$$J_{11} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_m} & \frac{\partial P_2}{\partial \delta_{m+1}} & \cdots & \frac{\partial P_2}{\partial \delta_n} \\ \vdots & & & & & \\ \frac{\partial P_m}{\partial \delta_2} & \cdots & \frac{\partial P_m}{\partial \delta_m} & \frac{\partial P_m}{\partial \delta_{m+1}} & \cdots & \frac{\partial P_m}{\partial \delta_n} \\ \vdots & & & & & \\ \frac{\partial P_n}{\partial \delta_2} & \cdots & \frac{\partial P_n}{\partial \delta_m} & \frac{\partial P_n}{\partial \delta_{m+1}} & \cdots & \frac{\partial P_n}{\partial \delta_n} \end{bmatrix}_{(n-1)\times(n-1)}$$

$$J_{12} = \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \cdots & \frac{\partial P_2}{\partial V_m} \\ \vdots & & \vdots \\ \frac{\partial P_m}{\partial V_2} & \cdots & \frac{\partial P_m}{\partial V_m} \\ \vdots & & \vdots \\ \frac{\partial P_m}{\partial V_2} & \cdots & \frac{\partial P_n}{\partial V_m} \end{bmatrix}_{(n-1)\times(m-1)} \qquad J_{21} = \begin{bmatrix} \frac{\partial Q_2}{\partial \delta_2} & \cdots & \frac{\partial Q_2}{\partial \delta_n} \\ \vdots & & \vdots \\ \frac{\partial Q_m}{\partial \delta_2} & \cdots & \frac{\partial Q_m}{\partial \delta_n} \end{bmatrix}_{(m-1)\times(n-1)}$$

$$J_{22} = \begin{bmatrix} \frac{\partial Q_2}{\partial V_2} & \cdots & \frac{\partial Q_2}{\partial V_m} \\ \vdots & & \vdots \\ \frac{\partial Q_m}{\partial V_2} & \cdots & \frac{\partial Q_m}{\partial V_m} \end{bmatrix}_{(m-1)\times(m-1)}$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}^{(n-1)\times 1}_{(m-1)\times 1} = \begin{bmatrix} \overset{(n-1)\times(n-1)}{J_{11}} & \overset{(n-1)(m-1)}{J_{12}} \\ \underset{(m-1)\times(n-1)}{J_{21}} & \underset{(m-1)\times(m-1)}{J_{22}} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \begin{matrix} \rightarrow (n-1)\times 1 \\ \rightarrow (m-1)\times 1 \end{matrix}$$

dimension of Jacobian $= (n+m-2)\times(n+m-2)$

For PV Bus.

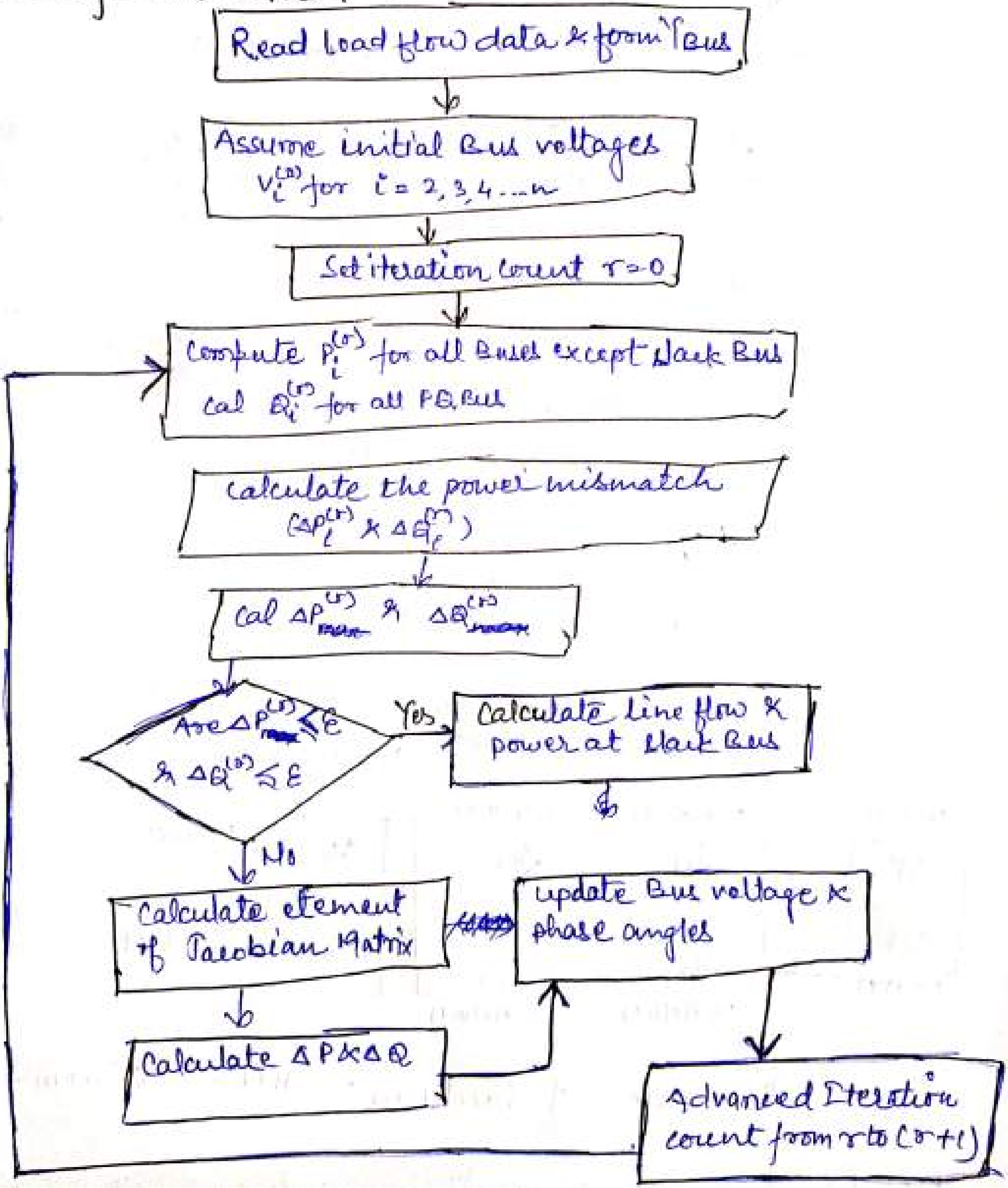
$$Q_i^r = -V_i \sum_{k=1}^{n} Y_{ik} V_k^r \sin(\theta_{ik} + \delta_k^r - \delta_i^r)$$

If $Q_{i\,min} \le Q_i^r \le Q_{i\,max} \longrightarrow$ PV Bus

else $\longrightarrow$ PQ Bus

$$V_i^r = V_i^{r-1} + \Delta V_i^{r-1}$$

Flow-chart for NR Method



Scanned by CamScanner

Qus. Consider the three Bus system as shown in fig. Each of the three Lines has a series impedance of 0.02+j0.08 pu and total shunt admittance of 0.02 pu. The specified quantities at the buses are tabulated below:

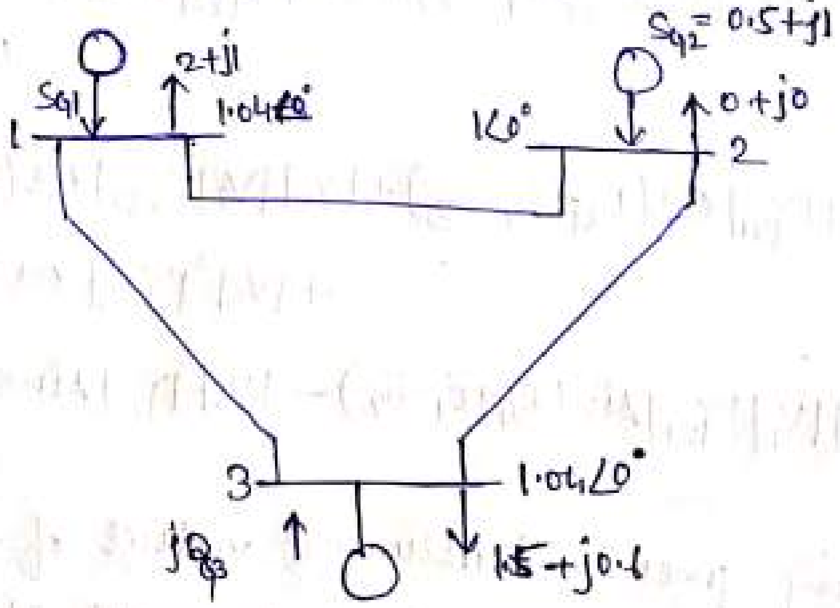
Bus	Real load Demand P_D	Reactive Load Demand Q_D	Real Power Generation P_G	Reactive Power Gen. Q_G	Voltage specification
1	2.0	1.0	Unspecified	Unspecified	$V_1 = 1.04+j0$ slack Bus
2	0.0	0.0	0.5	1.0	Unspecified (PQ Bus)
3	1.5	0.6	0.0	$Q_{G3} = ?$	$\lvert V_3 \rvert = 1.04$ PV Bus

Controllable reactive Power source is available at bus 3 with the constraint

$$0 \leq Q_{G3} \leq 1.5 \text{ pu.}$$

Find the load flow solution using the NR method. Use a tolerance of 0.01 for power mismatch.

Sol.



Using the nominal-π model for transmission lines, Y_{Bus} for the given system is obtained as follows:

For each line

$$y_{series} = \frac{1}{0.02+j0.08} = 2.941 - j11.764$$

$$= 12.13 \angle -75.96^\circ$$

Each off-diagonal term $= -2.941 + j11.764$

Each self term $= 2[(2.941 - j11.764) + j0.01]$

$$= 5.882 - j23.528$$

$$= 24.23 \angle -75.95^\circ$$

$$Y_{Bus} = \begin{bmatrix} 24.23\angle -75.95^\circ & 12.13\angle 104.04^\circ & 12.13\angle 104.04^\circ \\ 12.13\angle 104.04^\circ & 24.23\angle -75.95^\circ & 12.13\angle 104.04^\circ \\ 12.13\angle 104.04^\circ & 12.13\angle 104.04^\circ & 24.23\angle -75.95^\circ \end{bmatrix}$$

To start iteration choose $V_2^0 = 1+j0$ & $\delta_3^0 = 0$

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21}+\delta_1-\delta_2) + |V_2|^2|Y_{22}|\cos\theta_{22} + |V_2||V_3||Y_{23}|\cos(\theta_{23}+\delta_3-\delta_2)$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31}+\delta_1-\delta_3) + |V_3||V_2||Y_{32}|\cos(\theta_{32}+\delta_2-\delta_3) + |V_3|^2|Y_{33}|\cos\theta_{33}$$

$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21}+\delta_1-\delta_2) - |V_2|^2|Y_{22}|\sin\theta_{22} - |V_2||V_3||Y_{23}|\sin(\theta_{23}+\delta_2-\delta_3)$$

Substituting given and assumed values of different quantities, we get the values of powers as

$$P_2^0 = 1 \times 1.04 \times 12.13 \cos(104.04 + 0 - 0) + (1)^2 \times 24.23 \cos(-75.95)$$
$$+ 1 \times 1.04 \times 12.13 \cos(104.04)$$

$$= -3.06 + 5.88 - 3.06$$

$$= -0.24 \text{ pu.}$$

$$P_3^0 = \{1.04 \times 1.04 \times 12.13 \cos(104.04 + 0 - 0\} + \{1.04 \times 1 \times 12.13 \cos(104.04)\}$$
$$+ \{(1.04)^2 \times 24.23 \cos(-75.95)\}$$

$$= -3.18 - 3.06 + 6.36$$

$$= 0.12 \text{ pu.}$$

similarly.

$$Q_2^0 = -0.96 \text{ pu}$$

Power Residuals

$$\Delta P_2^0 = P_2^s - P_2^0$$
$$= 0.5 - (-0.24) = 0.73$$

$$\Delta P_3^0 = -1.5 - 0.12 = -1.62$$

$$\Delta Q_2^1 = 1 - (-.96) = 1.96$$

Changes in variables at the end of the first iteration are obtained as —

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}$$

Jacobian elements can be evaluated by differentiating the expressions given above for P_2, P_3, Q_2 w.r.t. δ_2, δ_3 & $|V_2|$

$$\begin{bmatrix} \Delta\delta_2^1 \\ \Delta\delta_3^1 \\ \Delta|V_2|^1 \end{bmatrix} = \begin{bmatrix} 24.47 & -12.23 & 5.64 \\ -12.23 & 24.95 & -3.05 \\ -6.11 & 3.05 & 22.54 \end{bmatrix}^{-1} \begin{bmatrix} 0.73 \\ -1.62 \\ 1.96 \end{bmatrix} = \begin{bmatrix} -0.023 \\ -0.0654 \\ -0.089 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^1 \\ \delta_3^1 \\ |V_2^1| \end{bmatrix} = \begin{bmatrix} \delta_2^0 \\ \delta_3^0 \\ |V_2|^0 \end{bmatrix} + \begin{bmatrix} \Delta\delta_2^1 \\ \Delta\delta_3^1 \\ \Delta|V_2|^1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.023 \\ -0.0654 \\ 0.089 \end{bmatrix} = \begin{bmatrix} -0.023 \\ -0.0654 \\ 1.089 \end{bmatrix}$$

$Q_3^1 = 0.4677^{\,0.24}$ (By formula)

$Q_{G3}^1 = Q_3^1 + Q_{D3} = 0.4677 + 0.6 = 1.0677$

which is within limit