bus variables is rendered difficult by fact that equ. (7) & (8) are non-linear algebric equations (bus voltages are involved in product form and sine & cosine Terms are present and therefore explicit sol.n is not possible. Sol.n can be obtained by iterative numerical techniques.

An approximate load flow Solution —

Following assumptions and approximations in the load flow equ. (7) & (8) have been made —

i) Line resistance — very small & neglected (shunt conductance of overhead line is always negligible) so, P_L - the active power loss of the system is zero. Thus, $\theta_{ik} = 90^\circ$ & $\theta_{ii} = -90^\circ$

ii) $(\delta_i - \delta_k)$ is small $(< \pi/6)$ so that $\sin(\delta_i - \delta_k) \simeq (\delta_i - \delta_k)$

iii) All buses other than the slack bus (numbered as bus 1) are PV buses, i.e. voltage magnitudes at all the buses including the slack bus are specified.

Now equ. (7) & (8) becomes —

$$P_i = |V_i| \sum_{k=1}^{n} |V_k||Y_{ik}|(\delta_i - \delta_k); \quad i = 2, 3, \ldots n \longrightarrow (9)$$

$$Q_i = -|V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k||Y_{ik}| \cos(\delta_i - \delta_k) + |V_i|^2 |Y_{ii}| \quad i = 1, 2, \ldots n \longrightarrow (10)$$

Gauss-Seidel Method :-

Let it be assumed that all buses other than the slack bus are PQ buses to explain how the GS method is applied to obtain the load flow solution,

Later we will include PV buses as well.

The slack bus voltage being specified, there are $(n-1)$ bus voltages starting values of whose magnitudes and angles are assumed. These values are then ~~upd~~ updated through an iterative process.

from eqn. (2)

$$I_i = \frac{(P_i - jQ_i)}{V_i^*} \longrightarrow (11)$$

from eqn. (4)

$$P_i - jQ_i = V_i^* \left[\sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k + Y_{ii} V_i \right]$$

$$V_i Y_{ii} = \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad i = 2, 3, \ldots n$$

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad i = 2, 3, \ldots n \longrightarrow (12)$$

During each iteration voltages at buses $i = 2, 3 \ldots n$ are sequentially updated through eqn. (12). Slack bus voltage being fixed & will not update (V_1)

Iteration are repeated till no bus voltage magnitude changes by more than a prescribed value during an iteration. ~~The compu process~~ Computation process is then said to converge to a solution.

If instead of updating voltage at every step of an iteration, updating is carried out at the end of a complete iteration, the process is known as the Gauss iterative method. It is much slower to converge and may sometimes fail to do so.

Algorithms for Load Flow Solution :-

All buses are PQ bus other than slack Bus

1. Load ~~plots~~ profile is known at each bus (P_{Di} & Q_{Di} known), allocate P_{Gi} & Q_{Gi} at all generating stations.

2. Assembly of bus admittance matrix Y_{BUS}

3. Iterative computation of bus voltages (V_i ; $i = 2, 3, .. n$) — A set of initial voltage values is assumed. Initially all voltages are set equal to $(1+j0)$ except the voltage of the slack bus which is fixed.

No. of equ. $(n-1)$ in complex no. to find. $(n-1)$ complex voltages $V_2, V_3 .. V_n$

$$A_i = \frac{P_i - jQ_i}{Y_{ii}} \qquad i = 2, 3, .. n$$