Newton-Raphson Method :

1) Faster than Gauss-Seidal method of load flow
2) Its a powerful method for solving non-linear algebraic eqn.

Consider a set of n non-linear algebric equations.

$f_i(X) = 0 \qquad i = 1, 2, \ldots n$ → no. of non-linear algebric eqn.

$$X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \qquad f_i(x_1, x_2, \ldots x_n) = 0$$

Assume Initial guess for unknowns

$$X^0 = \begin{Bmatrix} x_1^0 \\ x_2^0 \\ \vdots \\ x_n^0 \end{Bmatrix} \qquad \Delta X^0 = \begin{Bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{Bmatrix}$$

ΔX^0 → corresponding corrections → which are being added to initial guess

Algebric eqn. can be written as—

$$f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \ldots x_n^0 + \Delta x_n^0) = 0 \qquad i = 1, 2, \ldots n$$

So for all the eqn., using Taylor series expension we can write

$$f_i(x_1^0, x_2^0, \ldots x_n^0) + \left.\frac{\partial f_i}{\partial x_1}\right|_{X=X^0} \Delta x_1^0 + \left.\frac{\partial f_1}{\partial x_2}\right|_{X=X^0} \Delta x_2^0 + \ldots + \left.\frac{\partial f_i}{\partial x_n}\right|_{X=X^0} \Delta x_n^0 + \cancel{\begin{pmatrix}\text{Higher} \\ \text{order} \\ \text{terms}\end{pmatrix}}^{0} = 0$$

$$\begin{bmatrix} f_1(X^0) \\ f_2(X^0) \\ \vdots \\ f_n(X^0) \end{bmatrix} + \begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{X=X^0} \Delta x_1^0 & \left.\frac{\partial f}{\partial x_2}\right|_{X=X^0} \Delta x_2^0 & \cdots & \left.\frac{\partial f_1}{\partial x_n}\right|_{X=X^0} \Delta x_n^0 \\ \left.\frac{\partial f_2}{\partial x_1}\right|_{X=X^0} \Delta x_1^0 & \left.\frac{\partial f_2}{\partial x_2}\right|_{X=X^0} \Delta x_2^0 & \cdots & \left.\frac{\partial f_2}{\partial x_n}\right|_{X=X^0} \Delta x_n^0 \\ \vdots & & & \\ \left.\frac{\partial f_n}{\partial x_1}\right|_{X=X^0} \Delta x_1^0 & \left.\frac{\partial f_n}{\partial x_2}\right|_{X=X^0} \Delta x_2^0 & \cdots & \left.\frac{\partial f_n}{\partial x_n}\right|_{X=X^0} \Delta x_n^0 \end{bmatrix} = 0$$

$$\underbrace{\begin{bmatrix} f_1(X^0) \\ f_2(X^0) \\ \vdots \\ f_n(X^0) \end{bmatrix}}_{F^0} + \underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial x_1}\Big|_{X=X^0} & \frac{\partial f_1}{\partial x_2}\Big|_{X=X^0} & \cdots & \frac{\partial f_1}{\partial x_n}\Big|_{X=X^0} \\ \frac{\partial f_2}{\partial x_1}\Big|_{X=X^0} & \frac{\partial f_2}{\partial x_2}\Big|_{X=X^0} & \cdots & \frac{\partial f_2}{\partial x_n}\Big|_{X=X^0} \\ \vdots & & & \\ \frac{\partial f_n}{\partial x_1}\Big|_{X=X^0} & \frac{\partial f_n}{\partial x_2}\Big|_{X=X^0} & \cdots & \frac{\partial f_n}{\partial x_n}\Big|_{X=X^0} \end{bmatrix}}_{J^0} \underbrace{\begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \Delta x_3^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix}}_{\Delta X^0}$$

$$\boxed{F^0 + J^0 \Delta X^0 = 0}$$

$$\boxed{F^0 = -J^0 \Delta X^0}$$

Jacobian Matrix evaluated by taking Partial derivative w.r.t X (vector) & evaluated at X^0

$$\Delta X^0 = \frac{-F^0}{J^0} = -[J^0]^{-1} F^0$$

After I^st iteration $X^1 = X^0 + \Delta X^0$

After r^th iteration $\boxed{X^r = X^{r-1} + \Delta X^{r-1}}$

$$\boxed{\Delta X^{r-1} = -[J^{r-1}]^{-1} F^{r-1}}$$

Corresponding X^r

$$X^r = \begin{Bmatrix} x_1^r \\ x_2^r \\ \vdots \\ x_n^r \end{Bmatrix}$$

These are solutions.

After putting values —

$$\{ f_1(X^r), f_2(X^r), \cdots f_n(X^r) \}$$

tolerance $= \text{Max}\{ f_1(X^r), f_2(X^r) \cdots f_n(X^r) \}$

If tor $\leq \varepsilon$ (specified value)

$(\leq 10^{-3}, 10^{-2} \cdots$

N-R method for load flow :

Consider a power system having n-no. of Bus

1st - slack Bus

2-n - PQ Bus

$$S_i = P_i + jQ_i = V_i I_i^* = V_i \left(\sum_{k=1}^{n} \bar{Y}_{ik} \bar{V}_k \right)^*$$

$$S_i^* = P_i - jQ_i = V_i^* I = V_i^* \sum_{k=1}^{n} \bar{Y}_{ik} \bar{V}_k$$

$$\bar{V}_i = V_i \angle \delta_i$$

$$\bar{V}_k = V_k \angle \delta_k$$

$$S_i^* = P_i - jQ_i = \left[V_i \angle -\delta_i \sum_{k=1}^{n} (Y_{ik} \angle \theta_{ik})(V_k \angle \delta_k) \right]$$

$$= V_i \sum_{k=1}^{n} Y_{ik} V_k \angle (\theta_{ik} + \delta_k - \delta_i)$$

$$P_i = V_i \sum_{k=1}^{n} Y_{ik} V_k \cos(\theta_{ik} + \delta_k - \delta_i) \longrightarrow (1)$$

$$Q_i = -V_i \sum_{k=1}^{n} Y_{ik} V_k \sin(\theta_{ik} + \delta_k - \delta_i) \longrightarrow (2)$$

$$P_i = f(\delta, V)$$

$$Q_i = f(\delta, V)$$

Let us introduce two functions -

1) corresponding to P

2) Corresponding to Q.

$$f_i^\circ = \Delta P_i^\circ = P_i^s - P_i\left(\delta_2, \delta_3 \dots \delta_n, V_2, V_3 \dots V_n\right)$$

$P_i^s \downarrow$ specified value; $P_i(\dots) \hookrightarrow$ calculated value

$$g_i^\circ = \Delta Q_i^\circ = Q_i^s - Q_i^\circ\left(\delta_2, \delta_3 \dots \delta_n, V_2, V_3 \dots V_n\right)$$

$$F^{r-1} = -\left[J^{r-1}\right]\left[\Delta x^{r-1}\right]$$

For i^{th} PQ Bus

$$\begin{bmatrix} \Delta P_i^\circ \\ \Delta Q_i^\circ \end{bmatrix}_{2\times 1} = -\begin{bmatrix} \frac{\partial \Delta P_i^\circ}{\partial \delta_1} & \frac{\partial \Delta P_i^\circ}{\partial \delta_2} & \cdots & \frac{\partial \Delta P_i^\circ}{\partial \delta_n} & \frac{\partial \Delta P_i^\circ}{\partial V_1}, & \frac{\partial \Delta P_i}{\partial V_2} & \cdots & \frac{\partial \Delta P_i^\circ}{\partial V_n} \\ \frac{\partial \Delta Q_i}{\partial \delta_1} & \frac{\partial \Delta Q_i}{\partial \delta_2} & \cdots & \frac{\partial \Delta Q_i}{\partial \delta_n} & \frac{\partial \Delta Q_i}{\partial V_1} & \frac{\partial \Delta Q_i}{\partial V_2} & \cdots & \frac{\partial \Delta Q_i^\circ}{\partial V_n} \end{bmatrix}_{2\times 2(n-1)} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \vdots \\ \Delta\delta_n \\ \Delta V_2 \\ \Delta V_3 \\ \vdots \\ \Delta V_n \end{bmatrix}_{2(n-1)\times 1}$$

For All PQ Buses

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix}$$

Where

$$\Delta P = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \end{bmatrix}_{(n-1)\times 1} \qquad \Delta Q = \begin{bmatrix} \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix}_{(n-1)\times 1}$$

$$\Delta\delta = \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \vdots \\ \Delta\delta_3 \end{bmatrix}_{(n-1)\times 1} \qquad \Delta V = \begin{bmatrix} \Delta V_2 \\ \Delta V_3 \\ \vdots \\ \Delta V_n \end{bmatrix}_{(n-1)\times 1}$$

$$J_{11} = \left[\frac{\partial P}{\partial \delta}\right]$$

$$\frac{\partial \Delta P_i}{\partial \delta_j} = \frac{\partial}{\partial \delta_j}\left(\overset{\text{differenti}}{P_i^{s}} - P_i(\delta, V)\right)$$

$$\boxed{\frac{\partial \Delta P_i}{\partial \delta_j} = -\frac{\partial P_i(\delta,V)}{\partial \delta_j}}$$

$$\frac{\partial \Delta Q_i}{\partial \delta_j} = \frac{\partial}{\partial \delta_j}\left(Q_i^{s} - Q_i(\delta, V)\right)$$

$$\boxed{\frac{\partial \Delta Q_i}{\partial \delta_j} = -\frac{\partial Q_i(\delta,V)}{\partial \delta_j}}$$

similarly

$$\boxed{\frac{\partial \Delta Q_i}{\partial V_j} = -\frac{\partial Q_i}{\partial V_j}}$$

$$\boxed{\frac{\partial \Delta P_i}{\partial V_j} = \frac{-\partial P_i}{\partial V_j}}$$

$$J_{11} = \left[\frac{\partial P}{\partial \delta}\right] = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \cdots & \frac{\partial P_2}{\partial \delta_n} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \cdots & \frac{\partial P_3}{\partial \delta_n} \\ \vdots & & & \\ \frac{\partial P_n}{\partial \delta_2} & \frac{\partial P_n}{\partial \delta_3} & \cdots & \frac{\partial P_n}{\partial \delta_n} \end{bmatrix}_{(n-1)\times(n-1)}$$

$$J_{12} = \left[\frac{\partial P}{\partial V}\right] = \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_3} & \cdots & \frac{\partial P_2}{\partial V_n} \\ \frac{\partial P_3}{\partial V_2} & \frac{\partial P_3}{\partial V_3} & \cdots & \frac{\partial P_3}{\partial V_n} \\ \vdots & & & \\ \frac{\partial P_n}{\partial V_2} & \frac{\partial P_n}{\partial V_3} & \cdots & \frac{\partial P_n}{\partial V_n} \end{bmatrix}$$

$$J_{21} = \left[\frac{\partial Q}{\partial \delta}\right] = \begin{bmatrix} \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \cdots & \frac{\partial Q_2}{\partial \delta_n} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \cdots & \frac{\partial Q_3}{\partial \delta_n} \\ \vdots & & & \\ \frac{\partial Q_n}{\partial \delta_2} & \frac{\partial Q_n}{\partial \delta_3} & \cdots & \frac{\partial Q_n}{\partial \delta_n} \end{bmatrix}_{(n-1)\times(n-1)}$$

$$J_{22} = \left[\frac{\partial Q}{\partial V}\right] = \begin{bmatrix} \frac{\partial Q_2}{\partial V_2} & \frac{\partial Q_2}{\partial V_3} & \cdots & \frac{\partial Q_2}{\partial V_n} \\ \frac{\partial Q_3}{\partial V_2} & \frac{\partial Q_3}{\partial V_3} & \cdots & \frac{\partial Q_3}{\partial V_n} \\ \vdots & & & \\ \frac{\partial Q_n}{\partial V_2} & \frac{\partial Q_n}{\partial V_3} & \cdots & \frac{\partial Q_n}{\partial V_n} \end{bmatrix}_{(n-1)\times(n-1)}$$

$$P_i = V_i \sum_{k=1}^{n} Y_{ik} V_k \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$\left.\frac{\partial P_i}{\partial \delta_j}\right|_{i\neq j} = \frac{\partial}{\partial \delta_j}\left[V_i \sum_{k=1}^{n} Y_{ik} V_k \cos(\theta_{ik} + \delta_k - \delta_i)\right]$$

$$= \frac{\partial}{\partial \delta_j}\Big[V_i \Big\{ Y_{i1} V_1 \cos(\theta_{i1} + \delta_1 - \delta_i) + Y_{i2} V_2 \cos(\theta_{i2} + \delta_2 - \delta_i) + \cdots$$

$$\cdots + Y_{ij} V_j \cos(\theta_{ij} + \delta_j - \delta_i) + \cdots + Y_{in} V_n \cos(\theta_{in} + \delta_n - \delta_i)\Big\}\Big]$$

$$= V_i \left\{ 0 + 0 \cdots + (-Y_{ij} V_j \sin(\theta_{ij} + \delta_j - \delta_i)) + 0 \cdots + 0 \right\}$$

$$\boxed{\left.\frac{\partial P_i}{\partial \delta_j}\right|_{i\neq j} = -V_i Y_{ij} V_j \sin(\theta_{ij} + \delta_j - \delta_i)}$$

$$\left.\frac{\partial P_i}{\partial \delta_i}\right|_{i=j} = \frac{\partial P_i}{\partial \delta_i} = \frac{\partial}{\partial \delta_i}\left[V_i \sum_{k=1}^{n} Y_{ik} V_k \cos(\theta_{ik}+\delta_k-\delta_i)\right]$$

$$= \frac{\partial}{\partial \delta_i}\Big[V_i\Big\{Y_{i1}V_1\cos(\theta_{i1}+\delta_1-\delta_i)+Y_{i2}V_2\cos(\theta_{i2}+\delta_2-\delta_i)+\cdots$$

$$\cdots+Y_{ii}V_i\cos(\theta_{ii}+\delta_i-\delta_i)+\cdots+Y_{in}V_n\cos(\theta_{in}+\delta_n-\delta_i)\Big\}\Big]$$

$$= V_i\Big\{Y_{i1}V_1\sin(\theta_{i1}+\delta_1-\delta_i)+Y_{i2}V_2\sin(\theta_{i2}+\delta_2-\delta_i)$$

$$+\cdots+0+Y_{in}V_n\sin(\theta_{in}+\delta_n-\delta_i)\Big\}$$

$$= V_i\sum_{\substack{k=1\\k\neq i}}^{n} Y_{ik}V_k\sin(\theta_{ik}+\delta_k-\delta_i)$$

$$= V_i\sum_{k=1}^{n} Y_{ik}V_k\sin(\theta_{ik}+\delta_k-\delta_i) - Y_{ii}V_i^2\sin(\theta_{ii}+\delta_i-\delta_i)$$

$$= -Q_i - Y_{ii}V_i^2\sin(\theta_{ii})$$

$$\boxed{\frac{\partial P_i}{\partial \delta_i} = -Q_i - V_i^2 B_{ii}}$$

$$\bar{Y}_{ii} = Y_{ii}\angle\theta_{ii}$$
$$= Y_{ii}\cos\theta_{ii} + jY_{ii}\sin\theta_{ii}$$
$$= G_{ii} + jB_{ii}$$

J_{12}

$$\boxed{\left.\frac{\partial P_i}{\partial V_j}\right|_{i\neq j} = V_i Y_{ij}\cos(\theta_{ij}+\delta_j-\delta_i)}$$

$$\left.\frac{\partial P_i}{\partial V_j}\right|_{i=j} = \sum_{k=1}^{n} Y_{ik}V_k\cos(\theta_{ik}+\delta_i-\delta_i) + Y_{ii}V_i\cos\theta_{ii}$$

$$\boxed{\frac{\partial P_i}{\partial V_i} = \frac{P_i}{V_i} + V_i G_{ii}}$$

$$\boxed{\left.\frac{\partial Q_i}{\partial \delta_j}\right|_{i\neq j} = -V_i Y_{ij} V_j \cos(\theta_{ij} + \delta_j - \delta_i)}$$

$$\left.\frac{\partial Q_i}{\partial \delta_i}\right|_{i=j} = V_i \sum_{\substack{k=1 \\ i\neq j}}^{n} Y_{ik} V_k \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$= V_i \sum_{k=1}^{n} Y_{ik} V_k \cos(\theta_{ik} + \delta_k - \delta_i) - V_i^2 Y_{ii} \cos(\theta_{ii})$$

$$\boxed{\frac{\partial Q_i}{\partial \delta_i} = P_i - V_i^2 G_{ii}}$$

$$\boxed{\left.\frac{\partial Q_i}{\partial V_j}\right|_{i\neq j} = -V_i Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i)}$$

$$\left.\frac{\partial Q_i}{\partial V_j}\right|_{i=j} = -\sum_{k=1}^{n} Y_{ik} V_k \sin(\theta_{ik} + \delta_i - \delta_i) - V_i Y_{ii} \sin\theta_{ii}$$

$$\boxed{\frac{\partial Q_i}{\partial V_i} = \frac{Q_i}{V_i} - V_i B_{ii}}$$

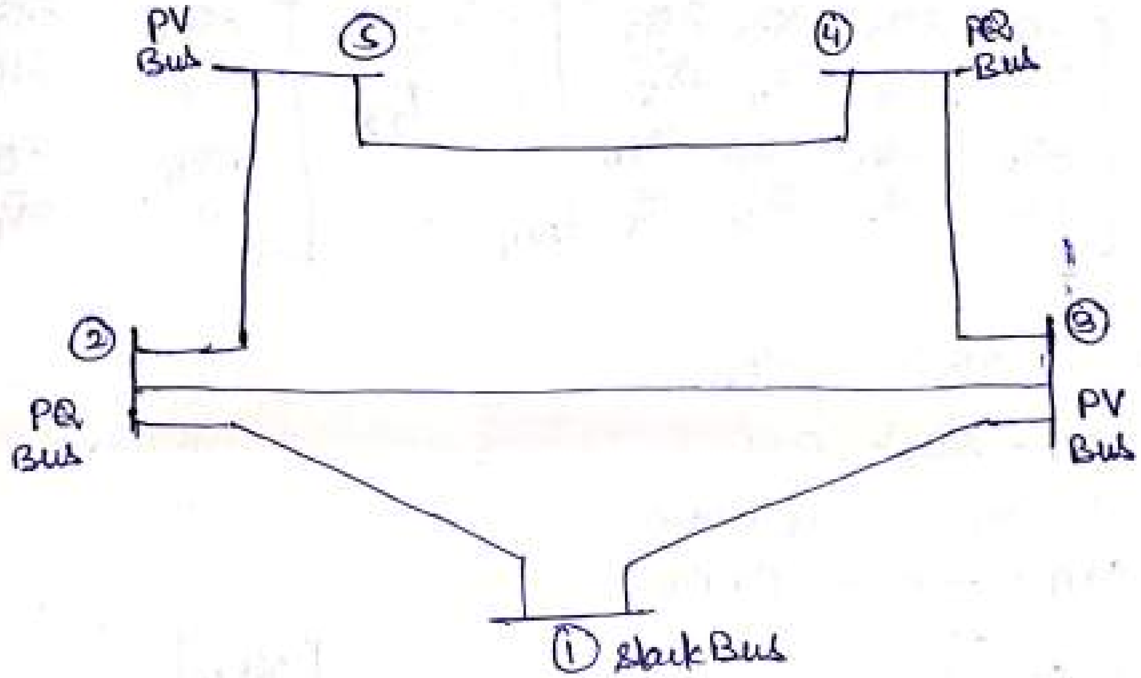
When PV Buses are available

$$\underbrace{\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}}_{\text{LHS}} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \underbrace{\begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix}}_{\text{RHS}}$$

for PV Buses

i) P is specified
Q is not specified
$\Rightarrow$ ΔQ will not appear on LHS

ii) V is specified
ΔV will not appear on RHS



Bus No.	Known (specified)	Unknown (Non-specified)
2	$P_2 Q_2$	$\delta_2 V_2$
3	$P_3 V_3$	$\delta_3 Q_3$
4	$P_4 Q_4$	$\delta_4 V_4$
5	$P_5 V_5$	$\delta_5 Q_5$

$$\Delta P = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \end{bmatrix} \quad \Delta Q = \begin{bmatrix} \Delta Q_2 \\ \Delta Q_4 \end{bmatrix} \quad \Delta \delta = \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta \delta_4 \\ \Delta \delta_5 \end{bmatrix} \quad \Delta V = \begin{bmatrix} \Delta V_2 \\ \Delta V_4 \end{bmatrix}$$

$$J_{11} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial \delta_4} & \frac{\partial P_2}{\partial \delta_5} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial \delta_4} & \frac{\partial P_3}{\partial \delta_5} \\ \frac{\partial P_4}{\partial \delta_2} & \frac{\partial P_4}{\partial \delta_3} & \frac{\partial P_4}{\partial \delta_4} & \frac{\partial P_4}{\partial \delta_5} \\ \frac{\partial P_5}{\partial \delta_2} & \frac{\partial P_5}{\partial \delta_3} & \frac{\partial P_5}{\partial \delta_4} & \frac{\partial P_5}{\partial \delta_5} \end{bmatrix}_{4\times 4} \qquad J_{12} = \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_4} \\ \frac{\partial P_3}{\partial V_2} & \frac{\partial P_3}{\partial V_4} \\ \frac{\partial P_4}{\partial V_2} & \frac{\partial P_4}{\partial V_4} \\ \frac{\partial P_5}{\partial V_2} & \frac{\partial P_5}{\partial V_4} \end{bmatrix}_{4\times 2}$$

$$J_{21} = \begin{bmatrix} \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial \delta_4} & \frac{\partial Q_2}{\partial \delta_5} \\ \frac{\partial Q_4}{\partial \delta_2} & \frac{\partial Q_4}{\partial \delta_3} & \frac{\partial Q_4}{\partial \delta_4} & \frac{\partial Q_4}{\partial \delta_5} \end{bmatrix}_{2\times 4} \qquad J_{22} = \begin{bmatrix} \frac{\partial Q_2}{\partial V_2} & \frac{\partial Q_2}{\partial V_4} \\ \frac{\partial Q_4}{\partial V_2} & \frac{\partial Q_4}{\partial V_4} \end{bmatrix}_{2\times 2}$$

Generalised n-Bus System

1 - Slack Bus

$2 - m \rightarrow$ PQ Buses

$(m+1) - n \rightarrow$ PV Buses

$$\Delta P = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_m \\ \hline \Delta P_{m+1} \\ \Delta P_{m+2} \\ \vdots \\ \Delta P_n \end{bmatrix}_{(n-1)\times 1} \begin{matrix} \left.\vphantom{\begin{matrix} a \\ a \\ a \\ a \end{matrix}}\right\} \text{for PQ Buses} \\ \left.\vphantom{\begin{matrix} a \\ a \\ a \\ a \end{matrix}}\right\} \text{for PV Buses} \end{matrix} \qquad \Delta Q = \begin{bmatrix} \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_m \end{bmatrix}_{(m-1)\times 1} \text{for PQ Buses}$$

$$\Delta V = \begin{bmatrix} \Delta V_2 \\ \Delta V_3 \\ \vdots \\ \Delta V_m \end{bmatrix}_{(m-1)\times 1} \qquad \Delta \delta = \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \vdots \\ \Delta P_m \\ \hline \Delta P_{m+1} \\ \Delta P_{m+2} \\ \vdots \\ \Delta P_n \end{bmatrix}_{(n-1)\times 1} \begin{matrix} \left.\vphantom{\begin{matrix} a \\ a \\ a \\ a \end{matrix}}\right\} \text{PQ Buses} \\ \left.\vphantom{\begin{matrix} a \\ a \\ a \\ a \end{matrix}}\right\} \text{PV Buses} \end{matrix}$$