#### BEAM

## Beam:

A beam may be defined as a structural member, supported at few points with one dimension (length) considerably larger than its other two dimensions (width and depth).

A beam is generally used to support the vertical loads.

Due to the applied loads, reactions develop at the supports. The system of forces consisting of applied loads and reactions keep the beam in equilibrium.

# **Types of Beam:**

Beams are mainly of two types.

- (i) Statically determinate beams
- (ii) Statically indeterminate beams

#### Statically determinate beams:

If the support reactions of a beam can be determined by the use of equations of static equilibrium, then such beams are called statically determinate beams. These beams are of three types.

- i. Simply supported beam
- ii. Over hanging beam
- iii. Cantilever beam

## Simply supported beam:

A beam whose ends are simply supported, (either by hinge or roller support) is known as simply supported beam.



Fig.-1

#### **Over-hanging beam:**

The beam whose one or both the ends are projected beyond the supports, is called overhanging beam.



#### **Cantilever beam:**

If a beam is fixed at one end and is free at the other end, it is called cantilever beam. In a cantilever beam at fixed end, there are three support reactions (horizontal reaction ( $R_H$ ), vertical reaction ( $R_V$ ), and moment (M)).



#### **Statically Indeterminate beams:**

If the support reactions of a beam cannot be determined by the use of equations of static equilibrium, then such beams are called statically indeterminate beams. These beams are of three types.

#### i. Fixed beam:

A beam whose both the ends are fixed, is called fixed beam.

#### ii. Continuous beam:

A beam which has more than two supports, is called continuous beam.

### iii. Propped cantilever beam:

A cantilever beam whose one end is fixed and the other end is provided with a simple support, in order to resist deflection is called propped cantilever beam.

# **Types of loading:**

Mainly three types of loads act on any beam;

- i. Concentrated load
- ii. Uniformly distributed load
- iii. Uniformly varying load



Fig 4: Various loads acting on a beam

## **Concentrated or Point load:**

If a load is acting at a point on the beam, then it is called concentrated or point load.

# Uniformly distributed load (UDL):

For finding the reactions, this load may be assumed as a point load acting at the center of gravity of the loading (mid-point).

Total load of UDL (W) = Load Intensity X Span of the UDL = (w)(x)

# Uniformly varying load (UVL):

If a load is varying linearly from Point A to point B and its intensity is zero at A and w N/m at B. Then the total load is represented by area of triangle and it acts at the centroid of the triangle (At a distance of 2x/3 from point A). Total load (W) = 1/2 (w). (AB)

#### **Shear Force:**

Force that tries to shear off the beam at any section is called shear force. It is equal to the algebraic sum of all the vertical forces either to the right or left of any section and is known as shear force at that section.

### **Bending Moment:**

Moment that tries to bend the beam at any section is known as bending moment at that section.

It is equal to the algebraic sum of the moments of all the vertical forces acting to the right or left of any section, about that section.

### Shear force diagram (SFD):

A Shear Force diagram is one, which shows the variation of the shear force along the length of the beam.

## Bending moment diagram (BMD):

A Bending Moment diagram is one, which shows the variation of the bending moment along the length of the beam.

## Steps for drawing shear force and bending moment diagrams:

In these diagrams, the SF or BM is represented by ordinates whereas the length of the beam represents abscissa. The following are the important points for drawing shear force and bending moment diagrams:

- i. Consider the left or right side of the section.
- ii. If the left portion of the section is chosen, a force on the left portion acting upwards is positive while force acting downwards is negative.
- iii. The +ve value of shear force and bending moment are plotted above the base line, and -ve value below the base line.

- iv. The S.F. diagram will increase or decrease suddenly i.e. by a vertical straight line at a section where there is a vertical point load. Same thing happens in the case of BMD if direct moment is acting at some point on the beam.
- v. In drawing S.F. and B.M. diagrams no scale is chosen, but diagrams should be in proportionate sketches.
- vi. The Shear force between any two vertical loads will remain constant, hence the S.F.D. will be horizontal line. The B.M.D. will be linear between these two loads.
- vii. For UDL, S.F.D. will be inclined line (linear) and the B.M.D. will be parabolic in nature.
- viii. For UVL, S.F.D. will be parabolic in nature while the B.M.D. will be a cubic in nature.
  - ix. The bending moment at the two supports of a simply supported beam and at the free end of a cantilever beam will be zero.
  - x. The B.M. is maximum at the section where S.F. changes its sign.

#### **Relation between load intensity (w), shear force(F) and bending moment(M):**

Consider a beam subjected to any type of transverse load of the general form as shown in fig 5. Take an element of length dx at a distance x from left end and draw its free body diagram as shown in fig 5. Since the element is of extremely small length, the loading over the beam can be considered to be uniform with intensity of w KN/m.



Fig. 5

The element is subjected to a shear force F on its left hand side while a shear Force (F+dF) on the right side. The bending moment M acts on the left side of the element and it changes to (M + dM) on the right side.

Apply the condition  $\sum V = 0$  for equilibrium, we obtain

$$\mathbf{F} - \mathbf{w}\mathbf{d}\mathbf{x} - (\mathbf{F} + \mathbf{d}\mathbf{F}) = \mathbf{0}$$

or

$$\mathbf{w} = \mathbf{dF}/\mathbf{dx}$$
(1)

It means that the intensity of loading is equal to rate of change of shear force with respect to x.

Taking moment about point C on the right side,  $\sum M_C = 0$ ;

M - (M + dM) + F.dx - (w.dx).dx/2 = 0

The UDL is considered to be acting at its C.G.

 $dM - Fdx + [W(dx)^2]/2 = 0$ 

The last term consists of the product of two differentials and can be neglected

$$d\mathbf{M} = \mathbf{F}d\mathbf{x}$$
, or  
 $\mathbf{F} = \mathbf{d}\mathbf{M}/\mathbf{d}\mathbf{x}$  ------ (2)

Thus the shear force is equal to the rate of change of bending moment with respect to x.

and

$$\mathbf{w} = \mathbf{dF}/\mathbf{dx} = \mathbf{d}^2 \mathbf{M}/\mathbf{dx}^2 \quad \dots \quad (3)$$

#### Point of contraflexure or point of inflexion:

The points in a beam (except ends) at which B.M. diagram changes its sign, are called points of contraflexure or inflexion.

Value of B.M. is zero, at the points of contraflexure.

The point of contraflexure generally occurs in overhanging beams.

**Note:** The point at which we get zero shear force, we get the maximum bending moment at that section.